

# Policy Uncertainty, Total Factor Productivity and Growth

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## Abstract

This note shows that differences (across sectors or regions) in the effective price of inputs results in low measured total factor productivity (TFP).

## 1 Introduction

The literature on factor taxation has shown that in cases in which tax rates are known —i.e. in the absence of uncertainty— taxes on physical or human capital reduce the growth rate of output (see e.g. [3], [6]). The precise magnitude of the negative impact of taxation depends crucially on the particular specification of the model, see for example [7].

In addition to the effect of mean tax rates, the recent growth literature has analyzed the effects on growth of the variance of policies (and shocks). From a theoretical point of view there are no clear cut results. As shown by [4] increases in the temporal variability of tax rates can either increase or decrease the growth rate of output depending on the specification of preferences and technology. From a quantitative point of view, for the most common specifications increases in the variance of tax rates result in higher growth rates of output, although the effect is small (see e.g. [8],[5], [1], [2], [4]).

What is the channel through which tax regimes affect growth? In standard models —an the ones cited above all belong in this category— distortionary tax regimes have a negative impact on factor accumulation and, through this mechanism, result in lower growth (or output). Thus, tax distortions *should not* result in low Total Factor Productivity (TFP). Thus, from a theoretical point of view, distortionary tax regimes of the type analyzed in the literature cannot “explain” low TFP.

Why is it that understanding low TFP is important? The basic reason is that many studies have estimated that changes in TFP are the *driving force* underlying many episodes of low or negative growth. For example, standard dynamic stochastic

models have to rely on low realizations of the Solow residual (TFP) to account for such episodes as the great depression, the U.S. productivity slowdown in the 70's and the poor performance of many Latin American economies in the 80s. Moreover, the appropriate policy response to an episode of low TFP is dependent on what causes changes in TFP. If variations in TFP are due to technology or terms of trade shocks which are independent of policy actions, a governmental response is justified only in the presence of distortions. This, of course, is a standard second best argument. However, if contrary to the traditional findings there are government policies that *cause* low TFP, the best response is quite different. In this case, removal of the policy-induced distortions is the appropriate action.

In this note I present a simple model of variable —across economic sectors or regions— sectoral incentives and I show that increases in the *cross-sectional variability* of the tax/subsidy scheme result in lower TFP. The model has the usual properties associated with models of taxes and growth: An increase in mean tax rates decrease the long run level of per capita output, but not measured TFP. However increases in the cross-sectional dispersion of tax/subsidy rates on capital and labor lower TFP. Moreover, the magnitude of the decrease is larger the more correlated (across sectors) are the distortions on capital and labor. The key mechanism is that variation across sectors in tax/subsidy schemes result in a static inefficiency in the allocation of resources. This, in the aggregate, appears as lower TFP. In addition to the static effect, there is a dynamic distortion that affects factor accumulation. Depending on parameter values, increases in the cross-sectional variability of tax/subsidy rates reinforce the negative effects induced by mean tax/subsidy rates. Thus, unlike standard models, distortions that affect different sectors in heterogeneous ways can account for low TFP.

In section 2 I present the model and show how the presence of cross-sectional variability of incentives results in lower TFP. Section 3 discusses the equilibrium implications of that class of policies and analyzes dynamic inefficiencies. Section 4 presents some extensions, and section 5 offers some concluding comments.

## 2 Static Distortions in a Simple Economy

Consider an economy that produces a large number of goods indexed by  $i$ .<sup>1</sup> Let aggregate consumption at time  $t$ ,  $c_t$ , be given by  $\int c_{it} di$ . Thus, effectively, all consumption goods are assumed to be perfect substitutes, or alternatively, the economy is open

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<sup>1</sup>Alternatively, it is possible to interpret  $i$  as an index of the location where the homogeneous good is produced. Both interpretations —different sectors or different regions— yield exactly the same results.

