Lecture Notes 6a: Intergenerational Transfers I

(reference: Kotlikoff and Spivak, 'The Family as an Incomplete Annuities Market,' JPE 1981.)

The greater extent of trust and common information that exists in the family, as compared with standard insurance markets, solves three of the major problems of such markets. Trust and information in the family can decrease or eliminate moral hazard, adverse selection and deception.

Insuring lost earnings from decreased labor demand or disability, and personal loans on which individuals may default, are particularly subject to problems of moral hazard, adverse selection and deception. Kotlikoff and Spivak argue that 'many family practices in dealing with these types of risks can be explained as implicit insurance contracts made ex ante by completely selfish family members.'

The paper we consider in this lecture is primarily concerned with family provision of insurance against the risk of outliving one's assets. Individuals face longevity risk, leading them in the absence of insurance markets to decide between consuming too much when young, and thus exposing themselves to the risk of poverty if they are long-lived, and consuming too little when young only to die early.

A complete annuity allows the individual to hedge longevity risk by exchanging initial resources for a stream of payments that continue as long as the individual survives.

Kotlikoff & Spivak (KS) consider the family as an annuities market, incomplete in that the parties to risk-sharing in the family are generally few.

KS find that even a small family can substitute by more than 70% for a complete annuity market in pooling longevity risk.

Family members are assumed to be completely selfish throughout the paper. One issue of concern is that of enforceability of contracts in the absence of altruism, which we will return to in our discussion.

We will first consider an individual who consumes optimally in the presence and absence of a complete annuity market. This yields benchmarks for individual welfare, and develops the intuition for the welfare gain from access to annuities markets.

Next we look at the market for annuities in small (2 or 3 person) families, who are of similar and dissimilar survival probabilities. We consider whether family formation with individuals of similar or different ages is more efficient. Optimal contracts in the families will require agreement on family members' consumption paths and commitment by each member to name the other members as sole heirs in her estate.

Individual Consumption Plan Without Fair Annuities

Here the individual maximizes expected utility from current and future consumption, (1.1), subject to the intertemporal budget constraint, (1.2).

$$EU = \sum_{t=0}^{D} P_t U(C_t) \tag{1.1}$$

$$\sum_{t=0}^{D} C_t R^{-t} = W_0 \tag{1.2}$$

The P_t's are probabilities in period *t* of living to period *t+1*. *D* is the maximum longevity. *R* is the discount factor, and is equal to 1+the interest rate. W_0 is initial wealth. There is no future stream of labor income or bequests.

Budget constraint (1.2) reflects the nonzero probability that the individual may live to period *D*, and in this case realized consumption must not exceed initial wealth. (1.2) is the BC *without* annuities.

Individual Consumption Plans *with* Fair Annuities

If the individual has access to actuarially fair annuities purchased in a complete market, then the relevant BC is

$$\sum_{t=0}^{D} P_t C_t R^{-t} = W_0 \tag{1.3}$$

Note that (1.3) requires only equality between the EPV of lifetime consumption and initial wealth.

We can consider the $P_t R^{-t}$'s as prices of future consumption. Since the future survival probabilities are less than one, the price of future period *t* consumption is less than the price of today's consumption by the probability of not surviving to *t*. In this manner, access to a fair annuity market decreases the price of future consumption.

For convenience, we assume an iso-elastic utility function

$$EU = \sum_{t=0}^{D} P_t \frac{C_t^{1-\gamma}}{1-\gamma} \alpha^t \tag{1.4}$$

γ is the CRRA parameter here, and α is the rate of time preference.

Without annuities, maximization of (1.4) s.t. (1.2) yields consumption path

$$C_t = \frac{W_0 (R\alpha)^{t/\gamma} P_t^{1/\gamma}}{\sum_{j=0}^{D} R^{j(1-\gamma)/\gamma} \alpha^{j/\gamma} P_j^{1/\gamma}} \tag{1.5}$$

and with fair annuities, maximization of (1.4) s.t. (1.3) yields

$$C_t = \frac{W_0 (R\alpha)^{t/\gamma}}{\sum_{j=0}^{D} R^{j(1-\gamma)/\gamma} \alpha^{j/\gamma} P_j} \tag{1.6}$$

Figure 1 compares (1.5) and (1.6), the consumption streams with and without fair annuities, for the case of $R = \alpha = 1$. The access to a fair annuities market may raise or lower initial consumption, depending on whether γ is greater or less than one.

