ENDOGENOUS POLICY CHOICE: THE CASE OF POLLUTION AND GROWTH

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Abstract

What determines the relationship between pollution and growth? Are the forces that explain the behavior over time of these quantities potentially useful to understand more generally the relationship between policies and growth? In this paper we make a first attempt to analyze the equilibrium behavior of two quantities --the level of pollution and the level of income-- in a setting in which societies choose, via voting, how much to regulate pollution. Our major finding is that, consistent with the evidence, the relationship between pollution and growth need not be monotone, and that the precise equilibrium nature of the relationship between the two variables depends on whether individuals vote over pollution taxes or directly restrict the choice of technology. Moreover, our analysis of the pollution problem suggests that, more generally, endogenous policy choices should be taken seriously as potential sources of heterogeneity when studying cross country differences in economic performance.

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1. **Introduction**

What determines the relationship between pollution and growth? Are the forces that explain the behavior over time of these quantities potentially useful to understand more generally the relationship between policies and growth? In this paper we make a first attempt to analyze the equilibrium behavior of two quantities --the level of pollution and the level of income-- in a setting in which societies choose, via voting, how much to regulate pollution. Our major finding is that, consistent with the evidence, the relationship between pollution and growth need not be monotone, and that the precise equilibrium nature of the relationship between the two variables depends on whether individuals vote over pollution taxes or directly restrict the choice of technology. Moreover, our analysis of the pollution problem suggests that, more generally, endogenous policy choices should be taken seriously as potential sources of heterogeneity when studying cross country differences in economic performance.

What is the evidence on pollution and growth? Recent empirical work suggests that the output of many pollutants is controlled as income grows (for a discussion of the direct evidence on pollution controls and the level of pollution and international movements of pollution in response to these controls, see Grossman and Krueger (1995), Low and Yeats (1992), Lucas, Wheeler and Hettige (1992) and Levinson (1994)). Although the evidence is sketchy at this time (see Baumol and Oates (1979) for a discussion on this), it supports the view that, after passing through a period of high and increasing pollution, eventually, society takes actions to control these external effects. Several patterns emerge from the data. For some pollutants, the relationship between income and pollution follows an inverted U, while for others a 'sideways mirrored' S (i.e., an inverted U followed by a sustained increase) is seen (see World Bank (1993) and Grossman and Krueger (1995)).

Is it possible that the nature of the institutions that are used to affect pollution determine whether one or the other paths are observed? To study this question, we develop a model of the joint determination of the rate of development of the economy through market interactions and the extent of pollution regulation through collective decision making. In the model, different technologies are associated with different amounts of pollution per unit of output. We describe the time paths of income and pollution implied by majority voting in two different settings: voting over (proportional) pollution taxes, and voting over direct regulation of technology, or choice of "minimum standards." We show that, when voting is over tax rates, the time path of pollution will display a "sideways mirrored" S. If, instead, individuals vote directly over the "dirtiest" allowable technology, the relationship between income and growth has an inverted U shape. Both these patterns fit well with the empirical evidence in Grossman and Krueger (1995). Finally, we study a class of optimal allocations. In this case, we show that the relationship between pollution and income is monotone increasing, but it becomes flat --actually pollution is bounded-- as income grows.

In the model, pollution is “internal” in the sense that it affects only the individuals who vote. A straightforward extension of the model shows that if the pollution is “external” to the political jurisdiction being modeled, the level of the pollutant will grow without bound. This suggests that some sort of political mechanism across countries is necessary to control these ‘global’ pollutants, but that once in place, this control will occur. Here one of the implications of the analysis is that heterogeneity in income levels across countries that are not fully integrated (capital is not mobile) will necessarily imply disagreement about the optimal regulation of global pollutants. Further, the analysis shows that low capital countries would prefer less regulation.

Although there is a large literature on the relationship between economic activity and the environment (see Baumol and Oates (1988) for a textbook treatment of the relevant static theory), there are few papers that explicitly model the equilibrium relationship between growth and environmental degradation. In the endogenous growth literature the recent work by Fisher and van Marrejwijk (1994) and Mohtadi (1994) is closest to our own. Both of these papers model the environment as an additional factor of production and concentrate on the impact of different criteria that a planner might choose to "allocate" this environmental input between households and firms. The key difference is that in our formulation, public policy is endogenously determined. In John, Pecchenino, Schimmelpfennig and Schreft (1994), the emphasis is on the analysis of the comparative statics properties of steady states of exogenous growth models in which the environment gets degraded by the act of consuming and resources can be used to improve it. Stokey (1995) studies economies in which a central planner chooses the optimal allocation when individuals are infinitely lived. She shows conditions for balanced growth and discusses the relative merits of tax and voucher schemes over direct regulation. She does not address the issue of implementation; that is, she does not study whether the optimal policies she describes can be supported as the equilibrium policies of some collective decision making mechanism.

Finally, this paper also contributes to the literature, being developed in recent years, on both the large differences in levels of income per capita across countries and in their growth rates. Our key source of cross country heterogeneity is policy endogeneity (see also Alesina and Rodrik (1994), Persson and Tabellini (1994), Glomm and Ravikumar (1992 and 1994) and Krusell, Quadrini and Rios-Rull (1997). We show that the equilibrium relationship between growth rates of capital and income levels displays an inverted U shape, when individuals vote over taxes. This theoretical implication, which is consistent with the findings of Easterly (1994) and Dongchul Cho (1995), provides an alternative interpretation to the standard view that such a relationship must be generated by models of nonconvexities and/or multiple equilibria. Overall the message from this research is clear: one the one hand our positive results make us optimistic about the ability of the model to explain stylized facts about pollution and growth. However, the sensitivity of the outcome to the details of institutions, suggests the effects of introducing endogenous policy choice are hard to predict in environments, like the one in this paper, in which income matters.

The paper is organized as follows. Section 2 lays out the basic model, and studies the equilibrium in which individuals vote over pollution taxes. Section 3 more fully describes the dynamic behavior of such an equilibrium. Section 4 studies the case in which individuals vote over minimum quality or environmental standards. Section 5 presents a class of optimal allocations. Section 6 suggests some possible extensions and modifications, and section 7 offers some concluding comments.

2. **The Basic Model with Pollution Taxes**

We consider an economy that has available a wide range of productive techniques that differ according to both their costs of production and their environmental effects. The choice, by firms, among techniques is influenced not only by their price but also by restrictions on use imposed by the government. To model this aspect of firm decision making, we adopt the assumption that the government sets technique-specific taxes with the aim of controlling pollution (in section 4, we consider the case in which the government institutes "minimum quality standards"). Interpreted broadly, however, these "environmental taxes" should be viewed as incorporating a wide array of regulations and other forms of control that result in an increase in the effective price of environmentally pernicious techniques.

We will use a growth model with two period lived overlapping generations. An implication of this is that voting decisions are made by individuals with short horizons, thus, the generational structure and the natural voting scheme that we study (more on this later) allow us to capture a realistic feature of actual economies. To guarantee that, in the absence of pollution, the equilibrium displays growth and is optimal, we study a two sector model (see Rebelo (1991) and Jones and Manuelli (1992)) with (potentially infinitely many) differentiated capital stocks. We assume that all forms or vintages of capital are perfect substitutes in production while they differ both in terms of cost of production and in how much pollution their use generates. In order to make the problem interesting we consider the case in which environmentally cleaner capital goods are more costly to produce.

The nature of pollution that we model is local in both the physical and temporal dimensions. Since we analyze a model in which voter's have full control over all polluting activities, it is probably best to interpret our results as applying to pollution that is local in nature (i.e., restricted to the voting region). In the temporal sense we first consider the case in which the level of pollution produced at time t affects only the members of generation t-1 without any direct generational spillovers. Although there are no direct effects, policy decisions on the part of one generation affect the well being of future generations through their impact on capital accumulation decisions. This second assumption is not critical and can be relaxed.

We assume that successive generations play a voting game in taxes on polluting activities. At time t, individuals born at t vote on pollution taxes that will be in effect during period t+1. Their realized utility, $V_t(\tau_1,\tau_2,...)$, is the voter's utility function over consumption and pollution paths evaluated at a competitive equilibrium given the sequence of taxes. The function V_t could (potentially) depend on the entire sequence of taxes chosen. We will examine subgame perfect equilibria of this voting game. As is well known, there are typically many equilibria in multiagent voting games. We consider a representative agent model, and simply select the equilibrium that corresponds to the best policy from the perspective of the representative agent. Even with this restriction, in general, very little can be said about the equilibrium outcomes of voting games of this type. Because of this, we will simplify the problem by going to a special OLG formulation. In essence, we will put enough restrictions on preferences and labor endowments so that V_t depends only on current and past tax rates. Moreover, for our specification, it will be true that the only way that previously chosen taxes matter for an individual of generation t is through their impact on the capital stock at the beginning of period t. In section 6 we discuss extensions to settings with heterogeneous agents.

To begin, we will describe the economic environment and the competitive equilibrium that results from an arbitrary sequence of taxes. This determines the form of V_t. We will then analyze the voting game given this indirect utility function over tax sequences.

Preferences

Consumer preferences are given by,

$$\log c_t^t + \beta \log c_{t+1}^t - u(s_{t+1}),$$

where c_j^i is consumption in period j of the individual born at time i and s_i is pollution in period i. We assume that individuals are endowed with one unit of labor in their first period of life and none in the second. Their endowments of all other goods are zero. Thus, individuals derive disutility from the level of pollution in their second period of life. Given the assumptions about voting, it is possible without changing the basic results to make preferences depend on the level of pollution when young if the separability assumption is maintained. We chose this simpler framework to simplify the presentation. We assume that u is increasing, convex and C^2, and that $u'(x)x$ is strictly increasing and C^1. These preferences imply that saving is not responsive to interest rates. This feature greatly simplifies our voting game, but we suspect it is not essential for the results. In particular, it implies that, as noted above, only current and past taxes will enter the voter's indirect utility function.

Consumers maximize utility by choosing consumption in each period of their lives taking after tax prices as given and beyond their control. It is standard to show that optimal decisions are completely summarized by a saving function which in this case is given by,

(2.1) $$b_t = (\beta/(1+\beta))w_t,$$

where w_t is the wage rate at time t and b_t is the level of savings. First period consumption at time t, c_t^t, is $(1/(1+\beta))w_t$. If we denote the rate of interest between period t and t+1 by $1+i_{t+1}$, second period consumption of an individual born at time t is $c_{t+1}^t=(1+i_{t+1})(\beta/(1+\beta))w_t$. It follows that the indirect utility function is,

$$U^t = \text{constant} + (1+\beta)\log w_t + \beta \log 1+i_{t+1} - u(s_{t+1}).$$

The equilibrium determination of w_t, i_{t+1}, and s_{t+1} depends on the details of the technology and is the focus of the remainder of this section.

Technology

On the production side we assume that there is one consumption good produced at each date and a (potentially infinite) number of different capital goods. Throughout we assume that there is a large number of firms in each sector that behave competitively. The consumption good is produced using a composite capital good, k_{ct}, and labor, n_t, according to the following production function,

$$c_t = Bk_{ct}^{\alpha}n_t^{1-\alpha},$$

where $k_{ct} = \int_0^\infty k_{ct}(z)dz$, and $k_{ct}(z)$ is the amount of capital of type z that is used in production of the consumption good. This formulation implies that the different capital goods are perfect substitutes in production and, hence, quality choice will be driven completely by cost considerations. From a formal point of view, we allow firms to choose either finitely or infinitely many qualities of capital by modeling the choice of firms as a measure over z's. Due to the nature of the cost functions that we will use below, it will follow that in equilibrium, only one quality level will be chosen in any period.

To simplify, we consider a situation in which capital is fully malleable. At the beginning of the period the existing stock of capital, k_t, can be used to produce either capital goods of type z to be used in the consumption sector or to produce "generic" capital that will be used in the investment sector. Formally, the investment side of the economy can be summarized by

$$(2.2)\quad \begin{cases} k_t = \int_0^\infty m(z)k_{ct}(z)dz + x_t \\ k_{t+1} = (1 - \delta)[k_{ct} + x_t] + A\, x_t. \end{cases}$$

One interpretation of this formulation is that, at the beginning of time t, capital is split into (basic) consumption capital and investment capital. We assume that the production of new capital is linear in the capital goods allocated to the investment sector, x--output of the investment sector is Ax_t. In this formulation, m(z) is the amount of general capital (i.e., k) that is necessary to produce one unit of type z capital. We assume that m(z) is increasing in z, convex and that m(0)=1. One implication of this formulation is that type z=0 consumption capital and investment capital can be exchanged one for one. The cost m(z) should be interpreted as a flow, that is, in every period if the firm wants to operate capital of type z, it must spend the "extra" m(z)-1 units of capital necessary to convert general capital into type z capital. The total amount of beginning of the period capital that is devoted to producing "consumption sector" capital is given by the integral $\int_0^\infty m(z)k_{ct}(z)dz$. Note that if we restricted quality choices to only be z=0, we would have a standard two sector growth model.