and their prices are fixed<sup>2</sup>. The production technology for sector  $i$  is,

$$y_{it} = Ak_{it}^\alpha n_{it}^\theta a_i^{1-\alpha-\theta} \quad 0 < (\alpha, \theta) < 1, \quad \alpha + \theta < 1, \quad (1)$$

where  $k_{it}$  and  $n_{it}$  are, respectively, the amount of capital and labor allocated sector  $i$ . The factor  $a_i$ , which I assume fixed, is interpreted as managerial ability, although it is a stand-in for all sector-specific factors. It is also a measure of size of each sector. It turns out that allowing the  $a_i$ 's to be jointly distributed with the distortions has no impact on the predictions of the model for the measurement of TFP. Thus, without loss of generality, I assume that each  $\theta_i = 1$  for all  $i$ .<sup>3</sup>

To model distortionary government policy I assume that through the use of sector specific taxes and/or subsidies the government affects the effective factor prices faced by each sector's firms. More precisely, if the price of a factor is  $p$  in the "open market" a producer in sector  $i$  faces a price equal to  $p/(1-\tau_i^j)$ , where  $\tau_i^j$  is the tax/subsidy rate faced by producers in sector  $i$  when purchasing input  $j$ . If  $\tau_i^j$  is positive, it corresponds to a tax, while if it is negative it is a subsidy. Note that although producers face an "effective" price given by  $p/(1-\tau_i^j)$ , factor owners receive only  $p$ . The difference is a tax (or subsidy) that accrues to the residual claimants. I assume that both inputs are mobile. However, the same results apply if capital is not mobile ex-post; that is, if capital is assigned to a specific sector before the sectoral realization of tax/subsidy rates is known.

The first order conditions corresponding to the optimal choices of capital and labor imply that they satisfy,

$$(1 - \tau_{it}^n)\theta k_{it}^\alpha n_{it}^{\theta-1} = w \quad (2a)$$

$$(1 - \tau_{it}^k)\alpha k_{it}^{\alpha-1} n_{it}^\theta = r \quad (2b)$$

It is straightforward to solve this system of equations and to obtain explicit solutions for the two inputs. Simple manipulations yield

$$k_{it} = (1 - \tau_{it}^n)^{\frac{\theta}{1-\alpha-\theta}} (1 - \tau_{it}^k)^{\frac{1-\theta}{1-\alpha-\theta}} C_k \quad (3a)$$

$$n_{it} = (1 - \tau_{it}^n)^{\frac{1-\alpha}{1-\alpha-\theta}} (1 - \tau_{it}^k)^{\frac{\alpha}{1-\alpha-\theta}} C_n \quad (3b)$$

where  $C_k$  and  $C_n$  are constants independent of  $i$ .

Equilibrium requires that the averages over all sectors of  $k_{it}$  and  $n_{it}$  from the previous two equations equal the economy-wide values denoted by  $k_t$  and  $n_t$ , respectively. It is convenient to view a particular pair  $(1 - \tau_{it}^n, 1 - \tau_{it}^k)$  as being drawn from some joint distribution. Given this interpretation, averages can be approximated by

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<sup>2</sup>This is not an essential assumption. It is possible to generalize this to preferences over a variety of goods by using some version of the Dixit-Stiglitz aggregator. This alternative formulation increases somewhat the algebraic complexity without any new insights.

<sup>3</sup>This is due to the assumption that the function is homogeneous of degree one in all factors. This implies that changes in  $a_i$  correspond to changes in the size of each sector.

expectations under the usual regularity conditions. Thus, imposing that  $E[k_{it}] = k_t$  and  $E[n_{it}] = n_t$ , it follows that sectoral allocations of capital and labor satisfy

$$k_{it} = k_t (1 - \tau_{it}^n)^{\frac{\theta}{1-\alpha-\theta}} (1 - \tau_{it}^k)^{\frac{1-\theta}{1-\alpha-\theta}} E[(1 - \tau_{it}^n)^{\frac{\theta}{1-\alpha-\theta}} (1 - \tau_{it}^k)^{\frac{1-\theta}{1-\alpha-\theta}}]^{-1} \quad (4a)$$

$$n_{it} = n_t (1 - \tau_{it}^n)^{\frac{1-\alpha}{1-\alpha-\theta}} (1 - \tau_{it}^k)^{\frac{\alpha}{1-\alpha-\theta}} E[(1 - \tau_{it}^n)^{\frac{1-\alpha}{1-\alpha-\theta}} (1 - \tau_{it}^k)^{\frac{\alpha}{1-\alpha-\theta}}]^{-1} \quad (4b)$$

These expressions show that sectoral demand for capital and labor depend on the difference between a sector's taxes and the economy-wide averages. They also reveal the source of static inefficiency associated with this policy: Differences across sectors in the effective price of inputs. To see this note that if all sectors faced similar tax rates,  $k_{it} = k_t$  and  $n_{it} = n_t$  and the capital-labor ratio would be the same for all sectors. This, of course, implies that TFP equal A in this economy.

