UNIVERSITY OF WISCONSIN

Economics 101 – Spring 2007

Professor Brown

Problem Set 10 - ANSWER KEY

Monopoly: Single Price vs. Perfect Price Discrimination

1) Suppose daily demand for airplane tickets from Madison to Chicago is given by the equation $Q_D=640 - 2P$.

   a. When finding the optimal price-quantity decision of a monopolist with single price policy (let's call this Q^M, P^M), we may proceed with 5 logical steps.

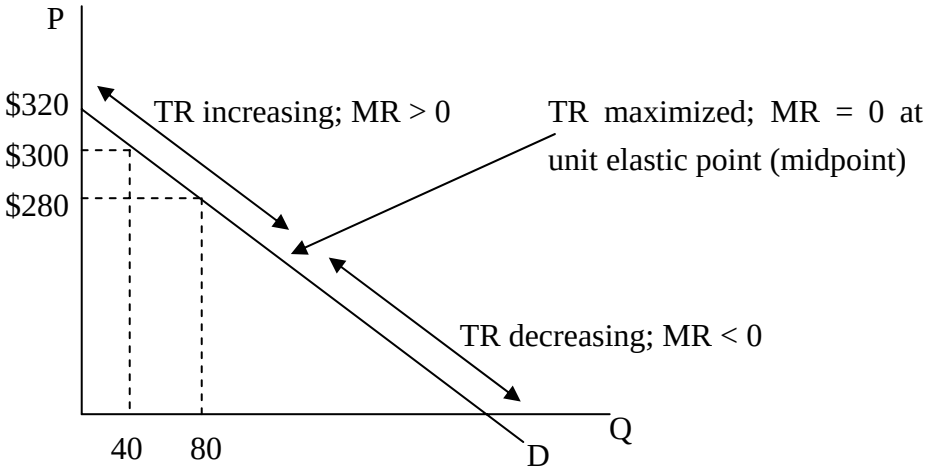
      i) Begin by remembering that a monopolist, like any other firm, maximizes profits by setting MR=MC.

      ii) Find marginal cost (MC). First notice that the equation for total cost may be written as the sum of fixed and variable costs. Here, TC = 35,000 + 20Q. MC is the slope of TC, i.e. the cost of putting one more passenger on the plane, and so here MC =$20.

      iii) Find marginal revenue (MR).

      Now this is the place to be careful. Remember that in perfect competition, the MR was just the market price, P*. But here, in a monopoly setting, there is no market price to take as given; the monopolist *is* the market! Assume, then, that for any quantity that the monopolist wants to sell, it charges the highest price consumers are willing to pay. For example, if it wanted to sell 40 tickets, the monopolist would charge $300. If it wanted to sell 80 tickets, the monopolist would charge $280, and so on. Thus we can see that <u>to sell another unit, the single price monopolist will have to lower its price to everyone</u>. On the upper half of the demand curve, we see that selling more increases total revenue (TR = PQ) since the price is decreased for relatively few buyers, but the additional unit sells at a high price. On the lower half of the demand curve, however, selling more decreases TR, since the increase in

revenue from selling another unit at a low price is small relative to the decrease in TR caused by decreasing the price for all of the other customers. That is, MR is positive on the upper half of the demand curve and negative on the lower half. Remember from earlier in the semester (see Problem Set 5) that TR is maximized at the unit elastic point on the demand curve, and that the unit elastic point on a linear demand curve is the midpoint. Now we're ready to draw in the MR curve for a single price monopolist.



Now that we know where that MR curve comes from, here's the trick to remember: The MR curve is the same as the demand curve (solved for P), but with twice the slope. Thus, in this example, since Demand: P = 320 – (1/2)Q, MR = 320 – Q.