At the end of the period, both the stock of capital that was used in the investment sector --x_t-- and that used in the consumption sector--k_{ct}--are depreciated at rate δ. Thus, capital at time t + 1 is $(1 - \delta)[k_{ct} + x_t] + Ax_t$. The assumption that all qualities have just a "flow" component substantially simplifies the algebra for the standard reasons: we do not have to keep track of all the distribution of old vintages in calculating the equilibrium. Moreover, since our period corresponds to a generation, δ is a large number. For δ = 1, there is no distinction between the flow interpretation and the case in which all the capital maintains its original vintage.[2]

Firms in the investment sector use general capital to produce capital available for use in the following period. These firms can use only the zero technology. The production function is linear: it uses type zero capital today to produce new (type zero) investment goods available tomorrow.

We assume that pollution --or smoke, which we denote by s-- is associated with the use of capital in the consumption sector. More precisely, the total amount of pollution is given by,

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[2] An alternative interpretation of this technology is that if a firm wants to use k_c units of capital operated at level z, then it has first to rent $m(z)k_c$ general capital, use $(m(z) - 1)\, k_c$ producing type z capital, and then --at the end of the period-- it returns to the market just k_c units. The difference --$(m(z)-1)k_c$-- was used up in the conversion process. This shows that operating capital at cleanliness level z is equivalent to allowing effective depreciation vary with z. Simple calculations show that the effective depreciation rate, δ(z), is given by $[m(z)-1 + \delta]/m(z)$.

(2.3) $s_t = \int_0^\infty \varphi(z)k_{ct}(z)dz,$

where $\varphi(z)$ is the amount of (flow) smoke generated by one unit of capital in the consumption sector operated at cleanliness level z. We assume that $\varphi(z)$ is convex and decreasing so that higher index goods (higher quality) are less polluting, but there is diminishing marginal reduction in the change of pollution per unit of capital.

There are several features of our technology that are worth emphasizing: First, it assumes that there are no "dirty" or "clean" goods per se; instead there are better or worse ways of producing goods. Thus, our emphasis is on the choice of technology and not on the composition of goods. Second, by being explicit about the role of technology in generating smoke, we move one step ahead of the "smoke in the production function" approach. This is important as the reader can check that our model does not easily resemble any reduced form in which smoke enters the production technology. Third, (2.3) implicitly assumes that the capital goods producing industry does not contribute to pollution. This is a rather extreme assumption and can be relaxed. However, in section 6 we discuss why some version of this assumption is necessary. Fourth, our assumption of a decreasing $\varphi(z)$, coupled with an increasing cost of producing "clean" capital goods, m(z), implies that a cleaner technology can be used only at the expense of fewer capital goods produced. Finally, our smoke variable depends on the amount produced today, that is, it is a flow variable (past smoke does not matter). Thus, decisions made by previous generations do not directly affect the amount of smoke breathed by generation t. This is restrictive and we discuss an extension to the case in which pollution is a cumulative or a capital-like variable in section 6.

We assume that a firm in the consumption sector that buys capital of type z has to pay a tax rate of $\tau_t(z)$. In order to fully capture the idea of a tax --as opposed to a highly non-linear scheme that is analogous to a direct quality choice-- we restrict the shape of the function $\tau_t(z)$ to be proportional to the amount of pollution produced, $\tau_t(z) = \tau_t\varphi(z)$. This implies that the representative firm producing consumption solves the following problem,

$$\max B(\int_0^\infty k_{ct}(z)dz)^\alpha n_t^{1-\alpha} - w_t n_t - \int_0^\infty (1 + \tau_t\varphi(z))r_{kt}(z)k_{ct}(z)dz,$$

where w_t is the wage rate, and $r_{kt}(z)$ is the (rental) market price of capital of type z. The firms producing new investment goods solve,

$$\max p_{kt}Ax_t - r_{kt}(0)x_t,$$

where p_{kt} is the price of new capital available for production at time t+1. To simplify the notation we use $r_{kt}(0)=r_{It}$. Finally, the firms that transform basic capital into its different types solve,

$$\max \int_0^\infty r_{kt}(z)k_{ct}(z)dz + r_{It}x_t - r_{It}k_t,$$

subject to,

$$\int_0^\infty m(z)k_{ct}(z)dz + x_t \leq k_{t.}$$

Finally, to keep the presentation simple we assume that the tax revenues (from the pollution taxes) are used to provide some good that enters separably in the utility function. We assume that individuals ignore the connection between tax revenues and this good. (Alternatively, we can assume that tax revenue is wasted.) In section 6 we discuss how the results would change under alternative uses of the tax revenue.

Equilibrium

For a given tax sequence $\{\tau_t\}$, the notion of equilibrium that we use is price taking equilibria. Formally we define an equilibrium by,

Definition: An equilibrium is a set of sequences [c_t^{t-1}, c_t^t, k_{t+1}, z_t, s_t, $k_{ct}(z)$, x_t, p_{kt}, $r_{kt}(z)$, r_{It}, $1+i_{t+1}$, τ_t , $t \geq 0$], such that,

(i) For all $t \geq 0$, c_t^t, c_{t+1}^t, solve the consumer's maximization problem,

(ii) c_0^{-1} is equal to $p_{k0}k_0$,

(iii) firms maximize profits,

(iv) $b_t = p_{kt}k_{t+1}$ (market clearing),

(v) $[p_{kt+1}(1-\delta) + r_{It+1}]/p_{kt} = 1+i_{t+1}$,

(vi) the tax τ_t is chosen, using a majority voting mechanism, by members of the generation born at t-1.

This definition does not require further elaboration except to point out that when making consumption-saving decisions individuals take as given all taxes, as well as the equilibrium level of pollution. Note, however, that separability of the utility derived from consumption and the disutility associated with pollution guarantees that consumption-saving decisions are made independently of the level of pollution. Thus, there is some myopia that is built into our specification of preferences. It seems reasonable to assume that aggregate pollution should not be a large determinant of individual consumption and this is precisely what our formulation captures.

Since the equilibrium outcome for a given tax sequence $\{\tau_t\}$ is rather standard we relegate (most of) its analysis to Appendix A. However, there is one aspect --the optimal choice of cleanliness level, z, as a function of the tax rate-- that is both nonstandard and important for our results. Even though it is possible to conduct the analysis at a fairly general level, the basic intuition can be fully captured in a simple setting in which we specialize the functions describing the flow cost of producing high quality goods, m(z), as well as the pollution per unit of capital z, $\varphi(z)$. Specifically, we assume,

(2.4a) $$m(z) = (1 + z)^{\theta},\ \theta > 0$$

and,

(2.4b) $$\varphi(z) = D(1 + z)^{-\nu},\ \nu > 0\ ,\ \ \nu > \theta \geq 1.$$

From the maximization problem of the firms in the consumption sector, it follows that they will choose the quality z for which the "full operating cost" is lowest (recall that different types of capital are perfect substitutes in production). Thus, any cost minimizing firm will pick z to solve:

(P1) $$\text{minimize}_z\ (1 + \tau_t\varphi(z))r_{kt}(z) = (1 + \tau_t\varphi(z))m(z)r_{It}.$$

Since r_{It} is independent of z, this is equivalent to

(P1′) $$\min_z\ (1 + \tau_t\varphi(z))m(z).$$

We summarize the results of this maximization in the following proposition.

Proposition 1. The solution to problem (P.1') is given by $z(\tau) = 0$ for $\tau \leq \tau_L$, and

$z(\tau) = \left(\dfrac{(\nu - \theta)\tau D}{\theta}\right)^{1/\nu} - 1$, for $\tau \geq \tau_L$, where τ_L is given by $\tau_L = \theta/[(\nu - \theta)D]$.

Proof: See Appendix B.

Proposition 1 shows that for tax levels that fall short of τ_L there will be no impact on the equilibrium level of the quality of capital good chosen. Hence, our setting implies a "kink" in the optimal choice of quality: for relatively low tax rates there is no quality upgrade (the optimal z is zero); however, as taxes increase --and they make operation of the dirtier forms of capital more expensive-- firms choose to upgrade their capital goods.

What are the factors that determine generation t's preferences over different τ_{t+1}? Recall that members of generation t vote on the pollution taxes that will be in place at time t+1. In doing so, voters are rational. By this, we mean that they understand how taxes will affect all variables of the economy (i.e., they know the equilibrium relationship between taxes, prices and quantities) and, of course, how this affects their individual welfare. Since we have assumed away all heterogeneity within a generation (see section 6 for a discussion of the role of heterogeneity) the median voter result applies trivially: the equilibrium tax will be the tax that maximizes the welfare of each member of generation t.

There are two avenues through which taxes affect utility of an individual born at time t. First, consumption in their first period of life is a constant fraction of wages. In appendix A it is shown that wages at t are independent of τ_{t+1}. Second, consumption in their second period of life is equal to the rate of return times saving. But savings --which are just wages minus consumption-- do not depend on the taxes they will choose. (It is at this point where our assumption that saving is inelastic with respect to the interest rate becomes quite convenient.) Thus, if higher taxes at time t+1 hurt generation t at all (and this is essential if we are to have a realistic model) it must be because they lower the rate of return between time t and t+1. Finally, on the positive side, higher taxes will both increase the quality of capital goods used in the consumption sector at time t+1 and reduce the quantity of capital allocated to the consumption sector.

How do taxes at time t+1 affect the rate of return? From the definition of equilibrium it follows that

$$1 + i_{t+1} = \frac{p_{kt+1}(1 - \delta) + r_{It+1}}{p_{kt}}.$$

Now, at the time of voting, p_{kt} is fixed, independent of τ_{t+1} (see Appendix A). Moreover, optimal behavior on the part of capital goods producers implies $r_{It+1} = Ap_{kt+1}$. Thus,

$$1 + i_{t+1} = \frac{p_{kt+1}(1 - \delta + A)}{p_{kt}}.$$

In this expression, the only element that depends on τ_{t+1} is p_{kt+1}. Thus, the key channel through which taxes affect consumption of voters is through their impact on the unit price of capital.

If the equilibrium choice of cleanliness level, z, at time t+1 depends just on τ_{t+1} (this is shown in Appendix A) , it follows that total pollution (see (2.3)), is

$$s_{t+1}(\tau_{t+1}) = \varphi(z_{t+1}(\tau_{t+1}))[k_{ct+1}(z_{t+1}(\tau_{t+1}))/k_{t+1}]k_{t+1} = \mu(\tau_{t+1})k_{t+1},$$

where we are using the result (see Appendix A for a derivation) that both z_{t+1} and the ratio

$k_{ct+1}(z_{t+1})/k_{t+1}$ depend on just the tax rate that will be chosen at t+1, τ_{t+1}.

To summarize this discussion, it follows that voters' utility function over taxes is given by:

$$V_t(\tau_1,...) = \log c_t^t (\tau_1,...) + \beta \log c_{t+1}^t (\tau_1,...) - u(s_{t+1} (\tau_1,...)).$$

First, note that the effect of τ_1 through τ_{t-1} on the voter's utility is completely summarized through their effect on k_t. Moreover, since all of these variables are predetermined and utility is separable, the term $\log c_t^t (\tau_1,...)$ is independent of the voter's choice. For similar reasons, both $c_{t+1}^t (\tau_1,...)$ and $s_{t+1} (\tau_1,...)$ can be written as a function of k_{t+1} and τ_{t+1} only. Finally, because of the independence of savings from the interest rate, it follows that k_{t+1} does not depend on τ_{t+1}. From all of this, it follows that the voter's problem is equivalent to maximizing

$$\beta \log c_{t+1}^t (\tau_{t+1};k_{t+1}) - u(s_{t+1} (\tau_{t+1};k_{t+1}))$$

where k_{t+1} is taken as given. Note that $c_{t+1}^t (\tau_{t+1};k_{t+1})=p_{kt+1} k_{t+1}$. The arguments in Appendix A show that $p_{kt+1} k_{t+1}$ is of the form $p_{k\tau}(\tau_{t+1})k_{t+1}^{\alpha}$, where the subscript τ is to remind the reader that these are the equilibrium values when individuals vote over tax rates. Because of our choice of log utility and the independence of k_{t+1} from τ_{t+1}, it follows that the voter's objective function is $V(\tau, k)$, which is given by,

$$V(\tau, k)= \beta \log [p_{k\tau}(\tau)] - u[\mu_\tau(\tau)k].$$

Thus, to analyze voting outcomes, the two critical pieces of the equilibrium that we need to characterize are the two functions $p_{k\tau}(\tau)$ and $\mu_\tau(\tau)$. Since the tax rate τ_L plays such a critical role in determining technology choice on the part of firms, it turns out to be convenient to work with a new variable, y, defined as $y = (\tau/\tau_L)^{\theta/\nu}$, instead of τ. Given this transformation, it follows that individuals can be seen as voting over y directly. Thus, the utility function of a voter when the economy chooses next period capital k, and his generation chooses (transformed) taxes given by y is just,

$$V(y, k)= \beta \log [p_{k\tau}(y)] - u[\mu_\tau(y)k]. \tag{2.5}$$

To describe the equilibrium of the voting game, it is necessary to determine the properties of $p_{k\tau}(y)$ and $\mu_\tau(y)$. These are summarized in Appendix A. From the point of view of the subsequent analysis, the key feature is that, although these functions are continuous, they are not differentiable at y =1 (this corresponds to $\tau = \tau_L$), and that the price of basic capital, $p_{k\tau}(y)$, is not necessarily decreasing in y. Some possible shapes are in Figure 1.

