Lecture 7: Divorce Policy and Families

reference: Flinn, "Modes of Interaction Between Divorced Parents," IER 2000.

Compliance with child support orders has been a major concern of family policy.

The first Figure 1a-c distributed with these notes is from Del Boca and Flinn AER 1995. Note that full compliance and zero compliance are common behaviors, and then several non-custodial parents pay positive child support amounts that are either below or above the court ordered amount.

In the Flinn IER 2000 model of divorced parenting, parents may determine expenditures on the child cooperatively or non-cooperatively.

Child support orders serve the purpose of dividing the rents from cooperation between parents, and parents recognize the order as a focal point for cooperation. If the value to cooperation exceeds the value of non-cooperation for each parent, then parents are able to cooperate.

Flinn considers the roles of the child support order and its enforcement in determining parents' equilibrium expenditures on the child.

Flinn takes as the barrier to cooperation between divorced parents the identification of a point on the efficient investment frontier. The institutional agent (whom we'll call the 'judge') may solve this problem.

Model

Divorced parents consume and interact over investments in the child.

They may behave cooperatively or non-cooperatively. Cooperation implies that efficient expenditures on the child are made.

The child functions as a public good in parents' consumption.

2 parents, *m* and *f*. Parental preferences are defined over own consumption $c_p,\ p\in\{m,f\}$, and the consumption of the child, *k*, and can be expressed

$$\alpha_p \ln c_p + (1-\alpha_p)\ln k,\ p\in\{m,f\}. \qquad (1.1)$$

The analysis in this paper assumes that the mother has legal and physical custody of the child. Thus the mother controls expenditures on the child. The father can only influence child expenditures indirectly, through transfers to the mother.

We therefore assume that *k* is the choice variable of the mother, and transfer *t* is the choice variable of the father.

The decision rules of the 2 parents in the 2 regimes will be denoted $\{k_j^*, t_j^*\},\ \mathrm{j}\in\{C,N\},$ where *C* is the cooperative regime and *N* the non-cooperative.

Noncooperative Behavior

The timing of the decision making is: (1) father transfers to mother (2) given the transfer from the father, the mother makes the child expenditure decision.

We solve for the mother's expenditure, conditional on the father's transfer, and then move back to the father's problem. The mother's problem is

$$\max_{k\in\{0,y_m+t\}} \alpha_m \ln(y_m+t-k)+(1-\alpha_m)\ln k, \qquad (1.2)$$

yielding decision rule for the mother in the noncooperative case

$$k_N^*(y_m+t)=(1-\alpha_m)(y_m+t). \qquad (1.3)$$

The father's problem, conditional on the mother's optimal choice in the noncooperative regime, is

$$t_N^*(y_m,y_f)=\arg\max_{t\in[0,y_f]}\{\alpha_f \ln(y_f-t)+(1-\alpha_f)\ln(k_N^*(y_m+t)). \qquad (1.4)$$

The father's optimal transfer rule can be explicitly stated as follows. Let $\tau(y_m,y_f)=(1-\alpha_f)y_f-\alpha_f y_m$. Then

$$t_N^*(y_m,y_f)=\begin{cases}0 & \text{if } \tau(y_m,y_f)\le 0\\ \tau(y_m,y_f) & \text{otherwise}\end{cases} \qquad (1.5)$$

There exists a unique Nash equilibrium to this Stackelberg game (in which the father acts as the leader), which is

$$(\tilde{k},\tilde{t})(y_m,y_f), \text{ where } \tilde{k}(y_m,y_f)=(1-\alpha_m)(y_m+t_N^*(y_m,y_f))$$
$$\text{and } \tilde{t}(y_m,y_f)=t_N^*(y_m,y_f).$$

The values to the mother and father of noncooperation are then

$$V_m^N(y_m,y_f)=\alpha_m \ln(y_m-\tilde{k}(y_m,y_f)+\tilde{t}(y_m,y_f))+(1-\alpha_m)\ln(\tilde{k}(y_m,y_f))$$
and
$$V_f^N(y_m,y_f)=\alpha_f \ln(y_f-\tilde{t}(y_m,y_f))+(1-\alpha_f)\ln(\tilde{k}(y_m,y_f)). \tag{1.6}$$

We will see that the noncooperative child expenditures must be (weakly) less than the cooperative child expenditures.

Observe that in the cooperative game the mother may have reason to expend less on the child than agreed upon by the parents in reaching the efficient frontier. (We might call this noncooperative behavior.) Flinn assumes in the cooperative case that the mother is able to commit to a cooperative expenditure, following the logic of the folk theorem in the repeated child expenditure game between divorced parents.

The father will transfer (weakly) less in the noncooperative than in the cooperative equilibrium.

Cooperative Behavior

Since the expenditure on the child is a public good, it may be possible to increase the welfares of both the mother and father (and the child, if child welfare is increasing in child expenditures) by implementing a cooperative solution.

