Economics 302 Menzie D. Chinn
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University of Wisconsin-Madison

The IS-LM Model

This set of notes outlines the IS-LM model of national income and interest rate determination. This involves extending the real side of the economy (described in the previous handout) and introducing a financial side (that involves money and bond markets). The real side of the economy is extended by introducing interest sensitive components of aggregate demand, namely investment and net exports. First the real side and the financial side are described. These are then put together to determine overall economic equilibrium. The impact of fiscal and monetary policy is then discussed, and policy multipliers derived.

1. The Real Side of the Economy

Eq.No.	Equation	Description
(1)	$Y = AD$	Output equals aggregate demand, an equilibrium condition
(2)	$AD = C + I + G + X$	Definition of aggregate demand
(3)	$C = a_o + bY_d$	Consumption function, b is the mpc
(4)	$Y_d \equiv Y - T$	Definition of disposable income
(5)	$T = TA_0 + tY$	Tax function; TA_0 is lump sum taxes, t is marginal tax rate.
(6)	$I = e_0 - dR$	Investment function ***(revised)***
(7)	$G = GO_0$	Government spending on goods and services, exogenous
(8)	$X = g_0 - mY - \tilde{n}R$	Net Exports ***(revised)***

The new parameters are $-d = \frac{\partial I}{\partial R}$, "the interest sensitivity of investment", and $-\tilde{n} = \frac{\partial X}{\partial R}$, "the interest sensitivity of net exports". The motivation for the interest sensitivity for investment can be seen from the following graph:

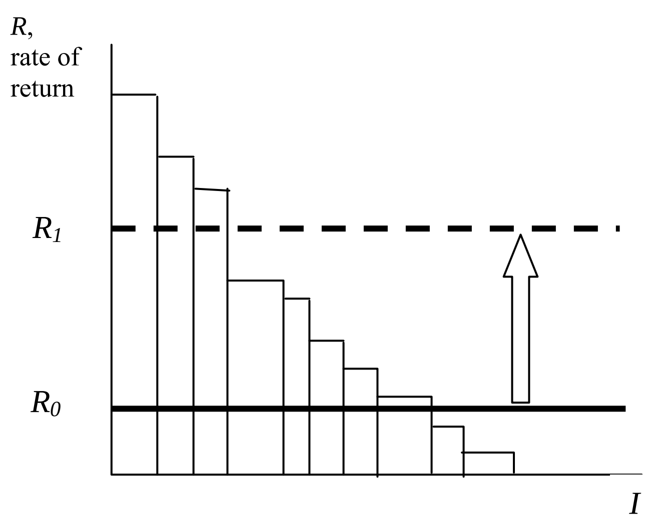

Figure 1. Investment and Interest Rates
Solving:

(10) $$Y = AD = (a_0 + b(Y - T)) + (e_0 - dR) + (GO_o) + (g_0 - mY - \tilde{n}R)$$

(11) $$Y = (a_0 + b(Y - (TA_0 + tY))) - mY + (e_0 - dR) + (GO_o) + (g_0 - \tilde{n}R)$$

Solving for Y as a function of R, one obtains:

(12) $$Y = \left(\frac{1}{1 - b(1-t) + m}\right)[A_0 - (d + \tilde{n})R]$$ <IS curve>

where $A_0 \equiv a_0 - b(TA_0) + e_0 + GO_o + g_0$.

This equation can be re-written as:

(13) $$\boxed{R = -\left(\frac{1 - b(1-t) + m}{d + \tilde{n}}\right)Y + \left(\frac{1}{d + \tilde{n}}\right)A_0}$$ <IS curve>

Note (12) is the same as equation (8.6) in the textbook. All points along the line defined by this equation are points where income and interest rates are such that aggregate demand equals income.

2. The Financial Side of the Economy

Eq.No.	Equation	Description
(14)	$\frac{M^d}{P} = \frac{M^s}{P}$	Equilibrium condition
(15)	$\frac{M^s}{P} = \frac{M_0}{P}$	Money supply
(16)	$\frac{M^d}{P} = \mu_0 + kY - hR$	Money demand

Substitute (11) and (12) into (10), and rearrange to obtain:

(17) $$R = \left(\frac{\mu_0}{h}\right) - \left(\frac{1}{h}\right)\left(\frac{M_0}{P}\right) + \left(\frac{k}{h}\right)Y$$ <LM curve>

Equation (13) is analogous to equation (8.8) in the textbook. The only difference is that money demand includes a constant μ. All points on this line represent the combinations of income and interest rates that equilibrate money supply and money demand.