In order to more precisely characterize the role of the cross-sectional variability in incentives it is useful to assume that the joint distribution of the log of  $(1 - \tau_{it}^n, 1 - \tau_{it}^k)$  is normal with mean  $(1 - \mu_n, 1 - \mu_k)$  and variance-covariance matrix with diagonal elements  $\sigma_n^2$  and  $\sigma_k^2$ , and off-diagonal elements given by  $\rho\sigma_n\sigma_k$ . Thus,  $\rho$  is a measure of how strongly the two sectoral distortions are correlated. A positive  $\rho$  indicates that sectors with high capital taxes (or subsidies) are also sectors with high labor taxes (or subsidies).

Substituting (4a) and (4b) in (1) and then taking again the average over all sectors to define  $y_t = E[y_{it}]$ , it follows that output is

$$y_t = \Delta(\sigma_n, \sigma_k, \rho) A k_t^\alpha n_t^\theta \quad (5)$$

where the function  $\Delta(\sigma_n, \sigma_k, \rho)$  satisfies

$$\Delta(\sigma_n, \sigma_k, \rho) = \exp\left\{-\frac{1}{(1-\alpha-\theta)}\left[\frac{\theta(1-\alpha)\sigma_n^2}{2} + \frac{\alpha(1-\theta)\sigma_k^2}{2} + \rho\alpha\theta\sigma_n\sigma_k\right]\right\} \quad (6)$$

In the absence of distortions, TFP in this economy is just A. Thus, it seems reasonable to interpret  $\Delta(\sigma_n, \sigma_k, \rho)$  as a measure of the TFP "gap." If either  $\sigma_n$  or  $\sigma_k$  are positive,  $\Delta(\sigma_n, \sigma_k, \rho) < 1$ , and *actual* TFP falls short *potential* TFP. In this setting distortionary government policies decrease TFP.

What drives the results? First, since the  $\mu_j$  do not enter in the expression for the TFP gap,  $\Delta(\sigma_n, \sigma_k, \rho)$ , changes in mean taxes (or subsidies) do not affect TFP. Second, increases in the variances of the two taxes (or the correlation between the two tax instruments) generates a *decrease* in measured TFP. Finally, the TFP gap is proportionally larger the higher the level of variability. This follows since the function  $\Delta(\sigma_n, \sigma_k, \rho)$  is strictly convex in  $(\sigma_n, \sigma_k, \rho)$ .

Can these effects account for substantial differences in measured TFP across countries or periods? The only possible way to determine the quantitative importance of the distortions analyzed in this paper is to numerically calculate  $\Delta(\sigma_n, \sigma_k, \rho)$ . As a first pass I consider an economy with shares of capital and labor equal to 0.4 (

$\alpha = \theta = 0.4$ ). This implies that if  $a_i$  is interpreted as returns to managerial ability, total returns to labor and managerial ability are 60% of output.<sup>4</sup> In addition to factor shares, it is necessary to specify the levels of cross-sectional variability and their correlation coefficient. Since I have no a priori information on whether capital or labor distortions are more severe I assume  $\sigma_n = \sigma_k = \sigma$ . For the lognormal distribution the coefficient of variation of a variable is approximately equal to the standard deviation of its log. Thus, the values of  $(\sigma_n, \sigma_k)$  should be interpreted as measures of the cross-sectional variability of incentives relative to the mean level of distortion. Thus, a value of 0.5 corresponds to the case in which the coefficient of variation is 50%. I considered values of  $\sigma$  in the interval  $[0.1, 0.7]$  with increment size equal to 0.1, and several values of the correlation coefficient  $\rho$ . The results are in Table 1