Any cooperative solution will have the father transfer a greater *t* than in the noncooperative case, and will have the mother expend a greater *k* on the child than in the noncooperative case.

While the noncooperative equilibrium in this model is unique, there exists a continuum of cooperative equilibria. Flinn is interested in the selection of a particular equilibrium in the cooperative set in this paper. We'll first characterize the full set of cooperative equilibria.

Tracing out the Pareto frontier: Suppose that the mother can choose both *t* and *k*, but her choice is constrained by the requirement that the father achieve a welfare of at least $\widetilde{V}_f$.

The mother's pseudo-choice problem is

$$\begin{aligned}&\widetilde{V}_m(y_m, y_f, \widetilde{V}_f) = \max_{k,t}\{\alpha_m \ln(y_m + t - k) + (1-\alpha_m)\ln k\} \\ &s.t.\ \ \alpha_f \ln(y_f - t) + (1-\alpha_f)\ln k \geq \widetilde{V}_f,\end{aligned} \tag{1.7}$$

where $\widetilde{V}_f$ is the utility guaranteed to the father under some unspecified mechanism for the division of the surplus from cooperating. Clearly the lower bound on allowable $\widetilde{V}_f$ in the interaction of the parents is $V_f^N(y_m, y_f)$, the outside value to the father of noncooperation.

Given strict concavity in parental utilities, the constraint in (1.8) will always be binding and the mother's welfare is monotone decreasing in that of the father.

We can define the maximum welfare attainable by the father under cooperation, $\overline{V}_f^C$, implicitly as the father's value that leaves the mother exactly her noncooperative welfare in the cooperative equilibrium, or

$$\widetilde{V}m(y_m, y_f, \overline{V}_f^C) = V_m^N(y_m, y_f).$$

Therefore the father's realizable value of cooperation must satisfy $V_f^C \in [V_f^N, \overline{V}_f^C]$.

Represent a particular division of the available surplus from cooperation by V_f^C.

Then solving for the cooperative *t* and *k*, given incomes and a division of the cooperation surplus:

The binding constraint on the mother's choice problem means

$$\alpha_f \ln(y_f - t) + (1-\alpha_f)\ln k = V_f^C. \qquad (1.8)$$

$$\Rightarrow \quad y_f - t = \exp(V_f^C / \alpha_f) k^{-\eta}$$

$$\Rightarrow \quad t = y_f - R(V_f^C) k^{-\eta},$$

where $\eta \equiv (1-\alpha_f)/\alpha_f$ and $R(V_f^C) \equiv \exp(V_f^C / \alpha_f)$.

So fixing the father's welfare in the cooperative case effectively fixes the transfer from the father to the mother.

Now we can substitute the expression for the father's transfer given his value of cooperation to determine the mother's *cooperative* child expenditure.

$$k^C(y_m, y_f, V_f^C) = \arg\max_k \{\alpha_m \ln(y_m - k + (y_f - R(V_f^C)k^{-\eta})) + (1-\alpha_m)\ln k\}$$
$$= \arg\max_k \{\alpha_m \ln(y_t - k - R(V_f^C)k^{-\eta}) + (1-\alpha_m)\ln k\}$$

where $y_t \equiv y_m + y_f$ is total parent income.

The equilibrium expenditure on the child conditional on surplus division V_f^C is defined implicitly by

$$k^C(V_f^C) = (1-\alpha_m)y_t - R(V_f^C) J[k^C(V_f^C)]^{-\eta}, \qquad (1.9)$$

where $J \equiv (1-\alpha_m(1+\eta)) = 1-\frac{\alpha_m}{\alpha_f}$.

The expenditure on the child in equilibrium may be increasing or decreasing in the value allocated to the father by the surplus division rule, depending on the relative selfishness of the parents.

$$\begin{aligned} \frac{\partial k^C}{\partial V_f^C} < 0 \;\;\Leftrightarrow\;\; \alpha_m > \alpha_f \\ \frac{\partial k^C}{\partial V_f^C} > 0 \;\;\Leftrightarrow\;\; \alpha_m < \alpha_f, \end{aligned} \tag{1.10}$$

with the partial derivative = 0 if parents are equally selfish.

The cooperation equilibrium transfer is

$$t^C(V_f^C) = y_f - R(V_f^C)[k^C(V_f^C)]^{-\eta}.$$

Given this equilibrium transfer and expressions (1.11), it follows that $\frac{\partial t^C}{\partial V_f^C} < 0.$ Conditional on incomes, the more value the surplus division guarantees the father the less he transfers to the mother in cooperation.

We know from the cooperative transfer expression that in equilibrium $y_f - t^C(V_f^C) = R(V_f^C)[k^C(V_f^C)]^{-\eta}$.

