Preliminary - Comments Welcome

Increases in Risk and the Probability of Trade

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1. Introduction

Economic situations in which agents perceive increased “uncertainty” about a variety of relevant economic variables lead, according to contemporaneous observers, to decreases in the volume of trade. Heymann and Leijonhufvud (1995) report that, during the Argentine hyperinflation of the late 80s, many retail shop owners refused to trade and posted signs that indicated they were “ 'Closed for Lack of Prices' ” (page 104). Similarly, J.P. Morgan (1998), describing the real estate situation in the East Asian countries affected by the 1997 crises, reports that among other factors, “the lack of property transactions reflects investor uncertainty.” This note describes a notion of “risk” and a class of models in which, under some conditions, increases in “risk” or “uncertainty” can lead to decreases in the probability of making beneficial exchanges, even if both agents are risk neutral. The main channel is not the effect upon curvature of preferences, but the impact upon each individual's prior of other agents information.

We study the link between “risk” and trade in two simple models of trade with asymmetric information. Both models share a similar structure, but differ in terms of what pieces of information are known to the parties. The *common* structure is that of a bargaining game in which a seller who meets a potential buyer (both risk neutral) announces a price, in terms of money, at which he is willing to sell one unit of an indivisible good. The buyer can accept or reject the offer. If the offer is accepted, the buyer makes a money payment to the seller. Both individuals use their residual money balances to buy a divisible good. The models *differ* in terms of the nature of the information that is unknown to the parties. In the first example, taken from Jones and Manuelli (2000), we study how an uninformed seller responds to an increase in “risk” in the value of the goods that receives in exchange for the indivisible good. This example captures situations in which increases in uncertainty about inflation affect the real value of money. The second example is the traditional “Lemon's problem” (see Akerlof (1970)), in which a seller faces a buyer who has an unknown (to him) valuation of the indivisible good, while the value of money is known to both parties. In this setting we study how changes in the "riskiness" of the buyer's valuation --induced, for example, by changes in tax policy-- influences the probability of trading the indivisible good.

The standard notion of changes in risk used in the economics literature is associated with the ordering of distributions according to the notion of second order stochastic dominance (see, for example, Rothschild and Stiglitz (1970)). Unfortunately, for the models that we are interested in, this notion is too broad, in the sense that following an increase in risk --according to this criterion-- the probability of trade can either increase or decrease. To partially solve this problem, we use the stronger notion of increases in risk, restricting our analysis to random variables that differ only in terms of *location and scale*.[2]

We show, in the first model, that and increase in "location and scale risk" decreases the probability of trade. In the second model our results are mixed. We show that when there are *large gains from trade*, in the sense that the seller's unconditional estimate of the buyer's value exceeds his own, increases in risk decrease the probability of trade, while in the case in which there are *small gains from trade* --the complement-- the opposite is true.

Section 2 describes the model when there is asymmetric information about the value of the divisible good (inflation uncertainty). Section 3 analyzes the case in which there is asymmetric information about the value of the good being traded. Finally, section 4 offer some concluding comments.

2. A Model with Volatile Monetary Policy

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[2] A more general transformation is described in Meyer and Ormiston (1989). The linear transformation --corresponding to changes of scale and location-- was originally studied by Sandmo (1971). For an application of these ideas to a number of economic problems see Meyer (1989), where the connection between location and scale and second order stochastic dominance is discussed. This class of distributions corresponds to what Feller (1970) defines as being of the *same type* (p. 45).

The description of the environment is taken from Jones and Manuelli (2000), where more details can be found. We assume that there are two types of agents: buyers and sellers. Both the buyer and the seller derive utility from two goods: an indivisible good which gives utility v^b to the buyer (if purchased) and v^s to the seller (if not sold), and another which we call general consumption. We assume that utility is additive and given by,

$$u^j (x,c)= v^j x^j + c, \qquad j = b \text{ or } s,$$

where x^j, restricted to be either 0 or 1, represents consumption of the indivisible good and c is the level of general consumption.

We take the valuations of both buyers and sellers as common knowledge, and assume that $v^b>v^s$. This assumption implies that trading the indivisible good is always the efficient outcome.