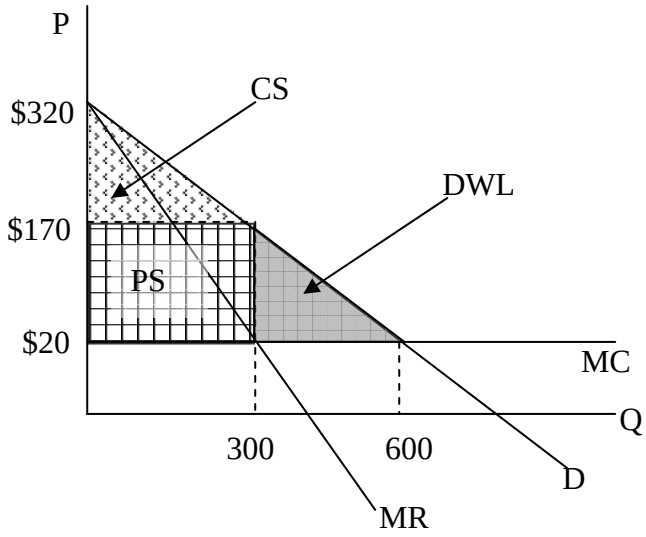
iv) Find Q^M. To do this, remember that a monopolist is maximizing profits where MR = MC. Thus,

$$MR = MC$$
$$320 - Q = 20$$
$$Q^M = 300$$

v) Find P^M. Remember that the monopolist will charge as much as it can – thus, find the price by substituting Q^M into the demand equation.

$$P^M = 320 - 1/2*(300) = \$170$$

Graphically, we've just found:



b. Profit is just: π = TR-TC = PQ – ($35,000 + $20*Q) = $170*300 – ($35,000+$20*300) = $51,000 - $41,000 = $10,000. Notice: Profit may also be written: π = PS – FC. (Profit is producer surplus less fixed costs.). With this way you get the same result: π = PS - FC= ($170-$20)*300 - $35,000 = $10,000.

   Consumer surplus is the area above the price, $170, and below the demand curve. So CS = ½(320-170)(300) = $22,500.

   Deadweight loss is the difference between the efficient outcome (the point where MB (or demand) = MC), Q = 600, and the realized outcome, Q = 300. Thus, DWL = ½($150)(600-300) = $22,500.

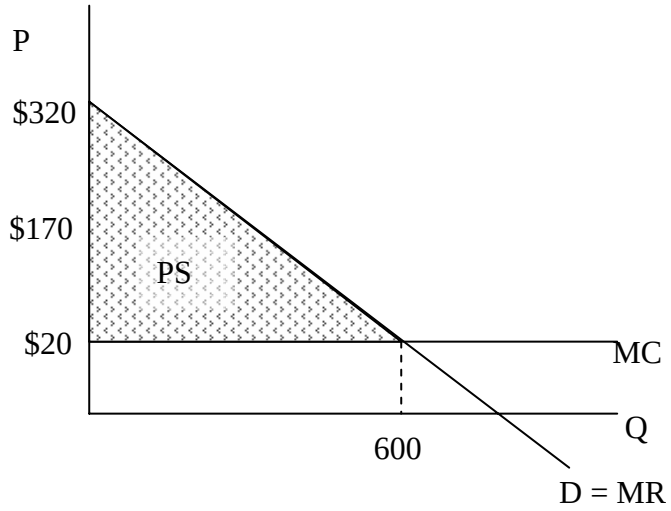
c. Perfect price discrimination: Notice that the only difference is in our definition of MR. Now if the monopolist wants to sell another ticket, it does not need to decrease the price to everyone else who's already buying. Thus, MR, the money the monopolist gets for selling one more unit, is just whatever the next person is willing

to pay. This means that *the MR curve is the demand curve*. Solving using steps 1-5 above:

$$MR = MC$$
$$320 - \tfrac{1}{2}Q = 20$$
$$Q = 600.$$

Prices range from $320 to $20.

Notice that the outcome is efficient here (there's no DWL, since the efficient outcome is where MB = MC – at 600 units!). All that's changed is that the monopolist gets the entire surplus, and the consumer surplus is zero (since every consumer pays exactly what he/she's willing to pay.) To find profit, remember that π = TR-TC. TR is all of the area under the demand curve, to the left of Q = 600, and TC is as before. Thus, that π = TR-TC. Alternatively (and equivalently), π = PS – FC. From below, producer surplus, PS = ½($300)(600) = $90,000. Thus, π = $90,000 - $35,000 = $55,000.

