

GAME THEORY

First Dose: Simultaneous Moves Games ¹

Game Theory is ideal for analyzing *Multi-Person Decision Problems*. We will use it to study strategic interactions among firms.

- A. What is a game?** A game is a situation where the players or participants' payoffs depend both on own actions as well as on rival's actions. Hence, players' optimal behavior depends on opponents' behavior. That is what we call *strategic interdependence*. My optimal behavior depends on what you do in the game. When players move simultaneously, they would like to form a belief (prediction) about what the opponent will do.

Example of strategic interdependence: The profitability of Airbus' Super Jumbo depends on Boeing's decision to introduce a Super Jumbo as well. If Boeing enters the market, both firms will probably lose money, as usually happens with aircrafts models that have a close substitute (eg DC10 vs L1011).

Four elements are needed to describe a game.

1. players
2. rules: when each player moves, what actions are possible, what is known to each player at the moment they move...
3. outcomes
4. payoffs as a function of the outcomes.

B. How to Represent a Game

We will represent games using a matrix or a tree. A matrix representation is good enough for static (simultaneous move) games. For dynamic games, when moves are taken one after the other, we will see that trees capture additional details about the game that are essential for us to consider.

Matrix Representation

1. Matching pennies: each player selects one side of a coin, if the coins match player 1 wins (gets one dollar from player 2), if they do not, payer 2 wins (she gets one dollar from player 1). The following matrix represents the relevant information about the game.

¹ We will cover only games in which players know their opponents preferences (namely their payoffs). These are called games of complete information. Information about your opponent's profits might be incomplete. For instance, a firm may not know how efficient the rival is. When buying a house you may have incomplete information about the seller's reservation price. Games of incomplete information bring additional challenges, which we should not worry in this course, but in principle they are analyzed in roughly the same way as games of complete information.

		Player 2	
		Head	Tail
Player 1	Head	1 , -1	-1 , 1
	Tail	-1 , 1	1 , -1

The matrix represents the players, their possible actions, the outcomes and payoffs. On the left of each cell (closer to Payer 1) we report the Player 1's payoffs, in this case 1\$ after HH and TT, and -1\$ after HT or TH. On the right of each cell we report Player 2's payoffs. Remember this convention to avoid mixing up the payoff of the players.

This is a zero-sum game, inherently there is a conflict of interests between the players. What one gains the other loses (payoffs sum zero).

- Meeting places: Two friends go on a trip to NY, they would like to meet but they had no chance of coordinating. Although preferences coincide (there is no conflict between their ranking of the possible outcomes) there is strategic interdependence (optimal behavior depends on what each thinks the other player does).

		Player 2	
		Empire State	Madison Sq. Garden
Player 1	Empire State	1 , 1	0 , 0
	Madison Sq. Garden	0 , 0	1 , 1

Examples of our interest are: firms setting prices, competing in R&D, introducing new products, choosing how much to advertise....

- Boeing-Airbus game: The companies have to decide whether to introduce a Super Jumbo to the market. The main elements of the game are: the players, their strategies and their payoffs.

		Airbus	
		Introduce the new product	Do not
Boeing	Introduce the new product	-50 , -50	80 , 0

	Do not	-20 , 100	0 , 0
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C. EQUILIBRIUM OR SOLUTION CONCEPTS

We need some criterion to form a prediction about the outcome of the game. The most used equilibrium concept is the one called Nash Equilibrium (NE). Lets start with a more basic concept, that of a dominant strategy, which will help us develop the NE.

Prisoners Dilemma

The story is that two individual are interrogated about a crime. If none confesses the police have very little evidence, both would be jailed for one period. If both confess, the police will use the confessions to hold them for 6 periods. If one player confesses, his confession is used in court against the other player sending him behind bars for 9 periods, while the cooperator walks free in exchange for his confession.

		Player 2	
		Don't Confess	Confess
Player 1	Don't Confess	-1 , -1	-9 , 0
	Confess	0 , -9	-6 , -6

Find a prediction for this game's outcome. What criterion have you used for finding it?

We want to predict the outcome of the game. We look for an obvious way of play. At least we would like to rule out those outcomes that are unlikely to happen.

In this example there is a strategy for each player which does worse than the other regardless of the rival's action. Hence we can conclude what each player will not do, which in turn determines the outcome of the game. One of the strategies is dominated by the other, it fares worse than the other no matter what the opponent does.

Dominated Strategy: a strategy is dominated if there exists another strategy that gives higher payoffs no matter what the opponent does.

It is safe to assume that a rational player will not choose a dominated strategy. In this case, we can predict the outcome (C,C) will arise just appealing to rationality. Notice (C,C) is an inferior outcome in comparison to (D,D) (inferior in what sense?). The problem is that there is a negative externality between the players.