$\sigma$	$\rho$					
	-0.8	-0.4	0.0	0.4	0.8	1.0
0.1	.99	.99	.98	.98	.98	.98
0.2	.98	.97	.95	.94	.93	.92
0.3	.95	.92	.90	.87	.85	.83
0.4	.91	.87	.83	.78	.74	.73
0.5	.87	.80	.75	.68	.63	.61
0.6	.82	.73	.65	.58	.52	.49
0.7	.76	.65	.56	.47	.41	.38

There are a couple of significant patterns. First, for high but not extreme levels of variability the model generates a substantial drop in TFP. Second, and more interesting, when distortions are correlated across sectors, even moderate cross-sectional coefficients of variation can result in large drops in productivity. From this numerical exercise, it is clear that the correlation of distortions across sectors is at least as important as their variability when it comes to estimating the output costs of policy distortions. It is also clear that high distortion countries (or periods) —as measured by  $\sigma_k$  and  $\sigma_n$ — are also low TFP countries (periods).

From a practical point of view it is very difficult to directly measure the distortions,  $(\tau_{it}^n, \tau_{it}^k)$ , since they capture a number of more specific policies such as: sectoral (or regional) credit subsidies or quotas, differences across sectors (or regions) in labor market legislation, and sector (or region)-specific promotional regimes, among others. It is then useful to use indirect evidence on the prevalence of distortions. In the

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<sup>4</sup>It is not clear how in the National Income and Product Accounts are the returns to fixed factors allocated. In some instances, special skills —e.g. owner’s organizational skills— are likely to be counted as profits and, hence, as part of the return to capital. This would argue for smaller values of  $\alpha$  and higher values of  $\theta$ . In other cases, the same skills are priced by the market and are counted as labor compensation.

context of the simple model described in this note there are two variables whose cross-sectional variability provide evidence of heterogeneity across sectors: the capital-labor ratio, and the return to managerial ability.

Let  $\kappa_{it}$  be the capital-labor ratio in sector  $i$  at time  $t$ . Simple calculations using (4a) (4b) show that,

$$\kappa_{it} = C \frac{k_t}{n_t} \frac{1 - \tau_{it}^k}{1 - \tau_{it}^n}$$

where  $C$  is a constant. Let the variance of the log of  $\kappa_{it}$  be denoted  $\sigma^2(\ln \kappa_{it})$ . Then it follows that,

$$\sigma^2(\ln \kappa_{it}) = \sigma_k^2 + \sigma_n^2 - 2\rho\sigma_k\sigma_n.$$

Thus, if there is no cross-sectional variability in tax/subsidies,  $\sigma^2(\ln \kappa_{it})$  should be zero. Evidence of variability, and especially changes over time, is indirect evidence for the presence of distortions.

Another variable that captures the relevant features of the tax code is the unit price of the sector-specific resource,  $a_i$ . In applications, this is a measure of profits in excess of the normal rate of return to capital and/or excess payments to managers. In the model of this paper the unit price of  $a_i$ , denoted  $p_i$ , is given by  $p_i = (1 - \alpha - \theta) \frac{y_i}{a_i}$ . Using (4a) and (4b) in (1) it follows that the variance of  $\ln p_{it}$  is given by,

$$\sigma^2(\ln p_{it}) = \left( \frac{\alpha}{1 - \alpha - \theta} \right)^2 \sigma_k^2 + \left( \frac{\theta}{1 - \alpha - \theta} \right)^2 \sigma_n^2 - \frac{2\alpha\theta}{(1 - \alpha - \theta)^2} \rho\sigma_k\sigma_n.$$

As in the case of the capital-labor ratio, variability across sectors in pure profits is indirect evidence of the presence of the type of distortions emphasized in this note.

### 3 Dynamic Equilibrium

The previous section showed that given stocks of capital and labor efficient allocation of resources using policy-distorted market prices can give rise to lower TFP in the presence of sector specific distortions. In this section I show that, in addition to the static distortion, cross-sectional dispersion of incentives affects output through capital accumulation. Moreover, it is shown that, in the steady state the degree of correlation of the two tax instruments does not matter. The notes also indicates how to extend the result to a setting in which the distribution of taxes is itself random. In that context, changes in the cross-sectional variability of tax rates is observationally equivalent —using aggregate data— to productivity shocks.