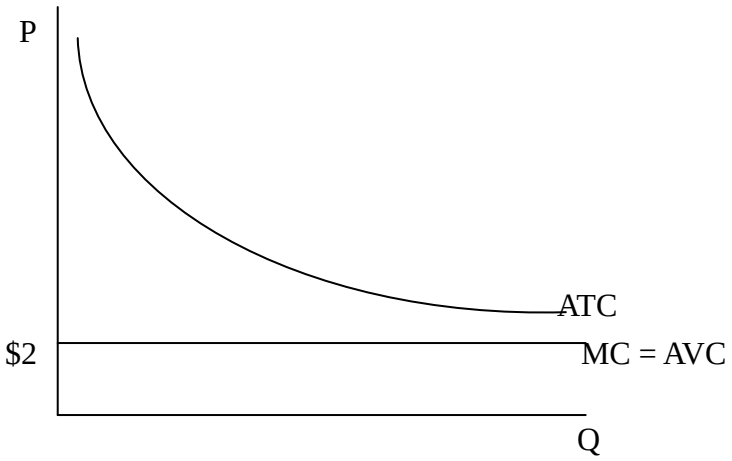


d. Recall that ATC = TC/Q. Thus, ATC = 35,000/Q + 20. At Q = 600, ATC = approximately $78. The airline will sell at a price of $20, even though this is below the ATC, since each additional passenger paying even a penny more than $20 is fully covering his own variable cost of $20 (peanuts, etc.) and is contributing to the fixed cost if only a little bit.

Natural Monopoly and Regulation

2) Suppose the annual cost of providing telephone service to the city of Springfield is given by the equation TC= 50 + 2Q, while annual demand for telephone services in Springfield is given by the equation $P = 42 - 2Q_D$.

   a. To show that this is a natural monopoly, just observe that ATC is decreasing everywhere. That is, there are always economies of scale. This means, roughly, that bigger is better. Here, ATC = 50/Q + 2:



   b. Since there are economies of scale for a natural monopoly, one provider for the whole market would have a lower ATC than two providers, each producing for only half of the market.

   c. Suppose the city government auctions off a license to be the monopoly provider of phone services for one year. The most we would expect a firm to pay for the right to be a monopoly provider is what it would make in profits, were it to have monopoly power in that market. Thus, all we need to do is to find monopoly profits for this market. Recall the steps from (1), and work through them more quickly this time.

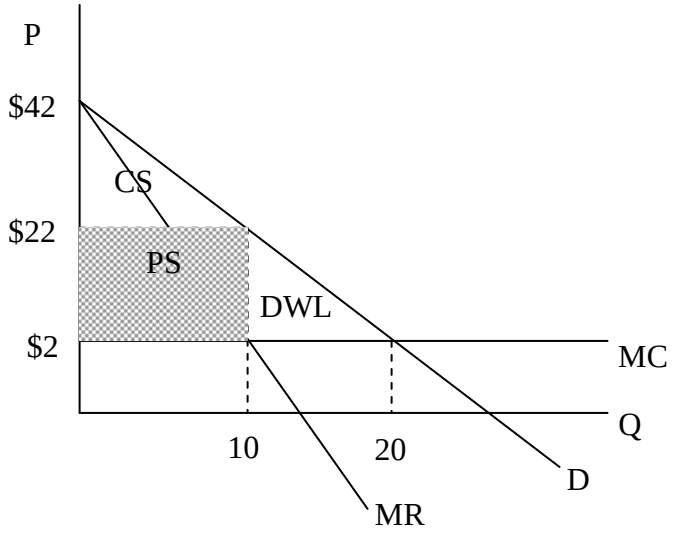
      - We want MR = MC.
      - MC = 2 from the TC function.

- Demand: P = 42 – 2Q, so MR = 42 - 4Q.
- Now set MR = MC to find Q^M.

  42 - 4Q = 2

  Q^M = 10.
- Finally, substitute into the demand curve to find P^M.