If the individual is risk averse (γ > 1), without annuities consumption is somewhat flat but declining over time. Note that it is everywhere below consumption with access to a complete annuities market: the welfare gains with fair annuities are very large for the most risk averse agents.

For less risk averse individuals, with no annuities consumption declines more steeply over time. These individuals are willing to consume more early on, responding to the discounting of future consumption through the probability of death.

With access to fair annuities markets, for any degree of relative risk aversion the individual consumes the same amount in every period. Consumption levels in Figure 1 also demonstrate that we expect access to the annuity market to be welfare enhancing.

This is obviously true for $\gamma > 1$, and is even true for $\gamma < 1$.

If we substitute (1.5) and (1.6) into (1.4), we get indirect utility functions with and without annuities, as functions of initial wealth, market return and survival probabilities.

$$H_0(W_0) = \frac{1}{1-\gamma} W_0^{1-\gamma} \left[\sum_{j=0}^{D} \alpha^{j/\gamma} R^{j(1-\gamma)/\gamma} P_j^{1/\gamma} \right]^{\gamma} \qquad (1.7)$$

$$V_0^*(W_0) = \frac{1}{1-\gamma} W_0^{1-\gamma} \left[\sum_{j=0}^{D} \alpha^{j/\gamma} R^{j(1-\gamma)/\gamma} P_j \right]^{\gamma} \qquad (1.8)$$

Equation (1.9) identifies the increment to initial wealth *M* that leaves the agent as well off in the absence of fair annuities as she would be with fair annuities at the initial wealth level.

$$H_0(MW_0) = V_0^*(W_0) \qquad (1.9)$$

The neat thing about using the iso-elastic utility function is that this calculation is then independent of initial wealth.

In Table 1, KS report values of *M* for various ages and risk aversions using both male and female survival probabilities from the SSA mortality tables. They cite research that has found that empirically relative risk aversion is > 1, and they take *D* to be 120.

Table 1 calculations indicate that the gains from annuity access can be large.

Gains are greater for older individuals, as younger individuals' consumption streams are more certain in the near future due to lower death probabilities. Note, however, that the required wealth gains are proportional. We might reasonably expect what's considered remaining lifetime wealth in this simple model to be larger for younger agents.

Men gain more than women, owing to higher male mortality rates.

More risk averse individuals gain more from access to the insurance market, as expected.

The choices of R and α matter. Raising the market rate of return from 1% to 5% for fixed α increases the age 55 wealth-equivalent factor from 46.90 to 55.57% at $\gamma = 0.75$.

Income, Substitution Effects and Unintended Bequests

KS: 'Without access to an annuity market a single, nonaltruistic individual will always die prior to consuming all his wealth and, accordingly, will make involuntary bequests.'

Reasonable parameterizations of the KS model yield quite large accidental bequests. We find the EPV of unintended bequests by multiplying the probability of dying at each age by the wealth at each age and discounting back to the first period.

At $\gamma = 0.75$, $R = 1.01$ and $\alpha = .99$, the EPV of unintended bequests for an age 55 male is 24.47% of initial wealth.

Increasing the risk aversion coefficient to 1.75 raises the EPV of unintended bequests to 35.83% of initial wealth.

Finally, the SSA mortality probabilities indicate a fairly rapid rate of consumption, even for the risk averse. With gamma = 1.75, a single male's consumption at age 85 is < 1/3 what it was at 55.

The Family as an Incomplete Annuities Market

Marriage: In marriage, spouses generally agree to pool resources while both partners are living, and to name each other as beneficiaries in their wills.

→ In this sense, the longevity risk of each spouse is hedged to some extent by her partner. If one spouse lives too long, it is very likely that the other has not, and has left the long-lived partner a bequest.

→ The two can further hedge longevity risk by choosing a joint consumption path that takes into account each spouse's expected bequest to the other.

Parent-child: Here a parent and child can share the longevity risk of the parent and the unintended bequest windfall for the beneficiary.

