Adjustment Costs and Consumption Behavior

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Housing services are different from other consumption goods. Moving typically entails substantial adjustment costs, so individuals adjust their consumption of housing services infrequently. Consequently, an individual's permanent income may rise or fall by substantial amounts without triggering a change in housing consumption. Consumer behavior thus has two components. Over intervals of time when housing is constant, the individual adjusts her nondurable consumption and her portfolio of financial assets continuously as her wealth fluctuates. At dates when she sells one house and buys another, her nondurable consumption and portfolio take discrete jumps.

This paper studies both kinds of behavior. Specifically, it looks at the implications of a transaction cost for housing adjustments for nondurable consumption and portfolio behavior. The analysis is quantitative, using simulations from a dynamic model with stochastic fluctuations in portfolio returns, and behavior is described both for periods when housing is fixed and at dates when it is adjusted.

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In the presence of adjustment costs, the elasticity of substitution between housing and nondurables becomes a critical parameter, even if relative prices are constant. To see this, suppose that preferences are homothetic, the relative price of housing is constant, and nondurable consumption and the portfolio are costlessly adjustable. Absent a transaction cost for housing, the individual chooses a constant mix of consumption goods. The mix depends on the relative price and the relative weights in preferences, but the elasticity of substitution is not important. With an adjustment cost for housing, however, nondurable consumption and the portfolio vary as permanent income varies, with housing constant. The magnitude of these fluctuations depends critically on the elasticity of substitution between the two components of consumption.

In particular, the lower the elasticity of substitution between nondurables and housing, the more weakly will consumption of nondurables respond to changes in permanent income during intervals when housing is fixed. Thus, in the short run consumption of both housing and nondurables may be unresponsive to fluctuations in permanent income, even if long run income elasticities are unity for both goods. In this sense household behavior in the model here is similar to behavior in setups where consumption has ‘fixed’ and ‘flexible’ components. It has the advantage, however, of being formulated in terms of fundamental—preferences and adjustment costs.

Adjustment costs also affect portfolio behavior. Portfolio behavior depends on the (local) risk aversion of the consumer’s value function for wealth, and the shape of this value function is distorted by the presence of a transaction cost. Thus, the relevant measure of risk aversion varies with total wealth and housing wealth, even if the underlying preferences display constant risk aversion. Specifically, the implicit risk aversion of an individual consumer changes depending on whether or not her wealth is near a threshold that will trigger an adjustment in housing.

The rest of the paper is organized as follows. Section 1 contains a brief review of the related literature. Preferences are described in section 2 and a model without transaction costs is studied briefly in section 3. The model with transaction costs is set out in section 4, the calibration is described in section 5, and the simulations are presented in section 6 Section 7 discusses the model's predictions about the equity premium puzzle, and section 8 concludes.

1. Related literature

There is a sizable literature, going back two decades, asking whether including durable goods can improve the fit of asset pricing models. Early attempts assumed that consumption of durables is flexible in the sense that there are no adjustment costs. In the absence of adjustment costs, homotheticity implies that the economy can be described, as usual, by a representative agent. That is, except for scale the behavior of all individuals and of the aggregate are identical. Under this assumption an Euler equation of the usual type holds for nondurables, and that equation can be tested on aggregate data. In this group are the papers by Dunn and Singleton (1986) and Eichenbaum and Hansen (1990). They found that including durables does little to improve the fit of the model.

The theoretical paper by Grossman and Laroque (1990) provided a framework for studying the behavior of an individual who consumes only one good, housing services, and faces adjustment costs for changing her level of consumption. They showed that the adjustment cost affects the consumer's portfolio behavior in a systematic way. Specifically, consumers who have recently adjusted their housing stock, and hence anticipate a long interval of time before another adjustment, are more risk averse than those who anticipate making an adjustment in the near future. Their model does not include nondurable consumption, however, so it provides no predictions about the behavior of standard Euler equations.

More recently, several papers have used models that incorporate nondurable consumption as well as adjustment costs for housing, to study nondurable consumption and portfolio behavior.[1]

Flavin and Nakagawa (2004) study a model similar to the one here that also includes life cycle effects and house price risk, and nests a habit persistence model as well. Using data from the PSID, they estimate various model parameters and test parameter restrictions corresponding to the adjustment cost model and the habit persistence model. They estimate a very low elasticity of substitution between housing and nondurables, and they find that while the habit persistence model can be rejected, the housing model cannot be.