  P^M = 42 – 2*10 = $22.
- Graphically, we've just found:



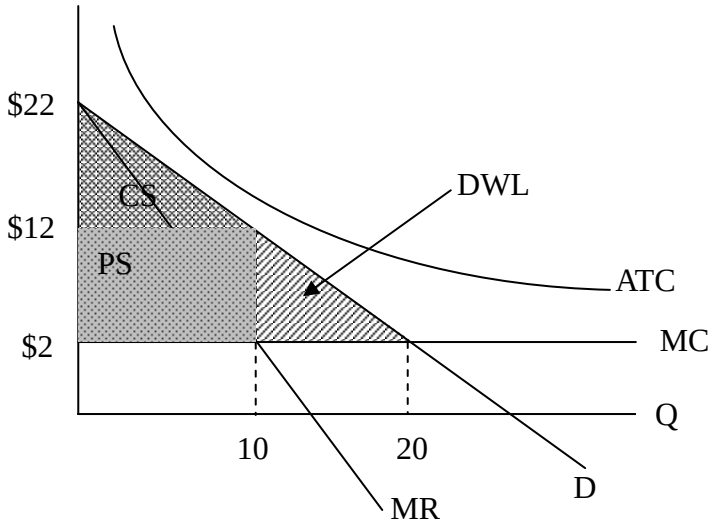
- Finally, π = TR-TC = $22*10 – ($50+$2*10) = $150.
- Or equivalently, π = PS – FC = $20*10 - $50 = $150.
- Thus, the maximum a firm would pay for monopoly rights to the market is $150.
- Finally, CS = ½($42-$22)(10) = $100 and DWL = ½($22-$2)(20-10) = $100

Monopoly: Short Run and Long Run

3) A monopolist in the salt market has demand given by the equation Q_D= 22-P and has total costs equal to TC= 110+ 2Q. Quantity is measured in ounces, and you may assume that units are infinitely divisible.

   a. Again, we'll run through the analysis quickly, since it's the same as in (1).

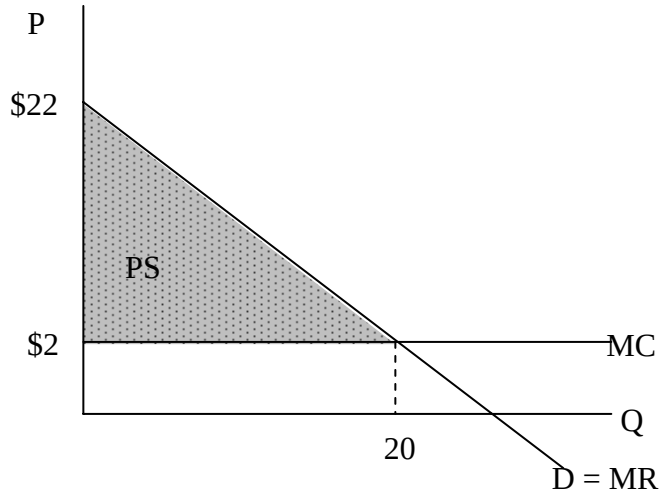
   - MC = 2
   - MR =22 – 2Q
   - So, MR = MC yields, 22-2Q = 2, such that Q^M = 10.
   - P^M = 22 - 10 = $12
   - π = TR – TC = 10*$12 – ($110 +$2*10) = $120 - $130 = -$10.
   - Graphically,



   b. The monopolist will stay open in the short run because total revenue from the production is covering its variable costs ($20). In the long run, however, the monopolist will shut down since its profit is negative (it's not covering all of the $110 fixed cost.)

c. Now suppose the monopolist can perfectly price discriminate. Recall that everything is the same except for our definition of MR. Now MR is the same as the demand curve. Thus,

- MC = 2
- *MR =22 – Q*.
- So, MR = MC yields Q = 20.
- Prices range from $22 to $2.
- π = PS – FC = ½($22-$2)(20) – $110 = $90.
- Graphically,



d. The output under perfect price discrimination is efficient, whereas that of a single price monopolist, clearly, is not.

Monopoly: Multiple-Choice Questions

4) d.

5) a.