The outcome of our voting equilibrium is, for each k, the values of taxes --as given by y-- that maximize the utility of the median voter. We summarize our results in Proposition 2. What we basically show is that, if a technical condition guaranteeing interiority is satisfied, for small values of k --equivalently a low income per capita country-- the equilibrium y is zero. As k increases, y increases, but stays in the region $y \leq 1$. These two results simply formalize standard intuition: when income is very low --and consequently consumption is low-- the equilibrium choice of pollution tax is zero. As income increases --for higher values of k-- the equilibrium level of taxes is positive but falls short of the minimum environmental tax τ_L. This induces a reallocation of capital between the two sectors --consumption and investment-- but induces no (costly) quality upgrade, i.e. z = 0. Finally, for sufficiently high levels of k, the equilibrium tax is greater than τ_L, and it grows without bound as a function of k. The transition from the low tax region ($y \leq 1$) to the high tax region ($y \geq 1$) is not smooth. Specifically, we show that there exists a level of k, denoted k_F, such that y(k) is discontinuous at k_F. The discontinuity is of a very simple form: y jumps upward at k_F. We postpone a more complete discussion of the economic intuition underlying this result until the next section.

Proposition 2. Assume that condition B* (in Appendix B) holds. Then, the maximum of V(y,k) is well defined for all $k \geq 0$. The maximizer, y(k) has the following properties,

(i) $\exists\, k_B > 0$, such that $\forall\, k \leq k_B$, $y(k) = 0$,

(ii) $\exists\, k_C$ such that for all $k \geq k_C$, y(k) is increasing, continuous and $\lim_{k \to \infty} y(k) = \infty$,

(iii) There exists at least one k such that y(k) jumps upwards at that point,

(iv) There exist k_D and k_E such that for all $k \in (k_D, k_E)$, changes in taxes do not induce any quality changes,

(v) If in addition to assumption B*, y(k), in the region in which $y \leq 1$, is differentiable, then there is a unique switch point, k_F, such that for all $k \leq k_F$, the equilibrium tax, $y(k) \leq 1$, while for $k \geq k_F$, $y(k) \geq 1$. Thus, the jump in the equilibrium y(k) can only occur once.

Proof: See Appendix B

It is easy to check that even though we assumed that u is convex, most of the analysis goes through as long as u is not ‘too concave.’ In particular, if u=log, the equilibrium tax rate is independent of k and pollution increases monotonically with income and the model converges in one period to a balanced growth path.

Even though Proposition 2 contains the basic result, it is useful to more carefully analyze what kind of time path for taxes and pollution are implied by it. We now turn to that task.

3. The Dynamics on the Equilibrium Path

It is of interest to analyze what kind of time paths for income and pollution --the variables selected by Grossman and Krueger (1995)-- are implied by this model. The previous section showed the nature of the equilibrium as a function of the aggregate capital stock. In Appendix A (see also section 6), we study the implications of the model for the growth rate of capital. Here, it suffices to say that, under standard assumptions (i.e., the productivity in the production of investment goods is sufficiently high) the model displays positive growth for all possible values of the tax variable y. Given this, the time path of capital is simple: it increases without bound. Thus, it is simple to describe the time path of the relevant endogenous variables using Proposition 2. Let $\gamma(y)$ be the growth rate of the capital stock when the tax rate is equal to y (see Section 6 for more details). We collect the main results in the following proposition,

Proposition 3: Assume B*, and that $\gamma(y) > \gamma(\infty) > 1$ for all y. Then, for a given initial capital k_0 which is assumed small,

(i) There is some t_0^* such that for $0 \leq t \leq t_0^*$:

(a) the equilibrium tax rate y_t is zero,

(b) the growth rate of capital is constant and equal to $\gamma(0)$,

(c) pollution is increasing.

(ii) There is a $t_1^* > t_0^*$, such that for all t between t_0^* and t_1^*:

(a) the equilibrium tax rate is increasing, but $y_t \leq 1$,

(b) the growth rate of capital is increasing over time,

(c) the level of pollution is decreasing.

(iii) At $t = t_1^*$,

(a) the equilibrium tax rate jumps up to a value y>1,

(b) the level of pollution jumps down.

(iv) For $t > t_1^*$,

(a) the level of pollution increases over time and converges to s_τ^*, where s_τ^* satisfies,

$$u'(s_\tau^*)s_\tau^* = \beta\alpha/(1+\nu/\theta),$$

(b) the tax rate is increasing over time and converges to ∞,

(c) the growth rate of capital is decreasing over time and converges to $\gamma(\infty)>1$.

(v) The asymptotic growth rate of consumption, $\gamma_c(\infty)$, is strictly greater than one.

Proof: See Appendix B.

An interesting implication of Proposition 3 is that the time path of pollution is not monotone. An example is shown in Figure 3. After an initial period in which pollution increases and taxes are zero, there is a period of relatively low taxes, no quality upgrades and decreases in the level of pollution. This is induced by a change in the composition of capital: more capital is allocated to producing investment goods, and less capital to producing capital goods used in the production of consumption goods. Since the latter are the pollution causing factors, total pollution decreases.

As income grows, voters choose to induce --through higher taxes-- a quality upgrade. In the model, this takes the form of a discrete increase, a jump, in the tax rate. In the period in which the jump occurs --recall that a period is roughly half a generation in this model-- output drops. In this period higher taxes induce a quality upgrade and pollution drops, as cleaner capital goods are used in the consumption sector.

After the transition period, the economy is in a new regime in which tax rates, as measured by y_t increase over time, and pollution per unit of capital decreases, as cleaner capital goods are used in equilibrium. What happens to measured pollution taxes? Recall that the actual tax rate per unit of capital is $\tau\varphi(z)$. As y increases without bound, τ increases as well, but z(y) increases and, hence, $\varphi(z)$ decreases. Thus, the endogenous response works in the direction of reducing the measured tax rate. For the example that we have analyzed, measured taxes, defined as $\tau\varphi(z)$, equal $\theta/(\nu-\theta)$ (see Appendix A). Thus, observations on measured taxes and pollution levels (or quality of the capital stock) could lead an observer to incorrectly infer that the price mechanism is not important to induce environmental protection when, in fact, exactly the opposite is true. In this region, both income and pollution grow, although pollution grows at a slower rate. In the long run, pollution is bounded and income is growing and hence pollution, as a fraction of income, disappears. Note that this does not imply that people do not care about pollution. Actually, the opposite is true: at high income levels, voters choose taxes to induce very high quality choices and, hence, they choose to pay a large penalty, in terms of resources used to operate cleaner technologies, relative to output.

Figure 3 shows the basic prediction of this model for the relationship between income and pollution. Initially a period of increasing pollution is followed by an era (a period in our model) of decreases in pollution. For higher levels of income, pollution increases. This stylized behavior is consistent with the evidence reported by Grossman and Krueger (1995) for some pollutants.

4. Regulating Quality Choice

An alternative institution that can be used to enforce environmental standards are quantitative restrictions. For example, in the U.S. there is a quantitative restriction on the number of leaded gasoline burning car engines that can be sold: it is zero. This type of quantitative restriction is common in many environmental regulations. One way of modeling quantitative restrictions is to allow the voters at time t to collectively choose a *minimum level of quality z*. In other words, at time t the old vote on a level of quality z_t and firms cannot use any variety z, with $z < z_t$. (Alternatively, one can interpret this scheme as a different simplification of the tax rules where $\tau=\infty$ for all $z<z_\tau$, for some z_τ, and $\tau=0$ for $z\geq z_\tau$.) It turns out that the equations describing the equilibrium for a given value of the tax rates hold with τ_t equal to zero. The reason for this is

simple: if the policy specifies that the lowest available quality is z_t, then firms will choose capital of quality exactly equal to z_t since all forms of capital are perfect substitutes. The market price of capital will be $m(z_t)r_{It}$, and firms optimize taking this as given.

It is useful to describe how the key variables vary with z. First, consider the price of capital. Simple calculations using (A.4) and (A.8) in Appendix A, show that $p_{kq}(z)$ (here the subindex q indicates that the collective decision is over qualities), the price of capital, is given by,

(4.1) $$p_{kq}(z)= M\, k^{\alpha-1} [a_0\, m(z) - a_1]^{1-\alpha}/m(z),$$

where $a_0 = 1 + [\alpha(1+\beta)(1-\delta+A)]/[(1-\alpha)\beta A]$, $a_1 = \alpha(1+\beta)(1-\delta)/[(1-\alpha)\beta A]$, and $M = (a_0-1)\alpha B/A$. If $a_1/\alpha a_0 > 1$, denote by z_M the value of z satisfying $m(z_M) = a_1/\alpha a_0$. If $a_1/\alpha a_0 \leq 1$, set $z_M=0$. As in the case in which the collective decision making is over taxes, there is a value of the policy variable --which we denote z_M-- that maximizes the value of capital and, hence, the value of consumption of the voters --the old. Note that, in equilibrium, the minimum allowable quality level will be always at least equal to z_M --voters will always extract the monopoly surplus. It can be easily verified that the function $p_{kq}(z)$ is decreasing for $z \geq z_M$. In order to avoid unnecessary notation, we will concentrate on the case $z_M=0$.

The amount of pollution per unit of total capital is given by,

(4.2) $$\mu_q(z) = D(a_0 - 1)\, [(1+z)^{\nu} (a_0\, m(z) - a_1)]^{-1}.$$

This function is strictly decreasing for all z's. Thus, the higher the minimum quality level the lower the amount of pollution per unit of output.

In this case, the rate of growth in total capital, is given by the appropriate version of (A.7), is,

(4.3) $$\gamma_{kq}(z) = (1-\delta+A)\, [1 + (a_0\, m(z) - a_1)/m(z)]^{-1},$$

which is decreasing for all z's.

Finally, the relevant voting function for this case is just,

(4.4) $$V_q(z,k) = \beta \ln(p_{kq}(z)) - u(\mu_q(z)k).$$

The next proposition summarizes the behavior of the relevant variables under this equilibrium.

Proposition 4. Assume that the initial capital stock k_0 is sufficiently small. Then,

(i) There is some t_0^* such that for $0 \le t \le t_0^*$:

(a) the equilibrium minimum quality z_t is zero,

(b) the growth rate is constant and equal to $\gamma_q(0)$,

(c) pollution is increasing.

(ii) For $t > t_0^*$,:

(a) the equilibrium minimum quality is increasing and it grows without bound,

(b) the growth rate of total capital is decreasing over time,

(c) the level of pollution is increasing and it converges to s_q^*, where s_q^* satisfies,

$$u'(s_q^*)s_q^* = \beta\alpha/(1+\nu/\theta)$$

Remark: If $z_M > 0$, then the chosen quality is strictly increasing for all values of k, and converges to z_M as k converges to zero. Thus, in terms of time paths, it is increasing and bounded below by z_M.

Proof: See Appendix B.

The behavior of pollution over time (as well as the minimum quality z_t) is shown in Figure 4. When individuals regulate technologies, the economy displays an ever increasing level of pollution. This level grows faster during the initial period in which the minimum quality is given by z_M. After t_0^* the increase in environmental standards decreases pollution per unit of output. However, the growth effect of output dominates, resulting in an increasing level of pollution. Finally, this scheme succeeds in controlling pollution even in a growth environment: the asymptotic level of pollution is finite; moreover, it coincides with s_τ^* --the limiting level of pollution under the scheme in which taxes are used to regulate pollution. Thus, although the time paths of the two regimes are quite different, their asymptotic or long run behavior is the same. From a practical point of view this example points out the limitations of using long run or balanced growth arguments to interpret data. In this case, if one were interested in using the evidence to distinguish between the two possible regimes, ignoring the transition phase would be equivalent to giving up the possibility of identifying the correct regime.