Martin (2003) also uses the PSID, and distinguishes households who are likely to make upward and downward adjustments in their housing from those who are unlikely to adjust. He finds evidence that relative to the last group, households that are likely to move to a larger house reduce their consumption of nondurables, and those likely to move to a smaller house raise their consumption of nondurables.

Siegel (2004) looks at both aggregate and micro data. He finds that aggregate data suggest that adjustment costs are important, and that household-level consumption data suggest that preferences are nonseparable between housing and nondurables.

Chetty and Szeidl (2004) examine portfolio behavior using data from the Survey of Income and Program Participation, taking housing tenure as a proxy for consumption commitments. They find that households who have moved recently, and hence are unlikely to move again in the near future, display more risk averse portfolio behavior than those with longer tenures at their current residence.

Kullmann and Siegel (2005) use the PSID to look at portfolios of homeowners,

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[1] In addition, many papers have studied other channels—like house price risk—through which housing affects portfolio choice and nondurable consumption. For example, see Campbell and Cocco (2005), Flavin and Yamashita (2002), Fukushima (2005), Lustig and Nieuwerburgh (2006), Piazzesi, Schneider, and Tuzel (2007), and Fillat (2007).

and find evidence of state-dependent risk aversion. Specifically, they find that higher ratios of house to net worth are correlated with lower stock market participation and reduced holdings of stocks and other risky assets.

Cocco (2005) studies a model similar to the one here that includes life cycle effects, house price risk, and fixed costs of stock market participation, and calibrates it to the PSID. He finds that younger and poorer households hold less equity and that house price risk crowds out equity holding.

2. Preliminaries

There are two consumption goods, housing services H and a single composite nondurable C. The flow of housing services H reflects both size and quality, including features like location, lot size, and other attributes. Preferences over the two goods have the CES form

$$U(C,H) = \begin{cases} \left[\omega C^{1-\zeta} + (1-\omega)\, H^{1-\zeta}\right]^{1/(1-\zeta)}, & \zeta \neq 1, \\ C^{\omega} H^{1-\omega}, & \zeta = 1, \end{cases}$$

where $\omega \in [0,1)$ is the relative weight on nondurables, and $1/\zeta$ is the elasticity of substitution.

The consumer's intertemporal utility function has the form

$$\mathrm{E}_0 \left[\int_0^{\infty} e^{-\rho t} \frac{\{U[C(t), H(t)]\}^{1-\theta}}{1-\theta} dt \right], \tag{1}$$

where θ is the coefficient of relative risk aversion and ρ is the rate of time preference.

The consumer's only income is the return on her portfolio. She holds two assets, a safe one with a constant rate of return r, and a risky one with mean return $\mu > r$ and variance $\sigma^2 > 0$. Define the ratio

$$\gamma \equiv (\mu - r)/\sigma^2,$$

so $\gamma > 0$ is the inverse 'price' of risk. We will assume that the return r on the safe asset is also the interest rate on mortgages.

The price of the nondurable is normalized to one. The purchase price of housing is constant, and housing units can be chosen so that this price is also one. The rental cost of housing then has three components: interest at the rate r, depreciation at the rate δ, and maintenance at the rate m. Hence for a renter the (flow) cost of housing services is $p_h = r + \delta + m$. Since the price of housing is constant, owners do not face any additional risk, and since the mortgage rate is r, for an owner wealth held in the safe asset can be interpreted as equity in her house. Hence p_h is also the direct cost of housing for an owner. Owners will also face an additional portfolio constraint, which will be discussed below.

Let Q denote the consumer's total wealth, H the value of her house, and A her holdings of the risky asset. Then $Q - A$ denotes wealth in the safe asset. The consumer faces constraints on her portfolio allocation A.

Under the parameter restrictions that will be used here, the consumer always chooses $A \geq 0$, so we do not have to worry about a lower bound on A. But we do have to worry about an upper bound. The constraint we will use has two parts.

First, there is an exogenously given minimal equity $\epsilon \in (0, 1]$ that an owner must hold in her house. Since the mortgage interest rate is the same as the return on the safe asset, this is equivalent to requiring that owner hold safe assets equal to ϵH. For a renter, $\epsilon = 0$.