Substituting this into (1.10), we find a very handy expression for expenditures as a function of parental incomes, the transfer and the surplus division rule.

$$k^C(V_f^C) = \theta_1 y_f + \theta_2 y_m + \theta_3 t^C(V_f^C),$$

where

$$\theta_1 = \frac{\alpha_m}{\alpha_f} - \alpha_m \tag{1.11}$$

$$\theta_2 = 1 - \alpha_m \text{ and}$$

$$\theta_3 = 1 - \frac{\alpha_m}{\alpha_f}.$$

Figure 1 plots some numerical examples of this linear relationship for parental incomes fixed at the sample means for mother and father. (see paper discussion pp 8-9 here)

Characterizing cooperation cases: Consider a level of transfers $\hat{t}$. We can use relationship (1.12) to characterize child expenditures with $\hat{t}$ as $\hat{k}(\hat{t}) = \theta_1 y_f + \theta_2 y_m + \theta_3 \hat{t}$.

Then the mother & father's welfares given transfer $\hat{t}$ are

$$\hat{V}_f(\hat{t}; y_m, y_f) = \alpha_f \ln(y_f - \hat{t}) + (1 - \alpha_f)\ln(\hat{k}(\hat{t}))$$

and

$$\hat{V}_m(\hat{t}; y_m, y_f) = \alpha_m \ln(y_m + \hat{t} - \hat{k}(\hat{t})) + (1 - \alpha_m)\ln(\hat{k}(\hat{t}))$$

So given any transfer $\hat{t}$, two parents with a given income cooperate if and only if *both*

$$\begin{aligned} &\hat{V}_m(\hat{t}; y_m, y_f) \geq V_m^N(y_m, y_f) \text{ and} \\ &\hat{V}_f(\hat{t}; y_m, y_f) \geq V_f^N(y_m, y_f). \end{aligned} \qquad (1.12)$$

Given this $(\hat{t}, y_m, y_f)$, we define $C(\hat{t}, y_m, y_f)$ as the set of pairs of preference parameters (α_m, α_f) such that (1.13) is satisfied. In other words, $C(\hat{t}, y_m, y_f)$ is the set of preference parameters for which a *cooperative* equilibrium exists given $(\hat{t}, y_m, y_f)$.

Implementing Cooperative Solutions

The judge, in determining the child support order, serves as a sort of ‘focal arbitrator’. She solves the parents’ problem of choosing a point on the Pareto frontier (when cooperation is possible) by setting child support order *s*.

If both the mother and the father prefer to behave cooperatively with transfer *s*, then the court-ordered child support is transferred from the father to the mother, and this dictates the division of the surplus from cooperation.

Figure 2 plots the cooperation sets in (α_m, α_f) space for given (y_m, y_f, s).

Figure 3 shows the child expenditures over the range of parents’ selfishness parameters. The expenditure surface shows a noticeable bulge (increase in child expenditures) over the set of preference parameters contained in $C(\hat{t}, y_m, y_f)$ for the transfer and incomes considered. Cooperating parents clearly expend more on children in the model than noncooperating parents.

Econometric model

[Here are some broad comments on the estimation. The likelihood itself will be derived in class.]

We note that when we observe a father who complies with the child support order, we learn that parents’ preference parameters are in the set $C(\hat{t}, y_m, y_f)$.

When we observe a noncompliant father, we learn something about the preference parameter of the father and conditional on this something about the preference parameter of the mother.

The parental preference parameters are assumed to follow joint distribution in the population *F*, governed by parameter vector θ.

6 specifications of the model are estimated:
→ In 1 & 2, *F* is assumed to be two independent power function distributions.
→ In 3 & 4, *F* is such that individual parental preference parameters are independently distributed as Betas.
→ In specifications 5 & 6, parental preferences are assumed to follow a bivariate normal.

Data

The data are administrative data on 222 divorce cases adjudicated in Wisconsin between 1980 & 1982.

[Note that the folks on the 3rd floor have collected and maintained child support policy data including Court Record Data (CRD) for Wisconsin since the 1980s.]

Flinn uses data on divorce cases involving only 1 child in which the mother was awarded sole legal custody and a child support order was in effect.

The data include reported monthly income of the mother and father, the child support order and the reported payment in the 5th month after the child support order took effect.

Results

Table 1 provides descriptive statistics for the data.

Tables 2 reports parameter estimates for the 6 specifications. Note the tight parameterization of the model.

Table 3's top panel allows us to compare means & SDs of mothers' & fathers' preferences parameters under various assumptions on their joint distribution.

Table 4 reports the results of a policy experiment in which Flinn varies child support enforcement from none to complete and studies the resultant child expenditure.

The model suggests a theoretically ambiguous effect of changes in the enforcement regime on child expenditures, and the policy simulation demonstrates an empirically small effect of the change from no to complete enforcement of child support payment.