5. A Class of Optimal Allocations

So far we have characterized the time path and the asymptotic behavior of pollution and income in two different equilibrium regimes. In both cases the old at time t vote on either pollution taxes or minimum quality standards to be implemented at t, and the collective decision is made using majority voting. There are several concerns that arise: Are the voters too myopic since they ignore the impact of pollution on other individuals? In general, how do equilibrium levels of pollution compare to optimal levels of pollution?

It is well known that in overlapping generations models it is difficult to fully characterize the class of optimal allocations. However, in order to get an idea of the nature of the optimal pollution paths we consider a simple utilitarian social preference function. Let U_t be the utility of generation t, and assume that the planner maximizes

(5.1) $$W = \sum_{t=0}^{\infty} \beta_P^t U_t,$$

subject to the feasibility constraints. The discount factor β_P is assumed to be between 0 and 1. The time path of the optimal solution is summarized in the next proposition.

Proposition 5. Assume that β_P (1-δ+A)>1 (to guarantee that the planner would be willing to grow), and that the initial capital stock k_0 is sufficiently small. Then, the solution to the planner's problem is such that,

(i) There is some t_0^* such that for $0 \leq t \leq t_0^*$:
  (a) the optimal level of quality z_t is 0,
  (b) the growth rate is constant and equal to $\gamma_p(0)$,
  (c) the level of pollution is increasing.
(ii) For $t > t_0^*$,:
  (a) the optimal quality level is increasing and converges to ∞ as t goes to ∞,
  (b) the growth rate of total capital is decreasing over time,
  (c) the level of pollution is decreasing and it converges to s_p^*, where s_p^* satisfies,
$$u'(s_p^*)s_p^* = (\beta+\beta_P)\,\alpha/(1+\nu/\theta).$$

Proof: See Appendix B.

There are some interesting differences between the planner's allocation and the allocations under either one of the two equilibrium regimes (choosing pollution taxes and regulating minimum quality levels). The basic details of the time path of the planner's choices of qualities and level of pollution are displayed in Figure 5. Note that the equilibrium path of pollution displays the U shape pattern that Grossman and Krueger (1995) find for some pollutants.

The planner's allocation is such that pollution converges to a long run level s_p^*. This level is strictly greater than the long run level in either one of the voting solutions (they are the same). Thus, it follows that the planner's allocation entails strictly more pollution per unit of output than the equilibrium allocation when people vote over minimum quality. It is more difficult to determine how the planner's solution compares with the equilibrium solution when taxes are the instrument. All that can be said is that for very high levels of capital (and, hence, income) the planner's choice of pollution is higher than the equilibrium. Thus, at least for high levels of income the models predict excessive conservation associated with the equilibrium solutions. The reason why the planner chooses more pollution in the limit is easy to ascertain: it cares about future generations more than voters do. In this setting in which there are no intergenerational spillovers of pollution, voters fail to take into account the impact of the more expensive capital on the welfare of future generations. Note that as the discount factor in the planner's problem is driven to zero all three solutions coincide in the long run.

Why is it that the market solution for a given sequence of minimum standards $\{z_t\}$ differs from the planner's allocation? More precisely, suppose that majority voting would have resulted in a sequence $\{z_t\}$ identical to the planner's choice of optimal $\{z_t\}$; even in this case the time path described in Figure 4 still describes the evolution of pollution: fast increases for low levels of income, and smaller but positive increases at higher levels. This is quite different from the inverted U shape in Figure 5. The resolution of this puzzle lies in the fact that the solutions to planner's problems like the one we used in this section are in general difficult to decentralize as laissez faire equilibria in overlapping generations models. Typically, it is necessary to implement a sequence of transfers so that the post-transfer equilibrium will implement the planner's allocation. An extreme example of this is the class of one sector endogenous growth models. In those models the planner's problem of the type considered in this section (which of course coincides with the equilibrium in an economy with infinitely lived agents) results in positive growth (under some assumptions on parameters), while it is well known (see Boldrin (1992) and Jones and Manuelli (1992)) that there is no equilibrium growth in an overlapping generations setting. Thus, the only way in which the planner's solution can be implemented as a competitive equilibrium is with the use of extensive intergenerational transfers. To some extent the same issues cloud the comparison of the time paths of pollution chosen by the planner and those chosen by voters in this setting. However, it is possible to show that even if the planner has a time varying discount factor β_{pt} (this is in general necessary to replicate the market solution), our asymptotic results about the lower than optimal pollution produced by the market obtain whenever the planner asymptotically gives some value to the future. More precisely, if $\lim_{t\to\infty} \beta_{pt}/\beta_{pt+1} > 0$, it is still the case that the planner asymptotically chooses more pollution than either of the equilibrium regimes.

6. Dynamics and Convergence

Even though our primary focus is on the relationship between pollution, income and growth, the model has implications about the relationship between income levels at t, and the growth rate between t and t+k. This is exactly the kind of relationship that has been the focus of the convergence literature. One position, usually ascribed to the proponents of endogenous growth models (for an early argument see Romer (1986)), is that, in models that have balanced growth potential, there should be no relation between income level and future growth. An alternative (see, for example, Barro and Sala-i-Martin (1995)) emphasizes that exogenous growth models imply conditional convergence, that is, that future growth is negatively correlated with current income level, once other factors are controlled for. The empirical evidence is inconclusive. The studies summarized in Barro and Sala-i-Martin seem to give support to the conditional convergence hypothesis, while Durlauf and Johnson (1995) and Quah (1996), using a different technique, find convergence clubs, that is, groups of countries that have similar behavior.

In this section we want to argue that endogenous policy choice can generate "empirical" convergence clubs, even though, in the long run, all countries will have the same policies and, hence, the same growth level. To see this, we concentrate on the tax equilibria of sections 3 and 4. From the calculations in Appendix A it follows that the mapping between the tax variable, y, and the growth rate of capital, γ, which we indicate by $\gamma(y)$, changes depending on whether the tax rate lies above or below the threshold τ_L (i.e. y is less or greater than one). The function $\gamma(y)$ is

$$\gamma(y) = \begin{cases} \dfrac{(1-\delta+A)\left(1+\dfrac{\theta}{\nu-\theta}y^{\nu/\theta}\right)}{\left(1+\dfrac{\theta}{\nu-\theta}y^{\nu/\theta}\right) + \dfrac{\alpha}{1-\alpha}\dfrac{1+\beta}{\beta}} & \text{for } y \le 1 \\[3ex] \dfrac{(1-\delta+A)\left(1+\dfrac{\theta}{\nu-\theta}\right)y}{\left(1+\dfrac{\theta}{\nu-\theta}\right)y + \dfrac{\alpha}{1-\alpha}\dfrac{1+\beta}{\beta A}\left((1-\delta+A)y-(1-\delta)\right)} & \text{for } y \ge 1 \end{cases}$$

First, it is easy to see that under some parameter restrictions --basically the magnitude of the productivity parameter A-- the model displays positive growth, in the sense that for all y, $\gamma(y)>1$. In addition, it is also possible to obtain that $\gamma(0) > \lim_{y\to\infty} \gamma(y) > 1$; that is, high taxes decrease growth --relative to zero taxes-- but do not stifle it altogether. In summary, the model is capable of displaying positive growth.

Inspection of $\gamma(y)$ shows that the growth rate of capital is increasing for values of τ that fall short of τ_L (y<1), while it is decreasing for values of $\tau>\tau_L$ (y>1). The intuition underlying this result is simple: for small values of y ($y\le 1$) a tax increase induces a "reallocation effect" as more capital is devoted to investment and less to the production of consumption goods (which is the activity that is taxed). This, of course, increases the growth rate. In the region $y\ge 1$, the higher level of taxes results in higher quality capital goods being demanded by the consumption sector (in smaller quantities though). Production of cleaner capital goods is a resource using activity and the net result is that the allocation of capital to the investment sector is decreased with the resulting negative effect on the growth rate. An example of the function $\gamma(y)$ is shown in Figure 6.

What does this model say about the relationship between the current level of income and its future growth rate? It turns out that this question is not easy to answer because the growth rate of income depends --in a complicated way-- on the changes in tax rates. However, it is relatively simple to describe the relationship between the level of income at time t and the growth rate of total capital between t and t+1 (half a generation in our model). Consider, as before, an economy that starts out at a low level of income (k is low). This economy chooses no pollution taxes (y=0) and the growth rate of capital is given by $\gamma(0)$ in Figure 6. For a period of time --more precisely between t=0 and t= t_0^*-- income grows at a constant growth rate. Thus, in this region, the model resembles the standard balanced growth path implications: growth rates are independent of income levels. However, after t_0^*, the equilibrium is such that "low" but positive taxes are chosen (this is the region in which $0<y_t<1$). Moderate taxation of capital used in consumption induces more investment, and a growth boom. Thus, in this region, the model implies that growth rates increase with the level of income and, from the point of view of the convergence literature, it looks like average income of low and middle income countries grow farther apart. Finally, the economy must enter the region in which taxes are high ($y_t>1$), a region that is never left. In this region the higher the initial level of income the lower the subsequent growth rate. Thus, in this region, the model predicts conditional convergence of growth rates of capital, that is, a convergence club for high income countries.

Even though this is a very simple model and emphasizes only pollution taxes, we suspect that the mechanisms that we describe are relevant in models that emphasize endogenous determination of utility producing publicly provided goods. To the extent that this is a good approximation, it suggests that care must be taken in the interpretation of "convergence" regressions. In particular, the convergence coefficient depends on initial income, and could take any value --including divergence-- depending on the mix of countries in the different regions. In addition, models of this type also suggest that it is possible to find "convergence clubs" in the data, without the usual implication associated with those findings. For example, a group of countries with income levels that put them in the region in which the equilibrium tax rate, y_t, lies between zero and one, would show divergence, while "rich" countries (those with y_t strictly greater than one) would form a convergence club.

Finally, the model is consistent with different policies "driving" differences in per capita income and growth across countries. Moreover, the model provides a theory for the differences. Basically, it emphasizes that --at different income levels- -countries choose different policies that result in different growth performances.

7. Extensions

The model in the paper is very simple, and designed to highlight the non-stationarity of equilibrium policy choices in an otherwise stationary environment. However, it is possible to extend the model in a variety of dimensions and, in a few case, the extensions do not affect the qualitative nature of the results.

a) *Heterogeneity* . The assumption of identical individuals is convenient but not essential. First, if individuals differed in terms of initial wealth --say due to differences in their labor endowment or government transfers-- our results in sections 2-4 go through as they are. The key observation is that the relevant voting function, even in this case, depends on just the amount of pollution and the unit price of capital. If individuals have different disutility from pollution (different u functions) then additional assumptions to guarantee that the median voter theorem holds are necessary. Given those assumptions the analysis proceeds as in sections 2 and 4. Finally, if the young cared about pollution but either cannot vote or are never the median voter, our positive analysis goes through without any changes. However, the normative analysis of section 5 needs to be revised. In particular, the conclusion that the asymptotic level of pollution in the planner's solution exceeds that of the market need no longer hold.

b) *The Nature of Pollution*. Throughout the paper we assumed that pollution is a flow: the amount of smoke generated today affects only today's utility. If instead, pollution is treated as a capital stock in which the amount produced at t increases that stock, the analysis in section 2 and 3, and hence the qualitative features of the model, remains basically unchanged. In this case, however, the welfare comparisons are more difficult.

c) *The Uses of Tax Revenue*. In the analysis of sections 2 and 3 we assumed that the proceeds from the environmental tax are thrown away (or, alternatively, used to finance a public good whose utility is separable with respect to the other variables). There are at least three simple alternatives: use the proceeds to "clean" the environment, make a transfer to the young or to the old. In the first case --and depending on how the cleaning up technology is specified-- the qualitative results remain the same. If the tax revenue is used to finance a transfer to the young, it is possible to obtain the puzzling result that higher taxes increase the growth rate. This is a standard result in overlapping generations models (see Boldrin (1992) and Jones and Manuelli (1992)). Finally, if the proceeds from taxing low quality capital are used to finance a transfer to the old, the problem becomes substantially more complicated: in this case the saving function will depend on the interest rate and hence the voting decision must be modeled as a complicated game with an uncountable number of agents. Even though this is an interesting (and realistic) situation it seems to add a level of complication that is hard to justify.

d) *Alternative Technologies*. In this paper we studied the case in which different varieties of capital used in the consumption sector generate pollution. An alternative would be to consider "clean" and "dirty" capital goods that can be used in both sectors. There are problems with these alternative specifications. If there are multiple capital goods in the production sector, it is possible that the voting problem is not well defined. The reason for this is simple: by increasing the price of capital (say through higher taxes) the old increase their consumption while at the same time reducing pollution. In this setting the optimal myopic private tax is infinite. Alternatively, one can consider multiple capital goods in the consumption sector. The problem in this case is that tax increases devote more resources to the capital sector and this increases the growth rate. Therefore, such a model would have the implication that higher pollution taxes are growth enhancing. It seemed that this is at odds with the evidence and, hence, we decided against this model.

e) *Global External Effects.* The model can be easily extended to one in which pollution is global. In this case, and for a small country, the level of pollution is independent of local activities. It is straightforward to show that in this case, no local pollution controls are enacted. Thus, control of global pollutants requires world-wide collective decisions making mechanisms, and it is not clear that a voting model is appropriate for this.