The buyer has an initial endowment of money m^b. If the seller announces the price p and the buyer purchases one unit of the indivisible good, the buyer's consumption of the general consumption good is $c^b = (m^b-p)/M$, where M is both the realization of the money supply (or the inflation rate) and --by suitable choice of units-- the price of the divisible good. Let the seller's endowment be m^s. Then, if he sells the indivisible good, his level of divisible consumption is $(m^s+p)/M$. Without loss of generality assume that $m^b=\lambda M$, while $m^s=(1-\lambda)M$, where $0\leq\lambda\leq 1$. The indirect utility functions, given that both parties know M and the price p has been announced by the seller, are,

$$u^b (x^b,p,M)= v^b x^b + \lambda - x^b p/M,$$

$$u^s (x^b,p,M)= v^s (1-x^b) +1-\lambda + x^b p/M,$$

where $x^b =1$ corresponds to the buyer consuming the indivisible good. We assume that the buyer knows the realization of M, while the seller does not. The distribution of M is common knowledge. Of course, the seller's announced price must be measurable function of his information and, in this case, a constant independent of M. The buyer's decision rule is summarized by,

$$(2.1) \qquad \begin{cases} \text{buy if} & p \leq Mv^b \\ \text{do not buy if} & p > Mv^b. \end{cases}$$

The seller knows this rule, and chooses p to maximize,

$$(2.2) \qquad V^s(p) = \int \left[\chi_{(p<Mv^b)}[1-\lambda+\frac{p}{M}] + (1-\chi_{(p<Mv^b)})[v^s+1-\lambda] \right] dG$$

where G is the cdf of the random variable M.

At this point, it is useful to discuss the notion of increases in risk. To capture increases in inflation risk, we look at a family of random variables, M_k, constructed from a "base case" denoted M. Let the cdf of M be G, and assume that it has a smooth density, g, strictly positive on the interval, (M_L, M_H), except possibly at the end points. Let $\mu = E(M)$ denote the mean of the random variable M, and σ be its standard deviation. We assume that the second moments of M and 1/M exist and are finite. For any $k > 0$, define the new random variable, M_k, by $M_k \equiv k(M - \mu) + \mu$. Thus, as indicated, this is a standard *change of location and scale* in statistics. Then, $E(M_k) = \mu$ and $E(M_k - \mu)^2 = k^2\sigma^2$. We take k as our index of risk, with increases in k corresponding to increases in risk. Since the transformation from M to M_k is linear, it can be shown, using the single crossing characterization of mean preserving spreads, that for any k' and k with k' > k, the distribution of $M_{k'}$ is a mean preserving spread of the distribution of M_k, that is, $M_{k'}$ is "riskier" than M_k.

Let $G(m; k) = Pr(M_k \leq m) = Pr(M \leq (m - \mu)/k + \mu) = G((m - \mu)/k + \mu;1) = G((m - \mu)/k + \mu)$ and hence, $g(m;k) = 1/k\, g((m - \mu)/k + \mu)$. Let the upper and lower bounds for M_k be denoted $M_j(k)$, j=L,H. The random variable M_k must be positive. Thus, the requirement that $M_L(k) = k(M_L - \mu) + \mu \geq 0$, puts restrictions on how big k can be.

It turns out to be more convenient --using (2.1)-- to formulate the seller's problem, (2.2), as one of choosing the cut-off money supply, M*, such that the buyer buys the object for all realizations of $M \geq M^*$, and does not buy below M*. Given M*, the seller must set the price at

$p^* = M^* v^b$. Thus, (2.2) is equivalent to[3]

$$(2.3) \qquad V^s(M^*) = v^s \int_{M_L(k)}^{M^*} 1\, dG(m;k) + M^* v^b \int_{M^*}^{M_H(k)} \frac{1}{m}\, dG(m;k)$$

subject to $M_L(k) \leq M^* \leq M_H(k)$. To simplify the presentation, let $v^s = (1-\epsilon)v^b$. It is immediate that the first order condition for an interior solution is,

$$(2.4) \qquad \int_{M^*(k)}^{M_H(k)} \frac{1}{m}\, dG(m;k) = \epsilon\, g(M^*(k);k).$$

Given a solution $M^*(k)$, the probability of trade is $\pi(k) = 1 - G(M^*(k),k)$. Equation (2.4) shows why second order stochastic dominance is not very informative. To see this, suppose that the solution to (2.4) satisfies $M^*(k) > \mu$. In this case, it is possible to increase risk --in the second order stochastic dominance sense-- by reallocating mass in the interval $(\mu-(M^*(k)-\mu),\mu+M^*(k))$. This, of course, does not affect either side of (2.4) and leaves both the solution and the probability of trade unchanged! The reason for this is simple: second order stochastic dominance requires that the mass of the distribution must be reallocated to the tails, but it is silent as to the location of the regions that gain or lose mass. The reader can follow a similar strategy to obtain either an increase or a decrease in the probability of trade.