3. Equilibrium in IS-LM: Algebraic and Graphical

The IS and LM equations constitute a two equation system with two unknowns. The unknowns can be solved for by substituting one equation into another. The two equations are:

$$(12) \qquad Y = \left(\frac{1}{1-b(1-t)+m}\right)[A_0 - (d+\tilde{n})R] \qquad \text{<IS curve>}$$

$$(17) \qquad R = \left(\frac{\mu_0}{h}\right) - \left(\frac{1}{h}\right)\left(\frac{M_0}{P}\right) + \left(\frac{k}{h}\right)Y \qquad \text{<LM curve>}$$

One way to solve this system is to substitute is to (17) in for R in (12).

$$(18) \qquad Y = \left(\frac{1}{1-b(1-t)+m}\right)[A_0 - (d+\tilde{n})\left\langle \frac{\mu_0}{h} - \frac{1}{h}\frac{M_0}{P} + \frac{k}{h}Y \right\rangle]$$

Notice that this can be solved for Y, by bringing the term in the (.) to the left hand side.

$$(19) \qquad Y(1-b(1-t)+m) = A_0 - (d+\tilde{n})\left\langle \frac{\mu_0}{h} - \frac{1}{h}\frac{M_0}{P} \right\rangle - (d+\tilde{n})\frac{k}{h}Y$$

Collect up the last term on the right hand side involving "Y" to the left hand side:

$$(20) \qquad Y[1-b(1-t)+m+\frac{(d+\tilde{n})k}{h}] = A_0 + (d+\tilde{n})\left\langle \frac{1}{h}\frac{M_0}{P} - \frac{\mu_0}{h} \right\rangle$$

Dividing both sides by the term in [.] to obtain:

$$(21) \qquad Y_0 = \hat{\alpha}\left[A_0 + \frac{(d+\tilde{n})}{h}\left(\frac{M_0}{P}\right) - \frac{(d+\tilde{n})\mu_0}{h}\right] \qquad \text{<equilibrium income>}$$

Where

$$\hat{\alpha} \equiv \frac{1}{1-b(1-t)+m+\dfrac{(d+\tilde{n})k}{h}}$$

Note further that

$$\frac{1}{1-b(1-t)+m+\frac{(d+\tilde{n})k}{h}} \equiv \hat{\alpha} \le \overline{\alpha} \equiv \frac{1}{1-b(1-t)+m}$$

Graphically:

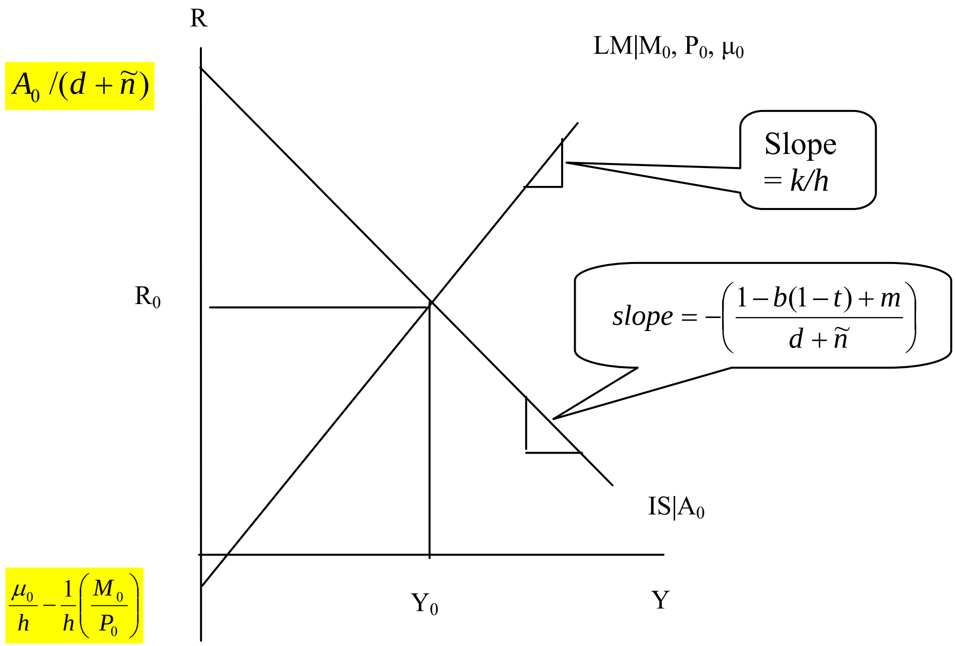

Figure 2: Equilibrium in IS-LM

4. Policy in IS-LM Model

The easiest way to see the impact of policy is to take the total differential of (21):

(21) $$Y_0 = \hat{\alpha}\left[A_0 + \frac{(d+\tilde{n})}{h}\left(\frac{M_0}{P}\right) - \frac{(d+\tilde{n})\mu_0}{h}\right]$$ <equilibrium income>

(22) $$\Delta Y = \hat{\alpha}\left[\Delta A + \frac{(d+\tilde{n})}{h}\Delta\left(\frac{M}{P}\right) - \frac{(d+\tilde{n})}{h}\Delta\mu\right]$$

For fiscal policy, one has to determine whether it is government spending that is changed, or lump sum taxes. Recall $A_0 \equiv a_0 - b(TA_0) + e_0 + GO_o + g_0$. If the fiscal policy involves only government spending, then:

$$\Delta Y = \hat{\alpha}\Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\alpha}$$

If it is lump sum taxes:

$$\Delta Y = -\hat{\alpha}b\Delta TA \Rightarrow \frac{\Delta Y}{\Delta TA} = -\hat{\alpha}b$$

If monetary policy is being used, the $\Delta A = 0$, so:

$$\Delta Y = \hat{\alpha}\left(\frac{d+\tilde{n}}{h}\right)\Delta\left(\frac{M}{P}\right) \Rightarrow \frac{\Delta Y}{\Delta(M/P)} = \hat{\alpha}\left(\frac{d+\tilde{n}}{h}\right)$$

How do these policies appear graphically? Below are fiscal (government spending) and monetary policies.