In order to simplify the presentation I will assume that leisure is not an argument in the utility function and that aggregate labor supply is equal to one. The notion of equilibrium is simply competitive equilibrium in which the representative household makes consumption and saving decisions taking the non-stochastic interest rate as given, while firms rent capital and labor in spot markets. The timing of events is

as follows: At time  $t$ , when saving decisions are made, consumers do not know the values of sector specific taxes  $(1 - \tau_{it+1}^n, 1 - \tau_{it+1}^k)$  but they know their distribution. After investment decisions have been made and the total available capital stock is determined, the values of  $(1 - \tau_{it+1}^n, 1 - \tau_{it+1}^k)$  are realized and capital and labor are allocated to each sector.

In this setting, firms solve static problems and households face truly dynamic optimization problems. The basic idea of the characterization of the equilibrium allocation follows [3]. The intuition of the argument is that a competitive equilibrium allocation solves a programming problem, although not a “standard” planner’s problem.

Consider first the representative household’s utility maximization problem. It is given by,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (7)$$

subject to,

$$c_t + s_{t+1} + \tau_t \leq (1 + r_{t+1})s_t + w_t \quad (8)$$

where  $s_t$  denotes savings carried from time  $t - 1$  to  $t$ ,  $r_t$  is the interest rate earned at time  $t$ ,  $\tau_t$  is the amount of (lump-sum) taxes or transfers paid by the household, and  $w_t$  is the wage rate at time  $t$ . In addition to this period-by-period budget constraint a transversality-like condition to prevent Ponzi games must be imposed. The household optimal consumption-saving decision is summarized by the Euler equation associated with the previous problem,

$$u'(c_t) = \beta u'(c_{t+1})[1 + r_{t+1}] \quad (9)$$

Consider next the decision by firms at time  $t+1$ . Profits are given by,

$$\pi_{it} = \max_{k,n} Ak_t^\alpha n^\theta a^{(1-\alpha-\theta)} - \frac{w_t}{1 - \tau_{it}^n} n - \frac{r_t + \delta}{1 - \tau_{it}^k} k \quad (10)$$

where each firm takes input “market” input prices,  $r_t$  and  $w_t$ , as given. Profit maximization implies that ,

$$(1 - \tau_{it}^k) \alpha k_{it}^{\alpha-1} n_{it}^\theta - \delta = r_t \quad (11)$$

Using (3a) and (3b) in (11) it is possible to derive that the equilibrium marginal product of capital satisfies,

$$\alpha Ak_t^{\alpha-1} \Upsilon(\mu_{kt}, \sigma_{kt}, \sigma_{nt}, \rho_t) = r_t - \delta \quad (12)$$

where the function  $\Upsilon(\mu_{kt}, \sigma_{kt}, \sigma_{nt}, \rho_t)$  satisfies,

$$\Upsilon(\mu_{kt}, \sigma_{kt}, \sigma_{nt}, \rho_t) = \exp\left\{1 - \mu_{kt} + \frac{\sigma_{kt}^2}{2}\right\} \exp\left\{\frac{1}{1 - \alpha - \theta} \left[ \frac{\sigma_{kt}^2}{2} \alpha (1 + \theta) - \frac{\sigma_{nt}^2}{2} \theta (1 - \alpha) + \rho \sigma_{nt} \sigma_{kt} \theta (1 - \alpha) \right]\right\} \quad (13)$$

In (13) all the parameters of the joint distribution of taxes are allowed to be time varying. The two terms on the right hand side of (13) have different interpretations. First,  $\exp\{1 - \mu_{kt} + \frac{\sigma_{kt}^2}{2}\}$  is just  $E[1 - \tau_t^k]$ . Thus, to investigate the effect of changes in cross-sectional variability holding mean taxes constant requires adjusting  $\mu_{kt}$  to keep this term unchanged. The second terms captures the “pure” effect of cross-sectional variability.