Increases in risk given by increases in k --our risk parameter-- gives definitive results. We summarize our findings in,

Proposition 1: Assume that the solution to (2.4) is interior. Then, the probability of trade, $\pi(k)$, is a decreasing function of k.

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[3] This objective function is not differentiable in the random variable, M. Thus, the results in Meyer and Ormiston (1989) do not apply.

Proof: See the Appendix

In this setting, Proposition 1 shows that increases in the variability of inflation can reduce the volume of trade, even if all agents are risk neutral. The key impact is that it changes, from the point of view of the seller, the expected value of the gain if the good is traded. In fact it increases it. Since the optimal solution is to equate marginal gains and loses, this induces the seller to change the price and, effectively, to reduce the chances of selling the indivisible good.

Our analysis assumed that the solution is interior. This however need not be the case. If v^b is large relative to v^s, the optimal strategy on the part of the seller is to sell with probability one. Even in this case, it is possible that an increase in k will switch the economy from a point in which the probability of trade is one, to another where it is less than one.

So far, we have assumed that the buyer is informed and the seller is not. If both parties share the same information --either both informed or both uninformed-- they trade with probability one (see Jones and Manuelli (2000)) and increases in risk have no effects. If the information pair consists of an informed seller and an uninformed buyer, there is a large number of equilibria, and in the equilibrium that maximizes the probability of trade, increases in risk decrease this probability.[4]

3. The Original Lemon's Problem

In this section we study the original lemon's problem in which the valuations of buyers and sellers are random (see Akerlof (1970), Samuelson (1984), and Myerson (1985)) but the value of money is known to both parties. In this case we show that, depending on the relative valuation of buyers and sellers, an increase in risk --measured, as before, as a change in location and scale--

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[4] In Jones and Manuelli (1998) it is shown that the probability of trade is given by $P[M_L(k) \leq M_k \leq M_L(k)(v^{b+}v^s)/(v^b-v^s)]$, and simple calculations show that this set shrinks as k increases.

can either increase or decrease the probability of trade.

We consider the case of a seller who is uninformed about the buyer's valuation of the indivisible good. The seller's valuation is denoted by v^s, and his beliefs about the distribution of v^b_k are summarized by the cdf $G(v|v^s,k)$ where, as before, k will be an index of risk, with increases in k corresponding to riskier distributions. Let μ^b be the mean of v^b --the "base" random variable. Then, using the notion of change of location and scale, it follows that $v^b_k \equiv k(v^b - \mu^b) + \mu^b$. Thus, for a given k, the distribution of v^b_k is given by $\text{Prob}[v^b_k \leq v|v^s,k] = G[(v - \mu^b)/k + \mu^b|v^s]$ where G is the distribution, conditional on v^s, of the "basic" variable v^b.

We study the case in which a potential buyer knows his valuation, v^b_k. The seller's beliefs about this valuation are given by $G(v|v^s,k)$, and the game is a standard bargaining game: the seller announces a price, p, and the buyer decides whether to accept or reject the offer.[5] Given a price p, the buyer's decision rule is to buy if and only if $p \leq v^b_k$. Given this rule, the seller's payoff from announcing price p is,

$$V^s(p) = v^s + (p - v^s)(1-G(p|v^s,k)).$$

In this setting, $1-G(p|v^s,k)$ is the probability of trade, which we denote $\pi(v^s,k)$. Clearly, the seller will choose a price $p \geq v^s$.

For a given value of v^s --which can be random-- we say that there are *large gains from trade* if $v^s < \mu^b$, while we say that there are *small gains from trade* if $v^s > \mu^b$. We can now state our main result,

Proposition 2: Let v^s be fixed, and assume that the bargaining game has an interior solution. Then an increase in risk in the sense of increasing scale:

i) Decreases the probability of trade, $\pi(v^s,k)$, if there are large gains from trade ($v^s < \mu^b$).

ii) Increases the probability of trade, $\pi(v^s,k)$, if there are small gains from trade ($v^s > \mu^b$).