→ The parent promises to name the child as beneficiary in her will, in exchange for the child's promise to care for the parent should the parent outlive her resources.

→ Because the child and parent have very different death probabilities during the life of the parent, both can gain through the sharing of consumption resources.

→ In some sense, the sharing of consumption resources allows the parent and child to gain from the value to the child of the parent's consumption resources in the state of the world in which the child survives but the parent does not.

<u>As the family grows large</u>: The view of the family as an incomplete annuities market becomes clear if we consider behavior when the family grows large.

Assume that family members have identical survival probabilities.

They enter the multiperson family with identical resources.

In the limit, as the family grows large the consumption path of a member of the family converges to that of a single individual with access to a complete and actuarially fair annuity market.

The two-person model

KS determine the efficient frontier of marriage contracts over joint consumption and bequests. The assumption of efficiency implies married partners behave cooperatively in determining the consumption streams of the partners.

The frontier of efficient contracts is the solution to the recursive dynamic programming problem

$$V_{t-1}(W_{t-1}) = \max_{W_t, C_{t-1}^H, C_{t-1}^S \geq 0, t=T,\ldots,1} [u^H(C_{t-1}^H) + \theta u^S(C_{t-1}^S)$$
$$+\alpha P_{t|t-1} Q_{t|t-1} V_t(W_t) + \alpha P_{t|t-1}(1-Q_{t|t-1}) H_t(W_t) \quad (1.10)$$
$$+\theta\alpha Q_{t|t-1}(1-P_{t|t-1}) S_t(W_t).$$

subject to

$$W_t / R + C_{t-1}^H + C_{t-1}^S = W_{t-1} \qquad (1.11)$$

where

$$V_T(W_T) = \max_{C_T^H, C_T^S} u^H(C_T^H) + \theta u^S(C_T^S).$$

In equation (1.10), $V_t(W_t)$ is the period *t* maximum-weighted expected utility of the two family members with joint wealth W_t.

H and *S* represent the two family members (Husband and Spouse??),

$C^H{}_t$ and $C^S{}_t$ are their period *t* consumptions,

u^H and u^S are their per-period utility functions,

$P_{t|t-1}$ and $\mathrm{Q}_{t|t-1}$ are *H* and *S*'s respective probabilities of surviving to *t* given survival to *t-1,*

and $H_t(\mathrm{W}_t)$ and $\mathrm{S}_t(\mathrm{W}_t)$ are the maximum expected utilities of each partner if he or she alone survives to period *t* with household wealth W_t.

The first 2 terms in the RHS of (1.10) represent *H* and *S*'s utility of their certain period *t-1* consumption. *S*'s utility weight relative to *H*'s in the family expected utility is θ.

The next term represents the family's utility of the future multiplied by the probability that both members survive to *t*.

The last two terms are the utilities when one member only survives, multiplied by the relevant lone survivorship probabilities.

Gains from Family Annuity Contracts

KS solve the maximization in (1.10) – (1.11).

[Impressively, they also solved the analogous 3 member family problem (in 1981--).]

<u>Marriage</u>

Assuming spouses have common rates of time preference and risk aversion, we use equation (1.10) to compare the consumption paths and welfare of married and single persons in the absence of a public annuities market.

Reasons that married couples' paths differ from singles':

- With common survival probabilities, the reduction in the risk of outliving one's assets through the spousal bequest arrangement effectively lowers the price of future consumption.

→ [the analogy here is to the $P_t R^{-t}$ consumption price effect in the case of fair and complete annuities markets (budget constraint (1.3)]

→ Therefore the consumption profiles for married agents lie between the no-annuity and full annuity single profiles in Figure 1.

- Differences in spouses' survival probabilities induces differences in the married and single consumption paths. Higher survival probabilities behave like lower rates of time preference in the determination of the consumption path.

→ For example, if an older man marries a younger woman then the two spouses compromise on the rate at which they spend down their assets while they are both alive. The old husband would prefer to spend more quickly than the young wife, all else equal.

→ Such an old husband will consume more slowly than an identical but single old man.

Table 2 reports gains from access to the marriage annuity arrangement in the cases of 2 and 3 person families. Members have identical survival probabilities, initial endowments and utility weights in the contract.