In addition, if the consumer—either an owner or a renter—is sufficiently risk tolerant, she may want to short the safe asset, i.e., to buy the risky asset on margin. We will allow her to do so, but will impose a margin requirement and assume that for an owner the minimal equity holding in her house cannot be used as collateral. Specifically, we will require

$$A \in [0, a_{ss}(Q - \epsilon H)], \tag{2}$$

where $a_{ss} \geq 1$ reflects the size of the margin requirement. If $a_{ss} = 1$, the consumer cannot buy the risky asset on margin.

Given C, H, A, Q, the change in the consumer's total wealth over a short interval of time dt is

$$\begin{aligned} dQ &= \left[r\left(Q - A - H\right) + \mu A - \left(\delta + m\right) H - C\right] dt + \sigma A dz \\ &= \left[rQ + \left(\mu - r\right) A - p_h H - C\right] dt + \sigma A dz, \end{aligned} \tag{3}$$

where z is a Wiener process. If $Q - A \geq H$ the consumer owns her house outright, and if the inequality is strict she has additional wealth invested at the risk-free rate.

The following parameter restrictions will be used throughout.

Assumption 1:

$$\begin{array}{llll} 0 < r < \mu, & \sigma^2 > 0, & \zeta > 0, & 0 \leq \omega \leq 1, \\ 0 \leq \epsilon \leq 1, & a_{ss} \geq 1, & \rho > 0, & \theta > 0, \quad \theta \neq 1, \\ & & \delta, m \geq 0, & \rho + \left(1 - \theta\right)\delta > 0. \end{array}$$

The case $\theta = 1$, which represents logarithmic utility, can be treated along similar lines. The last restriction will be used in section 4.

3. No transaction costs

A useful benchmark for comparisons is the model with no transaction cost. In this case the consumer's problem is to choose (C, H, A) to maximize (1) subject to the budget constraint (3) and the portfolio constraint (2), given initial wealth $Q_0 > 0$.

Since the objective function is homogeneous of degree $(1 - \theta)$ in (C, H, A, Q) and the constraints are homogeneous of degree one, the optimal ratios H/Q, A/Q, etc. are constant over time. Hence the consumer's problem can be written as

$$W(Q_0) = \max_{\substack{c \geq 0, h \in [0, 1/\epsilon] \\ a \in [0, a_{ss}(1 - \epsilon h)]}} \mathrm{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{\left[u(c) h Q(t)\right]^{1-\theta}}{1 - \theta} dt \right] \tag{4}$$

$$\text{s.t.} \quad \frac{dQ}{Q} = \left[r + a\left(\mu - r\right) - \left(p_h + c\right)h\right]dt + a\sigma dz,$$

where $c \equiv C/H$ is the ratio of nondurable consumption to housing services, $h \equiv H/Q$ is the ratio of housing to wealth, $a \equiv A/Q$ is the portfolio share in the risky asset, and $u(c) \equiv U(c,1)$ is the intensive form of the CES aggregator.

For any fixed (c,h,a), total wealth Q is a geometric Brownian motion with constant drift and variance. Hence $\mathrm{E}_0\left[Q(t)^{1-\theta}\right] = Q_0^{1-\theta}e^{\Gamma(c,h,a;\theta)t}$, where

$$\Gamma(c,h,a;\theta) \equiv (1-\theta)\left[r + (\mu - r)a - (p_h + c)h - \theta\frac{1}{2}(\sigma a)^2\right]. \tag{5}$$

Consequently, if $\rho > \Gamma$ the value function in (4) has the form $W(Q_0) = Q_0^{1-\theta}w^*$, where

$$w^* \equiv \max_{\substack{c \geq 0, h \in [0,1/\epsilon] \\ a \in [0, a_{ss}(1-\epsilon h)]}} \frac{\left[u(c)h\right]^{1-\theta}}{1-\theta}\frac{1}{\rho - \Gamma(c,h,a;\theta)}. \tag{6}$$

The next assumption insures that Γ satisfies the required condition.