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[5] This bargaining game implements the outcome of the mechanism that maximizes the seller's payoff. See Myerson (1985) and Samuelson (1984).

Proof: See Appendix.

First, note that the results can be generalized to the unconditional probability of trade if v^s is a random variable, provided that the support of its distribution lies completely on either side of μ^b. That is, if $\hat{\pi}(k) \equiv \int \pi(v^s,k)F(dv^s)$, for some distribution F, $\hat{\pi}(k)$ decreases (increases) in k if $supp(F) \subset [-\infty,\mu^b]$ ($supp(F) \subset [\mu^b,\infty]$). Second, it is not possible to say much without further assumptions, in the more general case in which the support of v^s is not restricted to lis on either side of μ^b. Moreover, it seems difficult to generalize the results to the case in which the increase in “riskiness” affects the seller's valuation. If the risk indices of v^b and v^s are k_1 and k_2, respectively, the unconditional probability of trade is $\hat{\pi}(k_1,k_2) \equiv \int \pi(v^s,k_1)dF(v^s,k_2)$. It follows that the effect of k_1 is just the integral of the cases discussed above. If $\pi(v^s,k_1)$ was a concave function of v^s the any mean preserving spread on v^s would reduce the probability of trade. Without stronger assumptions about the form of G it is not possible to give a determinate answer.

Are there situations in which policy changes can induce changes in risk according to our definition? It turns out that redistributive tax and spending policies fit our description perfectly. Suppose that a government decides to tax the return to buyers, v^b, at the rate τ and to give them a transfer, t. Balanced budget requires --with a large number of buyers-- $t = \tau\mu$. Thus, the after tax valuation of a buyer is $v^b(1-\tau) + \tau\mu = (1-\tau)(v^b - \mu) + \mu$. Thus decreases in the tax rate --and the degree of redistribution-- increase the riskiness of the valuations and --depending on the case-- can increase or decrease trade.

4. Conclusions

In this note we studied the effects of increases in risk on the probability of trading an indivisible good in settings in which agents are risk neutral, but asymmetrically informed. We show that, using a *change of location and scale* approach to defining risk, the effects of an increase in risk depend on the nature of the asymmetric information.

If the seller is uncertain is about the value of an outside good --for example, due to variable monetary shocks-- increases in risk decrease the probability of trade.

If, on the other hand, the source of uncertainty is the individual valuation of the indivisible good, increases in risk --which could be policy induced, as in the case of tax policy-- have effects that depend on the size of the unconditional gains from trade. If there are strong gains from trade, risk increases decrease the probability of trade, while if there are weak gains from trade it goes the other way. We hope these results will be useful in understanding trading volume in goods, as well as in markets --like the market for emerging country debts-- that are characterized both by high volatility and asymmetric information. Moreover, as our lemon's example shows, even redistributive tax policy can have an impact of the volume of trade.

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Appendix

Proof of Proposition 1: Assume that the solution is interior. In that case, the relevant first order condition is (2.4), and the second order condition --which we assume holds-- is:

$$\text{(A.1)} \qquad \frac{-\epsilon\, g'\!\left(\frac{M^*(k)-\mu}{k}+\mu\right)}{k} \le \frac{g\!\left(\frac{M^*(k)-\mu}{k}+\mu\right)}{M^*(k)}$$

Recalling the definition of g(;k) in terms of g, (2.4) can be written as:

$$\text{(A.2)} \qquad \int_{M^*(k)}^{M_H(k)} \frac{1}{m}\, g\!\left(\frac{m-\mu}{k}+\mu\right) dm = \epsilon\, g\!\left(\frac{M^*(k)-\mu}{k}+\mu\right).$$

Differentiating this last condition with respect to k gives:

$$\text{(A.3)} \qquad \frac{dM_H(k)}{dk}\,\frac{1}{M_H(k)}\, g\!\left(\frac{M_H(k-\mu)}{k}+\mu\right) - \frac{dM^*(k)}{dk}\,\frac{1}{M^*(k)}\, g\!\left(\frac{M^*(k)-\mu}{k}+\mu\right)$$

$$+\int_{M^*(k)}^{M_H(k)} \frac{1}{m}\, g'\!\left(\frac{m-\mu}{k}+\mu\right)\left(-\frac{(m-\mu)}{k^2}\right) dm = \epsilon\, g\!\left(\frac{M^*(k)-\mu}{k}+\mu\right)\left[(M^*(k)-\mu)\left(\frac{-1}{k^2}\right)+\left(\frac{1}{k}\right)\frac{dM^*(k)}{dk}\right].$$