Gains are calculated as the % increase in initial endowment required to make the single person as well of as she would be in the family arrangement.

Also reported are the gain as a fraction of the dollar gain from access to a fair and complete annuities market and the fraction of the utility gains from the family arrangement of the utility gains from access to the fair and complete market.

Family longevity risk pooling can be quite valuable even to young individuals. Table 2 shows an 11.7 to 13.6% dollar gain for an age 30 married male. Clearly gains increase with age, as mortality risk increases. At age 75, marriage is equivalent to increasing wealth by 30% when $\gamma = 1.25$.

Marriage does even better at substituting for the fair & complete annuities market when we consider utility gains.

> → For most cases considered in Table 2, marriage substitutes for considerably more than 40% of the fair & complete annuities market.

Note that marriage is a better substitute when young, as the probability that spouses will die at about the same age is particularly low when young.

Additionally, interactions between risk aversion and age lead marriage to be a better substitute when individuals are young and not very risk averse or old and very risk averse.

Over the range of parameters and ages considered in Table 2, 3 person families are able to substitute for the fair & complete annuities market by about 60%, using the utility gain measure.

KS calculate that a 4th family member would allow the family to substitute for about 70% of the fair & complete annuities market, in utility terms. [Note the complexity of this calculation.]

They also observe that diminishing returns to risk pooling set in fairly quickly.

→ The marginal dollar gain (as opposed to marginal utility gain) of adding a 3rd family member is 8.04%. For a fourth member, KS approximate it at 3.23%.

Parent-child risk sharing

Table 3 looks at the parent-child arrangement. It reports gains from an annuity agreement between two parents and one child (2P / 1K), and between one parent and two children (1P / 2K).

--All are assigned male survival probabilities
--Ages are 30 for the child(ren) and 55 for the parent(s).
--KS assume equal consumption by all family members but allow child endowments to vary.

In the 2P / 1K case, if the child's initial wealth is $35,000 and the parents' is $20,000, the equal consumption and bequest sharing arrangement is equivalent to a 32% increase in wealth for each parent and a 10.6% increase for the child.
→ Note that each family member is assumed selfish. Gains if not--

For the parents, this arrangement captures 71.2% of the utility gain from a fair & complete annuities market. For the child, it captures 45.4%.

For a 1P / 2K family, if each child contributes $35,000, the wealth gain to parents is 31.5% of the full annuities case, and for kids it's 14.6%. This implies similar gains for the parent and greater gains for the child than in the 2P / 1K case.

Note that differences in the table result from differences in the child's contribution and differences in the survival probabilities of the family members.

→ Clearly, the 2P / 1K family will consume its resources more quickly than the 1P / 2K family will.

Are marriages between more or less similarly-aged spouses the most efficient?

Suppose we have two women, one old and one young, and two men, one old and one young. For efficiency, are we better off pairing young and old or similarly aged couples?

What's the trade-off? Let's consider 20 & 90 year olds. If we marry the 90yo's together and the 20yo's together, then there is a high risk that both 90yo's will die in the near future, and the resources they've failed to consume will be of no benefit to the 20yo's in this case.

On the other hand, pairing the 20yo's with the 90yo's involves greater risk to one partner. This greater risk has a utility cost, and the cost may offset the gain from the increase in expected resources in a mixed marriage.

KS consider potential efficiency gains from mixed marriages (over same-age marriages) when the old spouse is 55 and the young is 30, with common gammas of .75. They find that weights of 1.7 on the old person's utility in determining spouses' consumption paths yield utility levels for young and old that exceed same-age marriage utilities.

The point is that mixed-age marriages can yield efficiency gains, but only when very skewed consumption arrangements can be made while the older spouse is still living. If this is impossible to negotiate, or if consumption is joint (eg housing), then equal consumption, same-age marriages will yield greater efficiency.

Of course, this assumes common initial endowments for all. If the young spouse enters the marriage with considerably greater assets, this may make up the difference in terms of expected future consumption available, and mixed marriages may then Pareto dominate same-age marriages.

Enforcement of Family Contracts and Altruism

Without altruism, can family members credibly commit to the bequest exchange and optimal consumption paths for each member required by the optimal risk-sharing contract?

