Economics 302
Spring 2010
University of Wisconsin-Madison

Menzie D. Chinn
Social Sciences 7418

Algebra for the Growth Accounting Formula

Suppose that each time period *A*, *N*, *K* grow by $\Delta A/A, \Delta N/N, \Delta K, K$. Further assume:
$Y = F(N,K,A) = Af(N,K)$

Take the total differential with respect to time *t*, set Δ*t* = 1, and manipulating yields:

$$\Delta Y/Y = \Delta A/A + \frac{\Delta f(N,K)}{f(N,K)} \qquad (5.6)$$

The last term in (5.6) is a differential with respect to the two arguments of the function, *N* and *K*. The differential of *f*(.) with respect to *N* is the marginal productivity of labor; the differential of *f*(.) with respect to *K* is the marginal productivity of capital. Let these be denoted by M_N and M_K respectively, and substitute these expressions into (5.6):

$$\frac{\Delta f(N,K)}{f(N,K)} = M_N \frac{\Delta N}{Y} + M_K \frac{\Delta K}{Y} \qquad (5.7)$$

(When there are constant returns to scale due to the assumption of a Cobb-Douglas production function.) Substituting (5.7) into (5.6) yields:

$$\Delta Y/Y = \Delta A/A + M_N \frac{\Delta N}{Y} + M_K \frac{\Delta K}{Y} \qquad (5.8)$$

Suppose (as we assumed in Chapter 11 under perfect competition), that firms hire factors of production up to the point where the marginal product of a factor equals the real factor price (e.g., capital is hired up to the point where rental cost of capital equals the marginal product of capital), *viz*.

$$M_N = \frac{W}{P}, M_K = \frac{R^K}{P} \qquad (5.9)$$

Substituting into (5.8) yields:

$$\Delta Y/Y = \Delta A/A + \left(\frac{W}{P}\right)\frac{\Delta N}{Y} + \left(\frac{R^K}{P}\right)\frac{\Delta K}{Y} \qquad (5.10)$$

Which can be rewritten as:

$$\Delta Y/Y = \Delta A/A + \left(\frac{WN}{PY}\right)\frac{\Delta N}{N} + \left(\frac{R^K K}{PY}\right)\frac{\Delta K}{K} \qquad (5.11)$$

Note that (*WN*/*PY*) is the labor share of income, and ($R^K K/PY$) is the capital share of output. These are measurable; in the United States, the former is 0.7, and the latter is 0.3, *on average*.

Substituting these values into (5.11) leads to equation (5.1) in the text, (5.12) in the appendix.

$$\Delta Y/Y = \Delta A/A + 0.7\left[\frac{\Delta N}{N}\right] + 0.3\left[\frac{\Delta K}{K}\right] \qquad (5.12)$$

One can re-express (5.12) in terms of output per unit labor (Y/N) by subtracting the growth rate of labor from both sides:

$$(\Delta Y/Y) - (\Delta N/N) = \Delta A/A + 0.7\times\left[\frac{\Delta N}{N}\right] + 0.3\times\left[\frac{\Delta K}{K}\right] - (\Delta N/N)$$

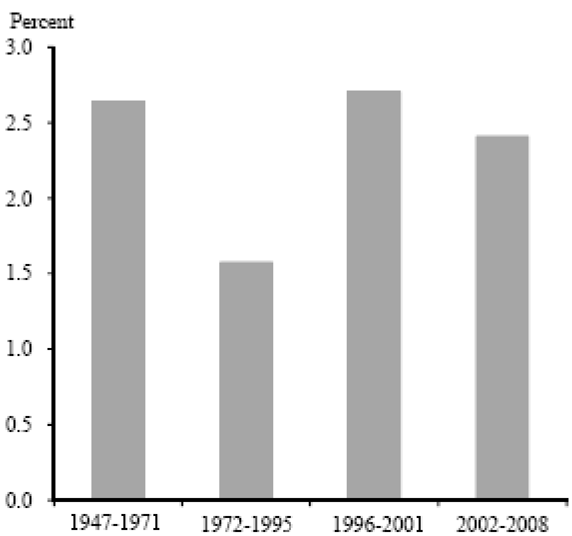
Which leads to:

$$(\Delta Y/Y) - (\Delta N/N) = \Delta A/A + 0.3\times\left[\frac{\Delta K}{K} - \frac{\Delta N}{N}\right]$$

Where the left hand side is the growth rate of labor productivity and the term in the [square brackets] on the right hand side is the growth rate of capital per unit labor.

This expression indicates that output per hour can rise because of improving technology (the A term) or because of capital deepening.

Figure 1: Average long-term labor productivity growth
Nonfarm business sector



Source: Bureau of Labor Statistics.

Figure 2: Labor productivity
Nonfarm business sector, year-over-year percent change

Source: Bureau of Labor Statistics.

Source: Mark Doms, 2009, The Outlook for Productivity Growth: Symposium Summary" Federal Reserve Bank of S.F. *Economic Letters*, March 20.