Recalling that $(M_H(k) - \mu)/k + \mu = M_H$ for all k, and using some simple algebra, we get:

$$\text{(A.4)} \quad \left[\frac{dM^*(k)}{dk} - \frac{(M^*-\mu)}{k}\right]\left[\frac{1}{k}\epsilon g'^{\,*} + \frac{1}{M^*}\, g^*\right] = \frac{M_H-\mu}{(M_H-\mu)k+\mu}\, g\!\left(\frac{M_H-\mu}{k}+\mu\right) - \frac{M^*-\mu}{M^*}\,\frac{g^*}{k}$$

$$-\frac{1}{k^2}\int_{M^*(k)}^{M_H(k)} \frac{m-\mu}{m}\, g'\!\left(\frac{m-\mu}{k}+\mu\right) dm.$$

where we have used the simplifying notation g* and g'* to denote the value of g((m-μ)/k+μ) and its derivative at m = M*. Next, integrate by parts the last term on the right hand side of (A.4) to get,

$$\text{(A.5)}\quad \left[\frac{dM^*(k)}{dk} - \frac{(M^*-\mu)}{k}\right]\left[\frac{1}{k}\epsilon g'^{\,*} + \frac{1}{M^*}g^*\right] = \frac{1}{k}\int_{M^*(k)}^{M_H(k)} \frac{\mu}{m^2}\, g\!\left(\frac{m-\mu}{k}+\mu\right) dm.$$

Given the second order condition, (A.1), it follows that this proves that dM*(k)/dk - (M*-μ)/k ≥ 0, with strict inequality if the second order condition holds as a strict inequality. Al that is left is to show that this implies that the probability of trade is decreasing. It suffices to show that our results imply that the probability of not trade is increasing. This is given by,

$$1 - \pi(k) \equiv \Pr(\text{NoTrade}\,|\,k) = \Pr(M_k \leq M^*(k)) = G\!\left(\frac{M^*(k)-\mu}{k}+\mu\right)$$

Differentiating the above expression one gets that the probability of no trade is increasing in k if and only if

$$\frac{g\!\left(\frac{M^*(k)-\mu}{k}+\mu\right)}{k}\times\left[\frac{dM^*(k)}{dk} - \frac{M^*(k)-\mu}{k}\right] > 0,$$

which is implied by (A.5).

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Proof of Proposition 2: Since we assume that the solution is interior, and the second order condition is satisfied, it follows that,

$$\text{(A.6)}\qquad 1 - G\!\left(\frac{p-\mu^b}{k}+\mu^b\,|\,v^s\right) = \frac{p-v^s}{k}\,g\!\left(\frac{p-\mu^b}{k}+\mu^b\,|\,v^s\right)$$

$$(A.7) \qquad 2g\left(\frac{p-\mu^b}{k}+\mu^b|v^s\right) + \frac{p-v^s}{k}g_1\left(\frac{p-\mu^b}{k}+\mu^b|v^s\right) \geq 0$$

where, g_1 indicates the partial derivative with respect to the first argument. Differentiating (A.6) with respect to k, and after some simple manipulations, we get,

$$(A.8) \quad \left(\frac{dp(k)}{dk}-\frac{p-\mu^b}{k}\right)\left[2g\left(\frac{p-\mu^b}{k}+\mu^b|v^s\right) + \frac{p-v^s}{k}g_1\left(\frac{p-\mu^b}{k}+\mu^b|v^s\right)\right] = \frac{g\left(\frac{p-\mu^b}{k}+\mu^b|v^s\right)}{k}(\mu^b-v^s).$$

It follows that, $dp(k)/dk - (p-\mu^b)/k \gtrless 0$ if and only if $\mu^b - v^s \gtrless 0$. Finally, it follows that

$$\frac{\partial\pi(v^s,k)}{\partial k} = -\frac{g\left(\frac{p-\mu^b}{k}+\mu^b|v^s\right)}{k}\left[\frac{dp(k)}{dk}-\frac{p-\mu^b}{k}\right].$$

Thus, this proves the proposition.

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