8. Concluding Comments

There are several conclusions that emerge from this analysis. It is possible to separate them into two categories: those specific to the issue of growth and the environment and those that are more general and point to the potential of models of endogenous policy to explain growth facts.

i) First, from a positive point of view, the voting model presented in this paper can match the observed relationship between income and pollution described in the empirical literature. However, the predictions of the model about this relationship depend on the nature of the instrument being chosen. If proportional pollution taxes are used, the relationship is not monotone and display drastic changes in short periods. If, instead, individuals vote over allowable production technologies, the model predicts a monotone, but bounded, relationship between income and growth. In both cases, pollution is bounded even though income grows without bound.

ii) Our analysis suggests that even in cases in which policy decisions about pollution control are made without consideration of future generations, it does not follow that the equilibrium level of pollution is excessive. In particular, the analysis in sections 2 and 3 suggest exactly the opposite: there are equilibrium allocations that --at least asymptotically-- result in too little pollution.

iii) From a more general perspective we have shown that in a stationary voting model with growth, the equilibrium policies are not stationary. This implies that this class of models have the potential to explain cross country differences in policies, and the fact that rich countries systematically pursue different policies from poor countries. In the specific example that we study, the model is capable of explaining some of the puzzling results in the convergence literature as being driven by endogenously chosen policies that change with the level of income.

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Appendix A: Equilibrium for a Given $\{\tau_t\}$

In this Appendix we describe the equilibrium of the model for a given sequence of taxes $\{\tau_t\}$. Consider the pricing of the different capital goods. If we let r_{It} be the rental rate for renting capital to use in the production of new capital (x_t), profit maximization requires,

(A.1) $$r_{kt}(z) \leq m(z) r_{It},$$

with equality if variety z is produced.

Consider next the decision of the capital producing firms. Let p_{kt} be the price of new capital produced at time t. Since this is a constant returns to scale activity, the standard zero profit condition implies,

(A.2) $$r_{It} / A \geq p_{kt} ,$$

with equality if new capital is produced.

From Proposition 1 we know that producers of consumption goods will choose, at time t, only one quality of capital which we denote either z_t, or $z(\tau_t)$, and its expression is given in Proposition 1.

For firms in the consumption sector, the rental price of capital (after taxes) must equal the marginal product of capital. That is:

(A.3) $$\alpha B k_{ct}(z_t)^{\alpha-1} = (1 + \tau_t \varphi(z_t)) r_{kt}(z_t) = (1 + \tau_t \varphi(z_t)) m(z_t) r_{It}.$$

Hence, using the equilibrium condition from the capital producing sector,

(A.4) $$p_{kt} = \frac{r_{It}}{A} = \frac{1}{A} \frac{\alpha B \ k_{ct}(z_t)^{\alpha-1}}{(1 + \tau_t \phi(z_t)) m \ (z_t)}.$$

Finally, to close the model we need to specify the equilibrium condition that ties consumer decisions and producer decisions. We do this by imposing the equality of savings with the value of capital taken into period t+1: $b_t = p_{kt} k_{t+1}$. Using the equilibrium condition that the wage rate is given by the marginal product of labor, it follows that,

$$p_{kt} k_{t+1} = \frac{\beta}{1 + \beta} w_t$$

and $w_t = (1 - \alpha)Bk_{ct}(z_t)^{\alpha}$.

This, in turn, implies,

$$k_{t+1}\frac{\alpha\beta B}{A}\frac{k_{ct}(z_t)^{\alpha-1}}{(1 + \tau_t\phi(z_t))m(z_t)} = \frac{(1 - \alpha)\beta B}{1 + \beta}k_{ct}(z_t)^{\alpha},$$

and hence

$$\text{(A.5)} \qquad k_{ct}(z_t) = \frac{\alpha}{1 - \alpha}\frac{1 + \beta}{\beta A}\frac{k_{t+1}}{(1 + \tau_t\phi(z_t))m(z_t)}.$$

Since, $m(z_t)k_{ct}(z_t) + x_t = k_t$, we can rewrite the law of motion for k as :

$$k_{t+1} = (1 - \delta + A)x_t + (1 - \delta)k_{ct}(z_t),$$

or,

$$k_{t+1} = (1 - \delta + A)k_t + k_{ct}(z_t)[(1 - \delta)(1 - m(z_t)) - Am(z_t)].$$

Using (A.5) in the above equation gives,

$$\text{(A.6)} \quad k_{t+1} = (1 - \delta + A)k_t + k_{t+1}\frac{\alpha(1 + \beta)[(1 - \delta)(1 - m(z_t)) - Am(z_t)]}{(1 - \alpha)\beta A(1 + \tau_t\phi(z_t))m(z_t)}.$$

It follows that the growth rate of capital between time t and t+1 depends on the tax rate chosen at time t. The precise expression is,

$$\text{(A.7)} \ \gamma(\tau_t) = \gamma_{kt} = \frac{k_{t+1}}{k_t} = \frac{(1 - \delta + A)}{1 + \dfrac{\alpha(1 + \beta)[(1 - \delta)(m(z_t) - 1) + Am(z_t)]}{(1 - \alpha)\beta A(1 + \tau_t\phi(z_t))m(z_t)}},$$

where the dependence of the equilibrium quality level, z_t, on the tax rate is described in Proposition 1. Note that (A.5) and (A.7) imply that the equilibrium relationship between the level of capital in the consumption sector, $k_{ct}(z)$, and the contemporaneous level of aggregate capital, k_t, is given by,

$$\text{(A.8)} \quad k_{ct}(z_t) = \frac{\alpha}{(1 - \alpha)}\frac{(1 + \beta)}{\beta A}\frac{(1 - \delta + A)k_t}{(1 + \tau_t\phi(z_t))m(z_t) + \dfrac{\alpha}{(1 - \alpha)}\dfrac{(1 + \beta)}{\beta A}[(1 - \delta)(m(z_t) - 1) + Am(z_t)]}.$$

Given the new variable $y = (\tau/\tau_L)^{\theta/\nu}$, the equilibrium level of z as function of y is given by,

$$z(y) = \begin{cases} 0 & \text{for } y \leq 1 \\ y^{1/\theta} - 1 & \text{for } y \geq 1 \end{cases}$$

Given this function z(y), the cost of capital in the consumption sector given a tax rate implied by y is,

$$m\ (y) = \begin{cases} 1 & \text{for } y \leq 1 \\ y & \text{for } y \geq 1 \end{cases}$$

while pollution per unit of capital in the consumption sector (again as a function of the tax rate) is,

$$\phi(y) = \begin{cases} D & \text{for } y \leq 1 \\ D\ y^{-\nu/\theta} & \text{for } y \geq 1 \end{cases}$$

With these two functions it is immediate to derive an expression for the after tax cost of capital in the consumption sector. It is given by,

$$(1+\tau\phi)m = \begin{cases} 1 + y^{\nu/\theta}\dfrac{\theta}{\nu - \theta} & \text{for } y \leq 1 \\ y\ (1 + \dfrac{\theta}{\nu-\theta}) & \text{for } y \geq 1 \end{cases}.$$

Using equations (A.5) and (A.6) evaluated at the equilibrium levels of all the variables as functions of the tax rate, y, it is possible to calculate the fraction of the total capital stock allocated to the consumption sector. This fraction is,

$$k_c(y)/k \quad = \begin{cases} \dfrac{\alpha\ (1+\beta)\ (1-\delta+A\ )}{(1-\alpha)\beta A\ [(1+\dfrac{\theta}{\nu-\theta}y^{\nu/\theta})\ +\ \dfrac{\alpha}{1-\alpha}\ \dfrac{1+\beta}{\beta}]} & \text{for } y \le 1 \\ \\ \dfrac{\alpha\ (1+\beta)\ (1-\delta+A\ )}{(1-\alpha)\beta A\ [(1+\dfrac{\theta}{\nu-\theta})y\ +\ \dfrac{\alpha}{1-\alpha}\ \dfrac{1+\beta}{\beta A}\ ((1-\delta+A\ )y-(1-\delta))]} & \text{for } y \ge 1 \end{cases} .$$

Finally, given the ratio k_c/k and φ, it follows that the unit price of capital, $p_{k\tau}(y)$, and pollution per unit of total capital, $\mu_\tau(y)$, are given by,

(A.9)
$$p_{k\tau}(y) \quad = \begin{cases} p_{k1}(y)\ =\ K\ \dfrac{[(1+\dfrac{\theta}{\nu-\theta}y^{\nu/\theta})\ +\ \dfrac{\alpha}{1-\alpha}\ \dfrac{1+\beta}{\beta}]^{1-\alpha}}{(1+\dfrac{\theta}{\nu-\theta}y^{\nu/\theta})} & \text{for } y \le 1 \\ \\ p_{k2}(y)\ =\ K\ \dfrac{[(1+\dfrac{\theta}{\nu-\theta})y\ +\ \dfrac{\alpha}{1-\alpha}\ \dfrac{1+\beta}{\beta A}\ ((1-\delta+A\ )y-(1-\delta))]^{1-\alpha}}{(1+\dfrac{\theta}{\nu-\theta})y} & \text{for } y \ge 1 \end{cases}$$

where K is a constant, and,

(A.10)
$$\mu_\tau(y) \quad = \begin{cases} \mu_1(y)\ =\ \dfrac{\alpha\ (1+\beta)\ (1-\delta+A\ )\ D}{(1-\alpha)\beta A\ [(1+\dfrac{\theta}{\nu-\theta}y^{\nu/\theta})\ +\ \dfrac{\alpha}{1-\alpha}\ \dfrac{1+\beta}{\beta}]} & \text{for } y \le 1 \\ \\ \mu_2(y)\ =\ \dfrac{\alpha\ (1+\beta)\ (1-\delta+A\ )\ D\ y^{-\nu/\theta}}{(1-\alpha)\beta A\ [(1+\dfrac{\theta}{\nu-\theta})y\ +\ \dfrac{\alpha}{1-\alpha}\ \dfrac{1+\beta}{\beta A}\ ((1-\delta+A\ )y-(1-\delta))]} & \text{for } y \ge 1 \end{cases}$$

Appendix B: Proofs

Proof of Proposition 1: The determination of the optimal quality as a function of the tax rate is given by the solution to,

$$\min_z (1 + \tau\varphi(z))m(z) \quad = \min_z(1 + \tau D(1 + z)^{-\nu})(1 + z)^{\theta}.$$

The first order condition is,

$$\theta(1 + z)^{\theta-1}(1 + \tau D(1 + z)^{-\nu}) + (1 + z)^{\theta}(-\nu)\tau D(1 + z)^{-(\nu+1)}$$
$$= (1 + z)^{\theta-1}[\theta(1 + \tau D(1 + z)^{-\nu}) - \nu(1 + z)^{-\nu}\tau D].$$

When $\tau = 0$, this is $(1 + z)^{\theta-1}\theta > 0$, for all $z \geq 0$. This implies that $z(\tau) = 0$ for sufficiently small τ, since the candidate interior solution yields a greater value of the objective function than the z=0 solution. This candidate solution is a local minimum. Note that, as defined, τ_L satisfies,

$$[\theta(1 + \tau_L D) - \nu\tau_L D] = 0.$$

For $\tau \geq \tau_L$, z is interior and given by the solution to,

$$\theta + \theta\tau D(1 + z)^{-\nu} = \nu\tau D(1 + z)^{-\nu}.$$

This condition implies, that z is given by,

$$z = \left(\frac{(\nu - \theta)\tau D}{\theta} \right)^{1/\nu} - 1.$$

■

Proof of Proposition 2:

a) *Preliminaries.*

Before stating the result we need to introduce some auxiliary notation. Since the function V is not differentiable (due to the non-differentiability of the functions p_k and μ) it is useful to define to differentiable functions that completely characterize V. Let,

$$V(y,k) = \begin{cases} V_1(y, k) & \text{for } y \leq 1 \\ V_2(y, k) & \text{for } y \geq 1 \end{cases}$$

where $V_i(y,k) \equiv \ln(p_{ki}(y)) - u(\mu_i(y)k)$. The maximized value of V, denoted $V^*(k) \equiv V(y(k),k)$, where y(k) is the optimal choice of y given k, is given by, $V^*(k)= Max_{\{i=1,2\}} V^*_i(k)$. $V^*_i(k)$ is just the maximum of V_i over the relevant domain for y, i.e. $0 \leq y \leq 1$ for i=1, and $y \geq 1$ for I=2. The maximizers are denoted $y_1(k)$ and $y_2(k)$, respectively. It follows that $V^*_i(k) = V_i(y_i(k),k)$, for i=1,2. Formally we have,

Next, we state assumption B*. This assumption guarantees that, along any equilibrium, the tax rate for small levels of capital lies in the region $(0,\tau_L)$. Let $\zeta(y)$ be defined by,

$$\zeta(y) = \beta\alpha + \beta[\alpha (1+\beta)/ (1-\alpha)\beta] [1 + \theta/(\nu-\theta)y^{\nu/\theta}]^{-1}.$$

Let k_A be the unique solution to $\zeta(1) = u'(\mu_1(1) k_A) \mu_1(1) k_A$, which clearly exists given our assumptions about u.