From the consumer’s Euler equation (9) and the firm’s marginal condition (12), it follows that in any equilibrium it must be the case that

$$u'(c_t) = \beta u'(c_{t+1})[1 - \delta + \Upsilon(\mu_{kt+1}, \sigma_{kt+1}, \sigma_{nt+1}, \rho_{t+1})\alpha A k_{t+1}^{\alpha-1}] \quad (14)$$

Following the arguments in [3], it is possible to show that the equilibrium allocation solves a pseudo planner’s problem. The key insight in constructing the correct programming problem is that it must deliver an equation like (14) as its first order condition, but it must also satisfy the consumer’s budget constraint or, alternatively, the economy-wide feasibility constraint. For this problem the appropriate pseudo-planner’s problem is,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (15)$$

subject to,

$$c_t + k_{t+1} \leq \Upsilon(\mu_{kt+1}, \sigma_{kt+1}, \sigma_{nt+1}, \rho_{t+1})A k_t^\alpha + T_t \quad (16)$$

where the sequence  $T_t$  must be chosen to guarantee that feasibility holds.<sup>5</sup>

What are the dynamic properties of the distorted economy? First consider time invariant policies. If  $\Upsilon(\mu_k, \sigma_k, \sigma_n, \rho) < 1$ , then the policy acts as an additional tax and, following standard arguments, it results in low capital per worker. Second, consider changes in policies. Any variations that result in increases in  $\Upsilon(\mu_k, \sigma_k, \sigma_n, \rho)$  will resemble positive TFP shocks, while changes that decrease  $\Upsilon(\mu_k, \sigma_k, \sigma_n, \rho)$  will resemble negative TFP shocks.

This analysis although suggestive is not complete since movements in  $\Upsilon(\mu_k, \sigma_k, \sigma_n, \rho)$  also result in movements in the sequence  $\{T_t\}$ . In order to determine the full impact of changes in policies it is necessary to jointly consider static and dynamic effects. In general, this is difficult to do analytically, and numerical methods are required. However, in this example it is possible to determine the effect of policies on steady state output per worker.

From the Euler equation (14) it follows that steady state capital per worker is given by

$$k^* = \left( \frac{\alpha A \Upsilon}{\beta^{-1} - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}$$

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<sup>5</sup>The sequence  $T_t$  satisfies  $(\Delta_t - \Upsilon_t)A k_t^{*\alpha}$ , where for brevity I suppress the arguments of the functions  $\Delta_t$  and  $\Upsilon_t$ . Moreover, the sequence  $\{k_t^{*\alpha}\}$  is the solution to the pseudo-planner’s problem. In [3] the details of the fixed point are spelled out.

and output per worker is just,

$$y^* = A \left( \frac{\alpha A}{\beta^{-1} - (1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} \Upsilon^{1-\frac{\alpha}{1-\alpha}} \Delta$$

where I have suppressed the arguments of  $\Upsilon$  and  $\Delta$ . Using (13) and (6), it follows that  $y^*$  is given by,

$$y^* = A \left( \frac{\alpha A}{\beta^{-1} - (1 - \delta)} \right)^{\frac{\alpha}{1-\alpha}} E[1 - \tau^k]^{\frac{\alpha}{1-\alpha}} \exp \left\{ \frac{1}{(1 - \alpha - \theta)} \left[ \frac{\sigma_k^2}{2} \frac{\alpha(2\alpha + \theta - 1)}{1 - \alpha} - \frac{\sigma_n^2}{2} \theta \right] \right\} \quad (17)$$

The impact of the policy on long-run output per worker is completely summarized by (17). As expected, the higher the mean tax rate on capital income the lower the level of steady state output per worker.<sup>6</sup> Increases in  $\sigma_k^2$  can, in principle, have positive or negative impact on output. A necessary and sufficient condition for an increase in  $\sigma_k^2$  to lower output is that  $\alpha < (1 - \theta)/2$ . This condition requires either low values of  $\alpha$  or, alternatively, low values of the labor share coefficient,  $\theta$ . It is not satisfied with  $\alpha = \theta = 0.4$ . On the other hand, increases in  $\sigma_n^2$  always result in lower output. Thus, the overall effect of increases in cross-sectional variability depends on the relative movement of the cross-sectional standard deviation of the two tax rates. In keeping with the example in section 2, I consider the case  $\sigma_k = \sigma_n = \sigma$  as the baseline situation. An increase in  $\sigma$  captures the effect of an *overall* increase in the cross-sectional variability in the tax regime. In this case, for reasonable parameter values—the precise condition is  $\alpha < 1/(2 - \theta)$ , and it automatically holds if capital’s share,  $\alpha$ , is less than 0.5—increases in  $\sigma$  reduce long run output per worker. Thus, in this case sectoral variability reinforces the impact of mean taxes and causes even further output losses.