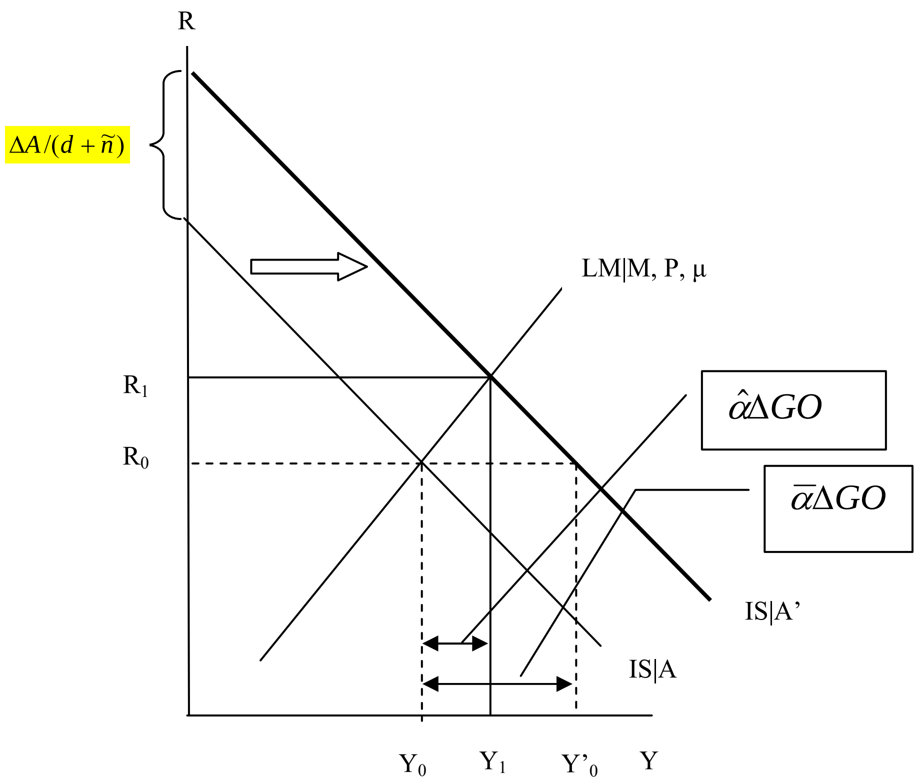

Figure 3: Fiscal (Govt. spending) Policy

Notice that fiscal policy is less powerful, in the sense of increasing income, than it was in the Keynesian cross model. It is important that you understand the intuition for why this result occurs: it is because the introduction of a financial sector means that as income rises, money demand rises (while the money supply is fixed); rising interest rates result in decreased investment and net exports and hence a lower income level relative to the counterfactual level of Y'_0.

You will notice, if you experiment with differing-sloped curves, that if the IS curve is steep because the parameters *d* or *n* are small, then fiscal policy will be more effective than when it is flat because these parameters are large. You should think about why that is the case. You will also find that when the LM curve is flat, the fiscal policy is also more effective than when it is steep.

Now consider monetary policy.

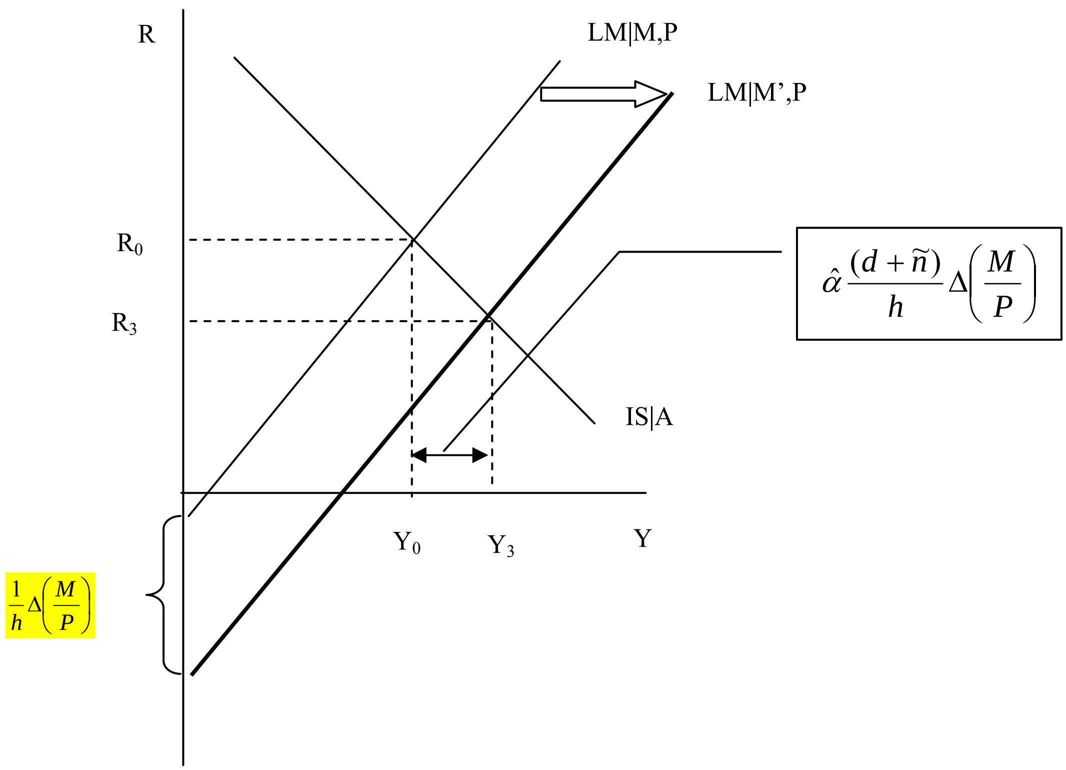

Figure 4: Monetary Policy

Monetary policy works by decreasing the interest rate, and thus spurring investment and net exports. Notice that when the LM curve is steep because *h* is small, monetary policy will tend to be powerful than when *h* is large. Monetary policy will also be more powerful, the flatter the IS curve is. You should consider why these outcomes occur.

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