*Assumption B**: There exists a (k,y) pair with $k<k_A$ and $0 \leq y<1$ such that,

$$\zeta(y) < u'(\mu_1(y) k) \mu_1(y) k.$$

b) *Proof*. It is convenient to restate Proposition 2 using the notation introduced above. To prove the Proposition, it suffices to prove, that the maximum of V(y,k) is well defined for all $k \geq 0$. The maximizer, y(k) has the following properties,

(a) $\exists k_B>0$, such that $\forall k \leq k_B$, $y(k)= y_1(k) = 0$,
(b) $\exists k_C$ such that for all $k \geq k_C$, $y(k) = y_2(k)$ which is strictly increasing, continuous and $\lim_{k\to\infty} y(k) = \infty$,
(c) For all k, $y_2(k) > y_1(k)$,
(d) There exist k_D and k_E such that for all $k \in (k_D,k_E)$, $y_1(k)$ is interior,
(e) If in addition to assumption B*, $y_1(k)$ is differentiable, there exists a unique k_F such that $y(k) = y_1(k)$ for $k \leq k_F$ and $y(k) = y_2(k)$ for $k \geq k_F$.

Note that (a)-(c) imply (i)-(iii), (d) implies (iv), and (e) implies (v). We are now ready to prove the result. First, we first characterize $y_1(k)$ and $y_2(k)$.

Consider $\max_{0\leq y\leq 1} V_1(y,k)$. Given that the function is continuous, existence and upper-hemicontinuity of $y_1(k)$ --the maximizer of V_1 in the region [0,1] as a function of k-- follows from the maximum theorem. The function V_1 is differentiable and it can be checked that $\partial^2 V_1/\partial y \partial k \geq 0$ (with the inequality being strict if k>0). Thus, the function is supermodular. The results of Topkis (1978) and Milgrom and Shannon (1994) show that the maximand y_1 is monotone increasing.

A straightforward calculation shows that,

$$\partial V_1/\partial y = (\mu_1'/\mu_1)\,[\beta(p_{k1}'/p_{k1})(\mu_1 /\mu_1') - u'(\mu_1 k)\mu_1 k],$$

or, equivalently,

$$\partial V_1/\partial y = (\mu_1'/\mu_1)\,[\zeta - u'(\mu_1 k)\mu_1 k],$$

where the argument y is omitted when there is no risk of confusion. In this derivation we used the fact that $\beta(p_{k1}'/p_{k1})(\mu_1 /\mu_1')=\zeta$, given the specific form of p_{k1} and μ_1. Next consider,

$$H(k) = \max_{0\leq y\leq 1} \{\zeta(y) - u'(\mu_1(y)k)\mu_1(y)k \}.$$

It follows that $\lim_{k\to o} H(k)>0$, and $\lim_{k\to\infty} H(k)<0$. Thus, there exists some k_{B1} such that $k\leq k_{B1}$ implies $H(k)\geq 0$. Since $\partial V_1/\partial y \leq (\mu_1'/\mu_1)\, H(k)$, this implies $y_1(k) = 0$ for $k \leq k_{B1}$. We will later show that y_1 and y coincide on an interval $[0,k_B]$ with $k_B \leq k_{B1}$.

Note that, as defined, k_A is the smallest k such that $y_1(k)$ is one. Given the monotonicity result it is also equal to one for $k\geq k_A$. That $k_{B1}<k_A$ is simply guaranteed by Assumption B*. In Figure 2 we show a candidate $y_1(k)$ function.

We now characterize the solution, $y_2(k)$, to the max of V_2. We need to consider two cases. If $y_M>1$ the function V_2 is clearly increasing in the region $[1,y_M]$. Thus in this case $y_2\geq y_M$. From an economic point of view y_M is a monopoly price of capital and the old would never choose taxes that result in a price lower than this monopoly price. The second case is when $y_M=1$. This corresponds to a p_{k2} function that is decreasing. In this case define k_{C1} as the largest value of k such that $y_2(k)=1$. Using the first order conditions for the maximization of V_2, one gets,

$$\partial V_2/\partial y = (\mu_2'/\mu_2)\,[\beta(p_{k2}'/p_{k2})(\mu_2 /\mu_2') - u'(\mu_2 k)\mu_2 k].$$

Then, k_{C1} satisfies,

$$\beta(p_{k2}'(1)/p_{k2}(1))(\mu_2(1) /\mu_2'(1)) = u'(\mu_2(1)k_{C1})\mu_2(1)k_{C1}.$$

It is easy to show that V_2 is supermodular as well and hence that y_2 is increasing. Moreover, inspection of the function $\beta(p_{k2}'(y)/p_{k2}(y))(\mu_2(y) /\mu_2'(y))$ shows that it is monotone increasing for $y\geq y_M$ ($y\geq 1$ when p_{k2} is everywhere decreasing), while $u'(\mu_2(y)k)\mu_2(y)k$ is decreasing. Since equality of these two functions is a necessary condition for an interior maximum ($y>1$) it follows that the maximizer is unique and hence continuous. Two possible y_2 functions --depending on whether $y_M >1$ or $y_M=1$-- are shown in Figure 2. Simple inspection implies that $y_2(k)\to\infty$ as $k\to\infty$.

We will now prove (iii).

We show that $k_{C1}<k_{B1}$. Direct calculation shows that $p_{k2}'(1)>p_{k1}'(1)$. From the definition of k_{C1} and k_{B1} and given that the functions p_{ki} and μ_i are continuous at 1, it follows that,

$$\mu_2'(1)u'(\mu_2(1)k_{C1})\mu_2(1)k_{C1} > \mu_1'(1)u'(\mu_1(1)k_{B1})\mu_1(1)k_{B1}.$$

Direct calculation shows that $\mu_2'(1)<\mu_1'(1)<0$. Thus,

$$\mu_2'(1)u'(\mu_2(1)k_{C1})\mu_2(1)k_{C1} > \mu_1'(1)u'(\mu_1(1)k_{B1})\mu_1(1)k_{B1}>\mu_2'(1)u'(\mu_1(1)k_{B1})\mu_1(1)k_{B1},$$

and this implies,

$$u'(\mu_2(1)k_{C1})\mu_2(1)k_{C1} < u'(\mu_1(1)k_{B1})\mu_1(1)k_{B1}.$$

Given the monotonicity of u'(x)x this implies $k_{C1}<k_{B1}$.

The key feature of the functions y_i in Figure 2 is that they do not intersect. Thus, if for some values of k the function V equals V_1 while for others V equals V_2 then at the switch points there will be a discontinuity in y(k). In particular, if as k increases the optimum switches from V_1 to V_2, then y(k) will display an upward jump. In Figure 2 such a k is denoted k_F (the location is somewhat arbitrary).

To complete the argument we need to show that for low values of k, $V_1^*>V_2^*$, while the opposite holds for large values of k (this is parts (i) and (ii)). First we show that for small k, $V_1^*>V_2^*$. Let $k\leq\min\{k_{B1}, k_{C1}\}$. Then,

$$V_1^*(k) = V_1(0,k) \text{ and } V_2^*(k) = V_2(1,k).$$

Thus, using the fact that $p_{k1}(1)=p_{k2}(1)$, we get,

$$V_1^*-V_2^* = \beta\ln(p_{k1}(0)/p_{k1}(1)) - [u(\mu_1(0)k) - u(\mu_2(1)k)].$$

Note that the first term is strictly positive, while the second converges to zero as k goes to zero. This completes the argument that $y=y_1$ for small k. Define k_B as the largest such k.

Next we show $V_2^*>V_1^*$ for k large. Let $k>\max\{k_A,k_{C1}\}$, and such that $y_2(k)>y^*>1$, and $y_1(k)=1$ (see Figure 2). Hence $V_2(y,k)>V_2(y^*,k)$. Thus,

$$V_2^*-V_1^*\geq V_2(y^*,k) - V_1(1,k) = \beta\ln(p_{k2}(y^*)/p_{k2}(1)) - [u(\mu_2(y^*)k) - u(\mu_1(1)k)].$$

Since the first term is bounded, it suffices to show that the second term (in square brackets) goes to -∞ as k grows. Since u is convex it follows that,

$$u(\mu_1(1)k) - u(\mu_2(y^*)k) \geq u'(\mu_2(y^*)k)\, \mu_2(y^*)k\, [(\mu_2(y^*) - \mu_1(1))/\mu_2(y^*)],$$

and the result now follows from our assumption about u'(x)x converging to ∞ as x goes to ∞.

This completes the proof of parts (i) and (ii).

To prove part (iv), consider the first order condition for an interior maximum of V_1. It is given by,

$$\partial V_1/\partial y = (\mu_1'(y)/\mu_1(y))\, [\zeta(y) - u'(\mu_1(y)k)\mu_1(y)k]=0.$$

It is sufficient to show that for some k and y, the term in square brackets is zero. From assumption A, this term is negative for some $k < k_A$ and some y<1. It is positive when evaluated at y=1 for this k. It follows that for this k, this term is zero for some y in (0,1). By continuity of u', it follows that this holds for an open set of k's.

To prove part (v), note that it is sufficient to show that if at some k, $y(k) = y_2(k)$, then, $y(k') = y_2(k')$ for all k'>k. To see that this holds, it is sufficient to show that $V_2^{*\prime} > V_1^{*\prime}$ for all k. To see that this holds, recall that $V_i^* = V_i(y_i(k);k)$ and so $V_i^{*\prime} = \partial V_i(y;k)/\partial y\, dy_i/dk + \partial V_i(y;k)/\partial k$. By the definition of y_i, it follows that the first term is zero and hence it is sufficient to show that $\partial V_2(y;k)/\partial k > \partial V_1(y;k)/\partial k$. Straightforward calculations show that this is equivalent to showing that $\mu_1(y_1(k)) > \mu_2(y_2(k))$ for all k. That this holds follows from the fact that both μ_1 and μ_2 are monotonically decreasing and they are equal at y=1.

■

Proof of Proposition 3: First, note that as the growth rate is strictly positive the stock of capital is monotonically increasing over time. Consider (i). From Proposition 2 if $k_0<k_A$, the optimal y(k) is zero. Thus, in this region, taxes are zero and growth is given by γ(0). Pollution is increasing and it can be verified that the ratio of s_t to I_t --where I is income-- is constant.

(ii) Even if the economy starts with $k_0<k_A$ a positive growth rate of capital implies that eventually the capital stock reaches k_A. At that point taxes start to increase smoothly (although they remain strictly below one) and the growth rate of capital increases (see in Figure 3 the region 0<y<1). From Proposition 2, the first order condition for an interior solution is,

$$\zeta(y_t) = u'(\mu_1(y_t)k_t)\mu_1(y_t)k_t.,$$

or, equivalently,

$\zeta(y_t) = u'(s_t)s_t$,

but, from the definition of $\zeta(y_t)$ (see Assumption A) it is a decreasing function of y, and y is increasing in t. Thus, in this region, $\zeta(y_t)$ is decreasing. Given our monotonicity assumption on $u'(x)x$ it follows that s_t must be decreasing.

(iii) When the capital stock reaches k_F (see Proposition 2) there is a regime switch and taxes jump upwards. Since the level of pollution is given by $s_t = \mu(y_t)k_t$, it must discontinuously decrease.

(iv) After the capital stock gets past the value k_F, $y_2(k)$ --and hence y(k)-- is increasing. More specifically, from the proof of Proposition 2 it follows that y_t solves,

(P.3.1) $\eta(y_t) \equiv \beta\, (p'_{k2}(y_t)/p_{k2}(y_t))(\mu_2(y_t)\, /\mu'_2(y_t)) = u'(\mu_2(y_t)k_t)\mu_2(y_t)k_t$.

