Midterm Examination II

Instructions:

Answer all questions in your bluebook.

Questions are equally weighted; they may not require the same amount of time to answer.

Show all your work.

1. The number of calls to a police dispatcher between 8:00 P.M. and 8:30 P.M. on Fridays is a Poisson random variable X with $\lambda=3.5$.

   (a) What is the probability that there are no calls during this period?

   (b) What is the probability that there are *exactly* three calls during this period?

   (c) What is the expected number of calls received by the dispatcher during this period?

   (d) What is the variance of the number of calls?

2. An economics department at a midwestern liberal arts college has six tenured and four untenured faculty members. A committee of n = 3 members is to be selected at random. Let X denote the number of committee members who are tenured.

   (a) What is the probability that *exactly* one of the committee members is tenured?

   (b) What is the probability that the number of tenured members on the committee exceeds the number of untenured members?

   (c) Obtain the expected number of committee members who are tenured.

   (d) Obtain the standard deviation of X.

3. Please state whether the following statements are TRUE, FALSE, or UNCERTAIN. Briefly, justify your answer. (No justification, no credit.)

   (a) Let X and Y be random variables. ***Claim***: E[X+Y] = E[X] + E[Y].

   (b) Assume random variables X and Y are independent with E[X] = 2, and E[Y] = 3. Define the random variable Z as Z=X/Y. ***Claim***: E[Z] = ⅔.

   (c) Let Y=aX+b, where X is a random variable and a, b are positive constants. ***Claim***: the covariance between X and Y equals a^2 Var(X).

   (d) Let Z=X+Y, where X and Y are random variables with zero means and with variances equal to σ_x^2 and σ_y^2, respectively. ***Claim***: Var(Z) $\geq \sigma_x^2 + \sigma_y^2$.

4. In restoring the exterior of a historic mansion, the required number of carpentry person hours (V) is assessed to be N(265,144). The required number of painting person hours (W) is assessed to be N(208,25). Assume V and W are independent random variables.

   (a) Define the random variable H, H = V + W. What does H represent here?

   (b) Completely describe the probability distribution of H.

   (c) The labor cost of either type of work is \$15 per hour. A sum of \$7000 has been budgeted for the total labor cost of carpentry and painting. What is the probability that the total labor cost will not exceed the budget?

   (d) Compared to your answer in (c), if V and W have a negative covariance will that increase, decrease, or leave unchanged the probability that the total labor cost will not exceed the budget? Briefly justify your answer.

5. Three brain teasers for your enjoyment.

   (a) If X has a Poisson distribution and Pr[X=0]=1/3, what is the numerical value of E[X]?

   (b) Suppose X is a binomial random variable with parameters n and π. Determine n and π if E[X] = 10 and Var[X] = 8.

   (c) Let X be a bernoulli random variable with parameter = π. What is the numerical value of $E[X^5]$?

   (d) Let X be an exponential random variable with parameter λ. If the median of X equals 1, what is the numerical value of Var(X)?

(To answer part (d) remember that the cumulative distribution function for an exponential random variable is: $\Pr(X \leq x) = 1 - e^{-\lambda x}$, and that $\mathrm{Var}(X) = 1/\lambda^2$. You may also find useful that the natural log of 1/2 is approximately -0.69.)