Exercise: Cigarette manufacturers often claim that the purpose of advertising is to attract smokers from the other brand, not to attract new smokers or increase the amount each consumer smokes. Let's suppose this claim is correct, and then construct a model of advertising decisions of the firms. To simplify the situation: (a) suppose there are only 2 firms: RJR and Philip Morris, and (b) firms have only 2 options: 'Advertise' and 'Not advertise'.

Let the cost of advertising be 30. Since advertising does not affect industry wide gross profits (pre-advertising expenses), assume that as you advertise your gross profits increase by 40, at the expenses of your competitor. Absent advertising each firm makes 160.

- i. Build the matrix describing the game.
- ii. What outcome do you predict?
- iii. Assuming the claim by the cigarette manufacturers (stated at the beginning of the problem) upon which our model is based is correct, should the companies lobby for or against that prohibits all cigarette advertising?

Notice: So far we only assumed each player is rational to reach a prediction for the game. Compare the use of rationality we made so far to the usual analysis of Competition and Monopoly cases.

Prisoners Dilemma II

	Do not Conf	Confess
Do not Conf	1,-1	-7,0
Confess	0,-9	-6,-6

In this modified prisoners' dilemma game, player 1 gets extra pride (enjoys an additional 2 units) from NOT cooperating with the police. As a result, now there is no dominated strategy for player 1, we cannot any longer claim that a rational player 1 would not play DC.

Can we reach a sensible prediction about the outcome of the game? Can we still get a prediction only appealing to rationality?

We will have to push **rationality** one step further. Before, we said no rational player would choose a dominated strategy. In this case player 2 will not chose DC. Now, beyond assuming Players 1 and 2 are rational, let us assume Player 1 knows Player 2 is rational.

Since player1 knows player 2 is rational she will not expect the latter to play DC, so she can cancel the first column of the matrix. Having done so, player 1 will chose C. The extra round of rationality enabled us to still provide a sharp prediction of the outcome of the game in spite of player 1 not having a dominant strategy.

Exercise:

1,0	1,2	0,1
0,3	0,1	2,0

1. Can we predict the outcome of the game by appealing to rationality?
2. How many levels of rationality did we need?

Exercise: consider 2 firms deciding simultaneously whether to offer rebates. Each firms' profits depend on both rebating decisions:

Firm A \ Firm B	Rebate	No Rebate
Rebate	80 , 60	150 , 10
No Rebate	40 , 70	200 , 40

Find a prediction. What are you appealing to in order to make your prediction?

Is there **always a clear prediction** after we iteratively delete the dominated strategies?

Example:

	L	C	R
T	0,7	7,0	5,3
M	7,0	0,7	5,3
B	3,5	3,5	6,6

To see that no action is dominated notice that each action of either player is a best response to some potential action the rival may take. For instance, against L the best player 1 can do is to play M, lets denote it M br L (for "M is the best response to L"). Since M is a best response to some action of the opponent, it cannot be a dominated strategy (dominated strategies do worse no matter what the opponent does; here there is an action for which M does best, hence is not dominated by anything). At the same time B br R and T br C. Hence Player 1 has no dominated strategies. For player 2: L br T, C br M and R br B. Hence, rationality or elimination of dominated strategies is not going to help us predicting the outcome of this game. We can push rationality further and further but it will not help us rule out any actions. Everything is a **best response** to something. Lets move into a sharper prediction concept, the Nash Equilibrium.

NASH EQUILIBRIUM

Def: a pair of strategies (s_1^*, s_2^*) form a Nash Equilibrium if:

$$s_1^* \text{ br } s_2^* \\ \text{and } s_2^* \text{ br } s_1^* .$$

In words, a Nash equilibrium is a pair of strategies (one for each player), such that each player is playing a best response to the other players' strategies.

No player has an (unilateral) incentive to deviate from a NE. Given your strategy I am doing my best, while given my strategy you are doing you best, nobody has an incentive to deviate. Notice: when we imposed **rationality** we assured that the players were behaving optimally against something. By pushing rationality one more level we imposed that each players behaved optimally against something that is optimal. The **Nash** eqm is a pair of strategies such that everybody is behaving optimally against the NE itself.

Hence, it is a prediction such that: a. each player is **behaving optimally** and b. each player is **correctly predicting** the outcome of the game.

Back to the previous examples, in all the outcomes that are not Nash, somebody was incorrectly predicting the outcome (was behaving optimally against something that will not happen, hence was getting surprised by the actual outcome of the game). Nash equilibrium is a stronger requirement than rationality. It is consistent with the idea of an equilibrium, in which nobody is mistaken or surprised all the time.

Why to use Nash Eqm for making predictions?

1. If the prediction is unique it has to be Nash. If the player will reach the same conclusion, and are behaving optimally...
2. Repeated games.
3. Self enforcing agreement - for example if there is pre-play communication.

Uniqueness

Is there a single Nash Equilibrium in every game? No, check the game Meeting Places.

Existence

Is there always a Nash Equilibrium? Yes, there always exist one.
(In spite of Matching pennies of the Penalty game).