Direct calculation shows that $\eta(y_t)$ is continuous, monotone increasing and satisfies $\lim_{y\to\infty}\eta(y)=\beta\alpha/(1+\nu/\theta)$. It follows from this equation (as well as from Proposition 2) that y_t is increasing (it is increasing in k_t and k_t is increasing over time). To show that it goes to ∞ suppose to the contrary that it converges to some value y*. This generates a contradiction in (P.3.1) as t goes to ∞, since the left hand side converges to $\eta(y^*)$, while the right hand side goes to ∞.

To study the behavior of pollution rewrite (P.3.1) as,

$\eta(y_t) = u'(s_t)s_t$.

Since $\eta(y_t)$ is increasing, so is s_t. Given that $\eta(y_t)$ converges to $\beta\alpha/(1+\nu/\theta)$, it follows that the limiting level of pollution also satisfies

$u'(s^*_\tau)s^*_\tau = \beta\alpha/(1+\nu/\theta)$.

Finally, consider the growth rate of consumption, γ_{ct}. From the production function it follows that $\gamma_{ct} = (\gamma_{kct})^\alpha$, where γ_{kct} is the growth rate of capital used in the consumption sector. From equation (2.13) (using the specific forms of m and φ) one gets,

$\gamma_{kct} = \gamma_{kt}\, (\gamma_{yt})^{-1}$,

where γ_{kt} is the growth rate of the capital stock, and γ_{yt} is the growth rate of taxes.

From the definition of s_t, after the equilibrium values of all functions have been substituted in, we get,

$\gamma_{kct} = \gamma_{st} (\gamma_{yt})^{\nu/\theta}$.

From these two equations it follows that

$\gamma_{kt} (\gamma_{yt})^{-1}=\gamma_{st} (\gamma_{yt})^{\nu/\theta}$,

where γ_{st} is the growth rate of pollution.

Let a starred variable denote the limit as $t\to\infty$ of any of these growth rates. It follows that,

$\gamma_k^* = 1 \text{ x } (\gamma_y^*)^{(\nu/\theta+1)}$,

since asymptotically s_t does not grow.

It follows that,

$\gamma_{kc}^* = \gamma_k^{*\ 1/(\nu/\theta+1)} > 1$.

This, of course, implies that the limiting growth rate of consumption is positive.

■

Proof of Proposition 4: (sketch)We first characterize how the optimal z varies with k. With this done, the assumption on parameters that imply that k is strictly increasing over time result in the behavior of time paths as stated in the proposition. Using (4.1) and (4.2) as well as the indirect utility function V_q, it follows that,

$$\partial V_q/\partial z = \beta m'(z)/m(z)\{[(1-\alpha)m(z)a_0/(a_0m(z)-a_1)]-1\} + u'(\mu_q(z)k)\ \mu_q(z)k\ [a_0m'(z) +\nu\ (1+z)^{-1}]/[a_0m(z)-1].$$

Given our assumptions about u, the second term converges to zero as k goes to zero. Thus, to establish that for small k the equilibrium z will be zero, it suffices to show that

$$\beta m'(z)/m(z)\{[(1-\alpha)m(z)a_0/(a_0m(z)-a_1)]-1\}>0.$$

Simple algebraic manipulations show that this inequality holds if and only if, $m(z) > a_1/\alpha a_0$.

Note that the assumption $z_M=0$ requires $1> a_1/\alpha a_0$, which completes the argument as

$m(z) \geq 1$.

Next we characterize the equilibrium z as a function of k when the solution is interior. Algebraic manipulation of the condition $\partial V_q/\partial z = 0$ show that it is equivalent to,

$$S(z) = G(z,k),$$

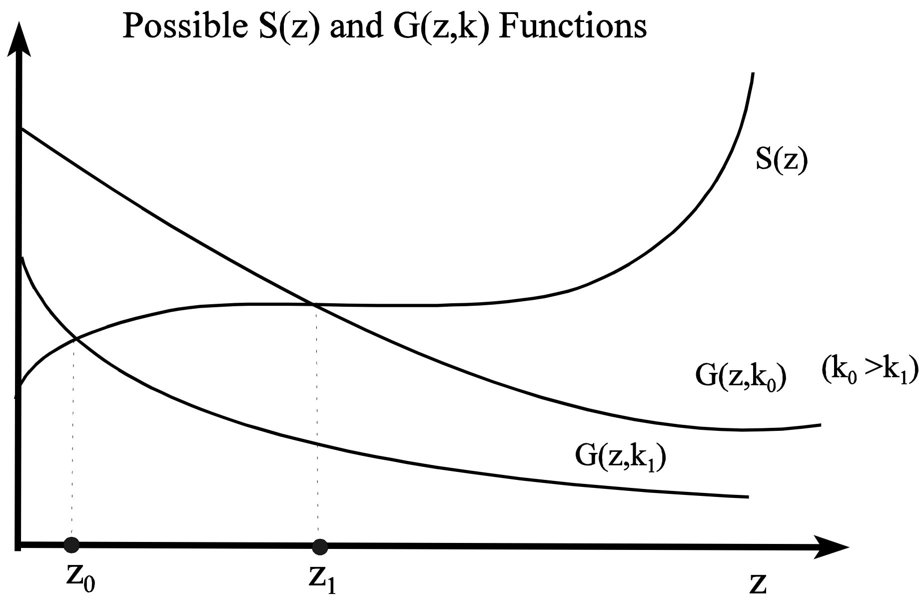
where,

$$G(z,k) = u'(\mu_q(z)k)\ \mu_q(z)k,$$

and,

$$S(z) = [(a_0 m(z)-a_1) - (1-\alpha)]/[(\nu/\theta)(a_0 m(z) - a_1) + a_0 m(z)].$$

It follows that G(z,k) is decreasing in z and increasing in k, while S(z) is increasing in z. The following Figure shows possible S and G functions. Note that the optimal z is unique, and that it increases as k increases. Moreover, as proved above, for small k, G(0,k) < S(0), and the equilibrium z is zero.



Proof of Proposition 5: The planner's problem is

$$\text{Max } W = \sum_{t=0}^{\infty} \beta_P^t \,[\, \ln(c_t^t) + \beta \ln(c_t^{t-1}) - u(s_{t+1})$$

subject to,

$$c_t^t + c_t^{t-1} \leq B\, k_{ct}^{\alpha}, \qquad (\lambda_{1t})$$

$$k_{t+1} \leq (1-\delta+A)k_t + [(1-\delta) - (1-\delta+A)m(z_t)]k_{ct}, \quad (\lambda_{2t})$$

$$\varphi(z_t)\, k_{ct} \leq s_t. \qquad (\lambda_{3t})$$

This statement of the planner's problem assumes that only one quality, z_t, is chosen at time t. A more general statement of the problem should allow for measures over z's as the relevant choice variable. Later in the proof we state the problem in this more general framework, and show that the optimum is such that only one quality is chosen at any given time.

The first order conditions for the planner's problem are,

(P.5.1) $(c_t^t)^{-1} - \lambda_{1t} \leq 0$

(P.5.2) $(c_t^{t-1})^{-1} - \beta_P \lambda_{1t} \leq 0$

(P.5.3) $- u'(s_t) + \beta_P \lambda_{3t} \leq 0$

(P.5.4) $\lambda_{1t}\, \alpha\, B\, k_{ct}^{\alpha-1} + \lambda_{2t}\, [(1-\delta) - (1-\delta+A)m(z_t)] - \lambda_{3t}\, \varphi(z_t) \leq 0$

(P.5.5) $-\lambda_{2t}\, (1-\delta+A)\, m'(z_t) - \lambda_{3t}\, \varphi'(z_t) \leq 0$

(P.5.6) $-\lambda_{2t} + \beta_P\, (1-\delta+A)\, \lambda_{2t+1} \leq 0$,

with the conditions holding at equality if the solution is interior. It is useful first to study the behavior of the solution in the interior case. Using the result that $\varphi'(z) = -\nu\, (1+z)^{-1}\, \varphi(z)$,and $m'(z) = \theta\, (1+z)^{-1}\, m(z)$, (P.5.5) is,

(P.5.7) $\qquad \theta\, \lambda_{2t}\, (1-\delta+A)\, m(z_t) = \nu\, \lambda_{3t}\, \varphi'(z_t)$.

For future reference we point out that if z=0 then the relevant version of (P.5.7) is,

(P.5.7') $\qquad \theta\, \lambda_{2t}\, (1-\delta+A)\, m(z_t) > \nu\, \lambda_{3t}\, \varphi'(z_t)$.

Using (P.5.1) , (P.5.2) and the feasibility constraint for the consumption sector, it follows that,

(P.5.8) $$1 + (\beta/\beta_P) = \lambda_{1t}\, B\, k_{ct}^{\alpha}.$$

Substituting in (P.5.8) in (P.5.4) and imposing (P.5.7) we get,

(P.5.9) $$1 + (\beta/\beta_P) = \lambda_{2t}\, k_{ct}\, [(1+\theta/\nu)(1-\delta+A)m(z_t) - (1-\delta)].$$

Next, use this expression (and (P.5.7)) in (P.5.3) to get,

(P.5.10) $$u'(s_t)s_t = \alpha\, (\beta + \beta_P)\, (\theta/\nu)\, (1-\delta+A)\, m(z_t)/[(1+ \theta/\nu)(1- \delta+A)m(z_t) - (1-\delta)].$$

Equation (P.5.10) completely summarizes the behavior of pollution in any interior solution. Direct calculation shows that the right hand side of (P.5.10) is decreasing in z_t. Thus, if z_t is increasing in equilibrium (we will show this to be the case), s_t is decreasing. Taking into account that $s_t = \varphi(z_t)\, k_{ct}$, it follows from (P.5.10) that if $\lim_{t\to\infty} k_{ct} = \infty$ (we will show this to be the case as well) then $\lim_{z\to\infty} z_t = \infty$. To show that $k_{ct}\to\infty$, suppose to the contrary that it remains in a bounded set. In this case (P.5.10) also implies that z_t remains in some bounded set. Using (P.5.5) it follows that λ_{2t} is bounded. However, since $\beta_P\, (1-\delta+A)>1$, (P.5.6) implies that $\lambda_{2t}\to\infty$, which gives rise to a contradiction. Thus, $k_{ct}\to\infty$ and, consequently $z_t\to\infty$.

Finally, the limit of the right hand side of (P.5.10) as $z\to\infty$ is $(\beta+\beta_P)\, \alpha/(1+\nu/\theta)$. Thus, it follows that s_t is decreasing (if $z_t>0$) and converges to s_p^* , where

$$u'(s_p^*)s_p^* = (\beta+\beta_P)\, \alpha/(1+\nu/\theta).$$

We next show that for k small, the optimal solution is a corner solution with z=0. For this to be the case it is sufficient that (P.5.7') hold. The appropriate version of (P.5.10) is,

(P.5.10') $$u'(\varphi(0)k_{ct})\varphi(0)\, k_{ct} < \alpha\, (\beta + \beta_P)\, (\theta/\nu)\, (1-\delta+A)/[(1+ \theta/\nu)(1- \delta+A) - (1-\delta)],$$

which clearly must hold for small k since k_c is bounded above by k, and the left hand side is converging to 0 as $k_c\to 0$.

To complete the proof we show that z_t is indeed increasing (and in passing we also show that only one z is chosen at time t). To do this, it is convenient to write the planner's problem as a dynamic problem with the choice variable being a measure, μ, over the space of possible qualities (which we denote Z), with the interpretation that $\mu(\{z\})$ is the amount of capital of type z that the planner allocates to the consumption sector. With this relabeling, the appropriate version of Bellman's equation for the planner's problem is (see Stokey and Lucas (1989) for details on this),

(P.5.11) $V(k) = Max_{\mu,x,k'} \{f(\mu) + \beta V(k')\}$

subject to,

$k \geq \int m(z)\, \mu(dz) + x,$
$k' \leq (1-\delta)[\int \mu(dz) + x] + Ax.$

Here, $f(\mu)$ is the indirect utility function given by,

$f(\mu) = \max \{\ln(c_y) + \beta \ln(c_o) - u(s)\}$

subject to,

$c_y + c_o \leq B[\int \mu(dz)]^{\alpha}$
$s \leq \int \varphi(z)\, \mu(dz),$

and where the maximization is over c_y, c_o and s. Straightforward calculations show that the function f is a strictly concave function of μ. Next a direct application of the arguments in Stokey and Lucas (1989) (Theorem 4.8) shows that the value function for the planner's problem, V(k), is strictly concave. This follows because the objective function is strictly concave and the constraint set is convex (in fact linear) in the choice variables. The arguments used by Benveniste and Scheinkman (1979) apply to our problem and establish that V is differentiable when the solution is interior (see the discussion in Stokey and Lucas (1989) Theorems 4.10 and 4.11).