How large are the quantitative effects of an increase in uncertainty on long run output per worker? The elasticity of  $y^*$  with respect to  $\sigma$  is  $\sigma^2 \left[ \frac{\alpha(2\alpha + \theta - 1) - (1 - \alpha)\theta}{(1 - \alpha)(1 - \alpha - \theta)} \right]$  which is increasing in  $\sigma$ . For the parameter values of the example used in section 2 this elasticity is  $-\sigma^2 1.33$ . Thus, for a coefficient of variation equal to 0.5, a 1% increase in overall variability (as measured by the coefficient of variation of both tax rates) reduces long run output per worker by 0.33%. For a given country, if there are significant changes in the level of cross-sectional variability over time, this can result in substantial movements in output per worker.

From a qualitative point of view there is one property of (17) that is worth emphasizing: The degree of correlation between the two tax instruments does not affect the steady state level of output per worker. This implies that two countries (or the same country at two different times) can have the same level of output per worker with different levels of TFP if the “driving force” are changes in the correlation coefficient  $\rho$ .

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<sup>6</sup>The reason why the tax rate on labor income does not affect output is because of the assumption of inelastic labor supply

Finally, it is possible —but quite difficult computationally— to consider the case in which the vector that defines the distribution of the tax policy is itself random. In this case, it can be shown that the pseudo-planner’s problem still describes the competitive allocation with expectations in place of realizations.<sup>7</sup>

## 4 Extensions and Concluding Comments

The model can be extended in several dimensions. First, the assumption that all production functions are identical and Cobb-Douglas allows for exact aggregation and for the derivation of closed form solutions. If, for example, all the sectoral production functions were of the CES variety, it is still possible to show that there exist functions similar to (4a) and (4b) such that  $k_{it}$  and  $n_{it}$  depend (linearly) on  $k_t$  and  $n_t$  and (nonlinearly) on sector specific tax rates and expectations of nonlinear functions of tax rates. To make progress with the more general formulation it is necessary to resort to numerical analysis.

In Section 2 a specific timing for the realization of tax rates was assumed. This, however is not essential. Two alternative timing schemes would have worked as well. In the first one the sequence of sector specific taxes is known. In this case the allocation is exactly the same as the one in the case discussed in the text. The reason is simple: ex-post, given the stocks of capital, the two realizations must coincide since this is implied by static efficiency at market prices. Since under this allocation the return to investment in all sectors is the same, investors who *know* the tax rates that will apply to each sector have no incentive to deviate from the proposed allocation. An alternative timing is one in which, as in the text, the particular values of the tax rates are not known until the beginning of a given period, but the capital allocation decision must be made in the previous period. In this case, it can be shown that similar results obtain and, as in the previous argument, the key observation is that ex-ante the rate of return on capital must be the same in all sectors. This takes into account that labor will move ex-post to equalize wages.

What are the implications of these ideas for the analysis of the growth process? First, the class of policies considered in this note provides one story that makes explicit the link between policies and TFP. Conditional on the model, TFP is not really a purely technological variable, but depends to some extent on government policies. The model is very explicit about the dimensions of government policy that affect TFP: it is the second moments of the incentive regime and not the means that can influence TFP.

From a practical perspective, this suggests that the variables that can reveal information about policy variability is the variability (across sectors or regions) in

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<sup>7</sup>To be precise, the fixed point argument in [3] goes through if the number of shocks —in this case the number of possible values of the parameters— is finite. In the infinite case, a new formal argument is necessary.

the return to the immobile factor and in the capital-labor ratio. Thus increases in the variability of sectoral profits or in capital-labor ratios can be interpreted as indirect evidence of increases in the cross-sectional variability in tax/subsidy rates. Unfortunately, there are other factors (e.g. changes in relative prices) that can have similar effects. Thus, the analysis needs to be supplemented with a careful discussion of sectoral and regional policies. Finally, it is important to note that the model predicts that sectoral or regional factor returns are equalized. Thus, changes in the variability of these observations is not an indication of changes in the tax regime. It is possible to allow for slow adjustment of one factor. This, on the one hand, weakens the impact of policy variability. On the other hand it implies that changes in the variability of sectoral factor prices can be evidence of changes in policy variability.

## References

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