KS tell us that the answer is yes. They first consider the case of the common survival probability, equal consumption contract between married partners. If each spouse *controls her or his own wealth* and consumes at the same rate as her or his spouse, then each individual will prefer continuing in the contract at each point in time.

Similarly, asset control can support parent-child risk-sharing arrangements. Instead of allowing parents to use up all of their resources by old age, leaving children no incentive to support them, children can contribute to parents in each period in exchange for the claim on the parent's bequest.

<u>Digression: Estate planning incentives</u> The US gift and estate tax system imposes marginal tax rates of up to 60% on bequests that exceed the exempt ceiling (formerly $600,000 for a single and 1.2 million for a couple, now up to $1 million for a single and $2 million for a couple & statutorily still climbing.)

Additionally, inter-vivos gifts of up to $11,000 annually per recipient are tax-exempt. There is a clear incentive for wealthy parents to give regular $11,000 gifts to their heirs. An existing empirical puzzle is why most wealthy parents fail to take advantage of this tax break.

KS hypothesize that the parent-child risk-sharing scenario explains some of this behavior. Parents hold assets until the end of life to give non-altruistic children the incentive to support or care for them.

See UW's own Karl Scholz, along with others like Poterba and McGarry, for interesting references on estate planning behavior and tax incentives.

[returning to contract enforcement] We now prove the assertion that, with the possibility of transfers and maintenance of own wealth, families can support such risk-sharing contracts.

We begin with equation (1.10). Given initial *t-1* endowments, W_{t-1}^{H} and W_{t-1}^{S}, family members choose a relative utility weight for S of θ^{*} such that the contracted consumption path

$$[C_{t-1}^{H}(\theta^{*}),C_{t-1}^{S}(\theta^{*}),C_{t}^{H}(\theta^{*}),C_{t}^{S}(\theta^{*}),...,C_{D}^{H}(\theta^{*}),C_{D}^{H}(\theta^{*})]$$

is in the core (so no party to the contract will opt out of the contract in favor of optimal own consumption) at time *t-1*.

The time t consumption plan,

$$[C_{t}^{H}(\theta^{*}),C_{t}^{S}(\theta^{*}),...,C_{D}^{H}(\theta^{*}),C_{D}^{H}(\theta^{*})],$$

is the period t Pareto efficient contract that corresponds to the initially chosen θ^{*}. This plan must be in the core for some set of period *t* endowments for family members S_t, which are W_t^H and W_t^S satisfying $W_t^H + W_t^S = W_t$.

To ensure the continuation of the contract (which is preferred by all to the single optima in *t-1*) from time *t-1*, voluntary side payments are made in *t-1* to leave period *t* endowments in the set S_t.

Since the initial contract $[C_{t-1}^H(\theta^*), C_{t-1}^S(\theta^*), C_t^H(\theta^*), C_t^S(\theta^*), ..., C_D^H(\theta^*), C_D^H(\theta^*)]$ was in the core, each family member has an incentive to make side payments to support the continuation of the contract.

A couple of final concerns about commitment: A spouse may gain if she can covertly promise her bequest to a third party in exchange for some similar commitment of bequest transfer if her spouse does not learn of the arrangement. Additionally, cheating may occur because spouses credibly commit to leave bequests to each other, but then covertly overconsume, decreasing the value of the bequest in any given period.

KS argue that these cheating behaviors are les likely to be commitment problems for spouses, but may be serious concerns for relatives or friends who are physically separated.

They investigate the solution to the Nash-bargained noncooperative version of their risk-sharing model, in which family members credibly exchange bequests but cheat by overconsuming.

KS find that the dollar-equivalent utility gain for two 55 year olds from this Nash consumption-cheating partnership is 19.9, 22.2 and 23.5%, for gammas of .75, 1.25 and 1.75, respectively. While the gains from risk-sharing in the consumption cheating case are considerably lower, they are still meaningful relative to utility gains with full annuities.

Altruism Alternatively, equation (1.10) can be taken as the family optimization problem when all members are altruistic and all agree on the utility weights of each family member in the optimization problem. Clearly, risk-sharing can be supported in the altruistic case. Utility gains with risk sharing over optimal individual behavior can't be calculated here, since optimal individual behavior coincides with risk sharing.