Even though we stated the planner's problem in a general way by allowing measures to be the choice variable, it is easy to see that the optimal measure would be a Dirac measure that puts mass k_{ct} on the set $\{z_t\}$, where k_{ct} and z_t are the variables characterized in the planner's problem at the beginning of the proof. To prove this note that both m(z) and $\varphi(z)$ are convex ($\varphi(z)$ is strictly convex given our assumptions about relevant parameter values), and a direct application of Jensen's inequality establishes that a point mass measure on some z_t dominates any measure that puts positive mass on more than one point. Thus, in the planner's problem stated at the beginning of the proof, the assumption that there is a unique z_t is without loss of generality.

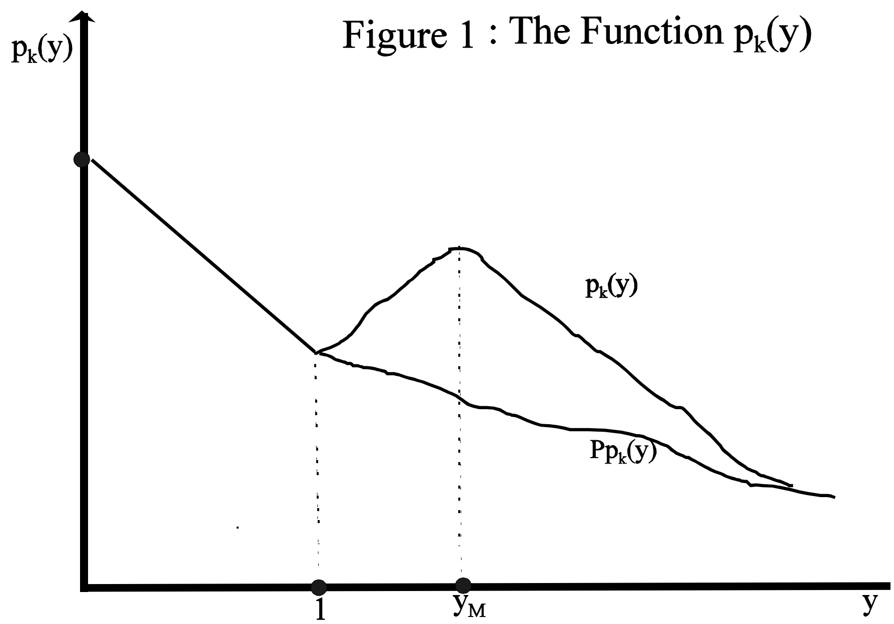
Finally, to establish that the sequence z_t is increasing when it is interior (we already showed that there is a region in which it is zero), rewrite the planner's's problem as,

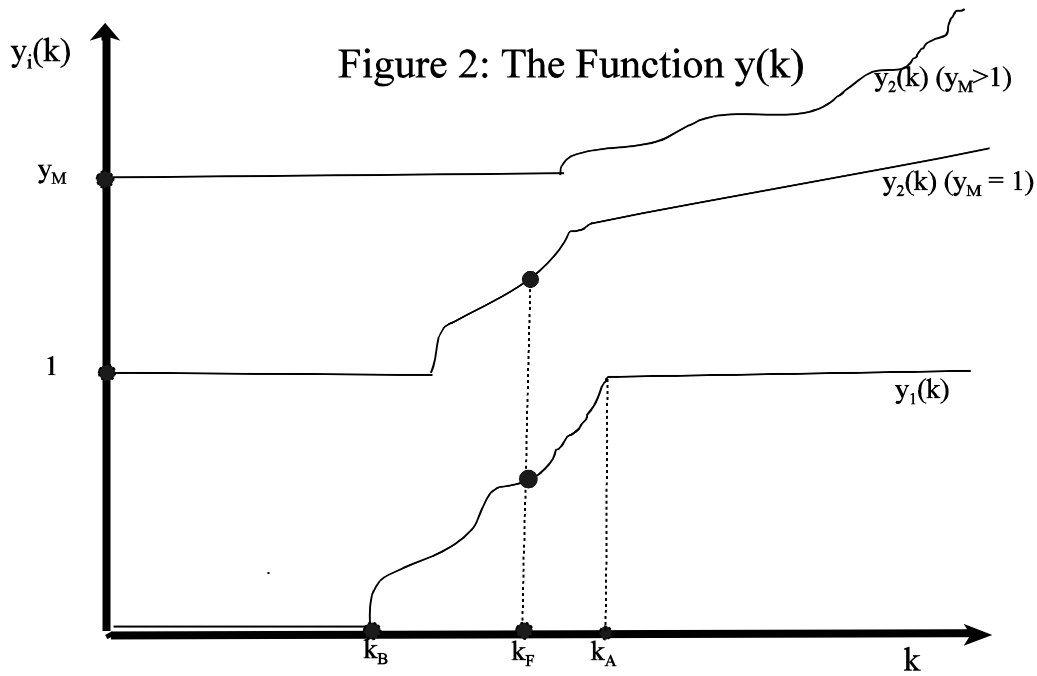
$\max \{C_0 + \alpha(1+\beta) \ln(k_c) - u\, (\varphi(z)k_c) + \beta\, V[(1-\delta)k_c - (1-\delta+A)(k - m(z)k_c]\},$

where C_0 is a constant. This formulation already incorporates the result that the choice of z is unique. Let the function inside set brackets be denoted $M(k_c,z;k)$. It follows from the concavity of V that the function is supermodular (see Topkis (1979) and Milgrom and Shannon (1994)) and hence that higher values of k result in higher values of z. However, since the growth rate is positive, the sequence k_t is increasing. Thus, the sequence z_t increases as well.

■

Figures





$y_i(k)$
Figure 2: The Function y(k)
$y_2(k)$ $(y_M>1)$
y_M
$y_2(k)$ $(y_M = 1)$
1
$y_1(k)$
k_B
k_F
k_A
k

Figure 3: The Time Path of Pollution and Taxes

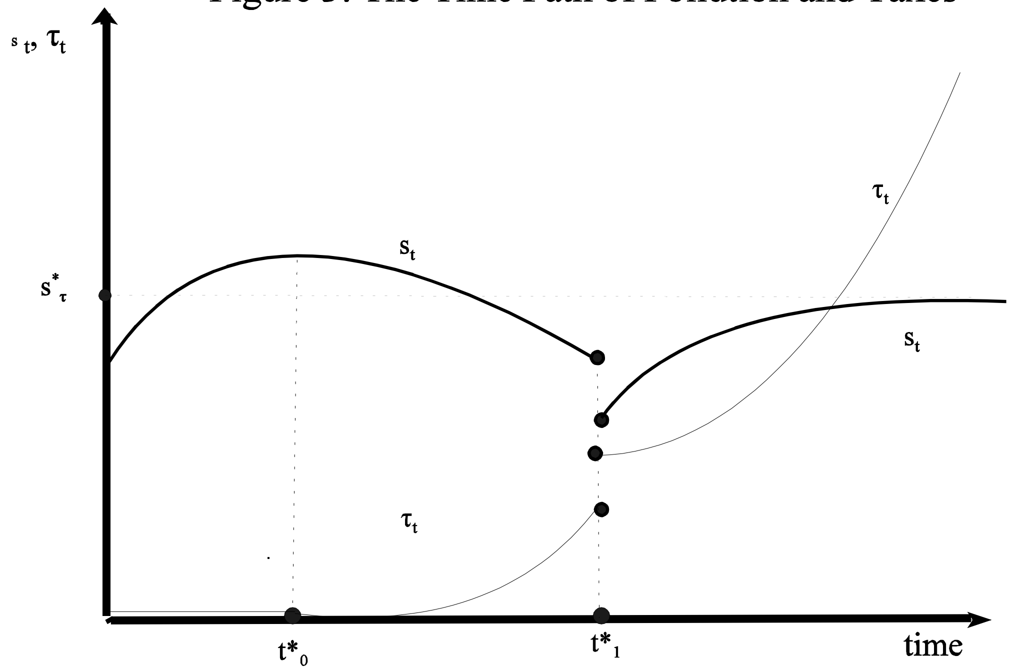


Figure 4: Voting over Qualities: Pollution and z

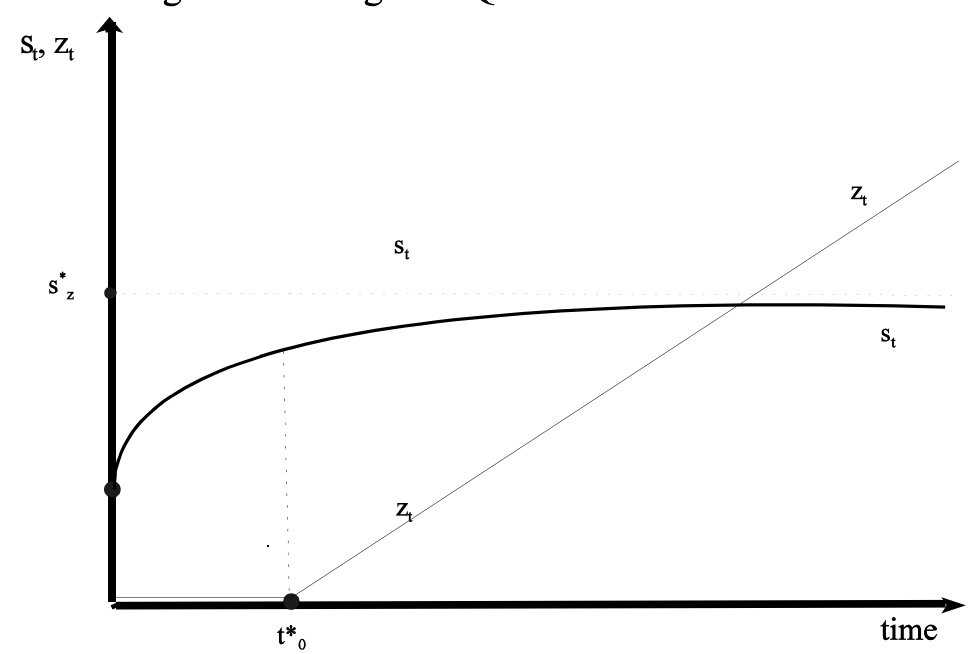


Figure 5: Planner's Problem: Pollution and z

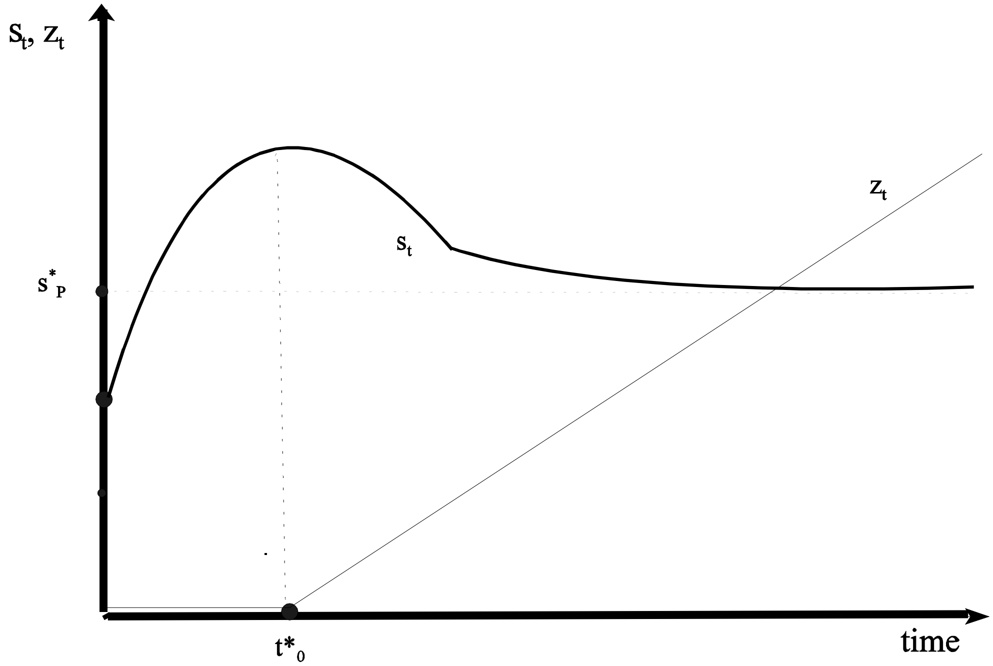


Figure 6: Growth Rate of Capital and Taxes