Assumption 2: If $0 < \theta < 1$,

$$\rho > (1-\theta) \times \begin{cases} \left[r + (\mu - r)a_{ss} - \theta a_{ss}^2\sigma^2/2\right], & \text{if } \theta < \gamma/a_{ss}, \\ \left[r + (\gamma/\theta)(\mu - r)/2\right], & \text{if } \theta \geq \gamma/a_{ss}. \end{cases}$$

The following proposition characterizes the solution for renters ($\epsilon = 0$) and buyers ($\epsilon > 0$).

Proposition 1: Let Assumptions 1 and 2 hold.

(a) For $\epsilon = 0$ the unique solution to the problem in (6) is

$$\begin{aligned} a_R(\theta) &= \min\left\{\gamma/\theta,\ a_{ss}\right\}, \\ c_R &= \left(\frac{\omega p_h}{1-\omega}\right)^{1/\zeta}, \\ h_R(\theta) &= \frac{1}{c_R + p_h}\frac{1}{\theta}\left\{\rho - (1-\theta)\left[r + \sigma^2\gamma a_R(\theta) - \theta\frac{1}{2}\sigma^2 a_R^2(\theta)\right]\right\}. \end{aligned} \tag{7}$$

Moreover, h_R is strictly concave in θ, reaching a maximum where

$$\frac{r-\rho}{\sigma^2} = \left(\frac{\theta}{2}a_R - \frac{\gamma}{\theta}\right)\theta a_R.$$

(b) For $\epsilon > 0$, the unique solution to the problem in (6) is as in (7) if $a_{ss}\left[1-\epsilon h_R(\theta)\right] \geq \gamma/\theta$. Otherwise

$$\begin{aligned} \left[c_B(\theta,\epsilon), h_B(\theta,\epsilon)\right] &= \arg\max_{c,h} \frac{\left[u(c)h\right]^{1-\theta}}{1-\theta} \frac{1}{\rho - \Gamma[c,h,a_{ss}\left(1-\epsilon h\right)]}, \\ a_B(\theta,\epsilon) &= a_{ss}\left[1-\epsilon h_B(\theta,\epsilon)\right]. \end{aligned} \tag{8}$$

In this case

$$h_B(\theta,\epsilon) < h_R(\theta), \qquad \text{and} \qquad c_B(\theta,\epsilon) > c_R.$$

PROOF: See the Appendix.

For renters the share of wealth in the risky asset $a_R(\theta)$ is strictly positive and depends only on γ/θ and a_{ss}. For renters who are sufficiently risk averse the solution is interior, at $a_R = \gamma/\theta$, while for those who are sufficiently risk tolerant the constraint binds and the solution is $a_R = a_{ss}$.

For renters the ratio c_R of nondurable consumption to housing depends on the rental price p_h and the parameters ω and ζ, but not on $\theta, \rho, \gamma, \sigma^2$.

For renters the ratio of expenditure to wealth, $(c_R + p_h)\, h_R(\theta)$, depends on $\theta, \rho, \gamma, \sigma^2$ but not on p_h, ω or ζ. The function $h_R(\theta)$ has an inverted U-shape. If $r = \rho$ and a_{ss} is large, then $h_R(\theta)$ peaks at $\hat{\theta} = 2$. The peak occurs at a lower value if $\rho > r$.

The hump shape is the result of two opposing forces. Consumers with lower θ are more risk tolerant, so they hold portfolios with higher expected rates of return. Hence the prefer lower ratios of expenditure to wealth. But consumers with higher θ have have a stronger incentive to smooth consumption over time. Hence they also prefer lower ratios of expenditure to wealth. The first force predominates for values of θ below a certain threshold, and the second for θ above that threshold, leading to the hump shape for $h_R(\theta)$.

Note that the expenditure share for housing is $p_h / (p_h + c)$, and the ratio of expenditures to wealth is $(1 + c)\, h$. Proposition 1 implies that for a renter the former is constant and the latter is hump-shaped.

For a portfolio-constrained owner, the house/wealth ratio is lower than for a renter. For a constrained owner housing services have an extra cost at the margin, the incremental portfolio distortion. The constrained renter chooses a higher ratio of nondurables to housing.

4. With transaction costs

Suppose the consumer must pay a transaction cost of λH when she adjusts her consumption of housing services, where $\lambda > 0$. Then she will adjust her housing consumption only occasionally, by discrete amounts, and her budget constraint has two parts. At dates when she adjusts her housing consumption, her wealth falls by the amount of the transaction cost. At all other times the durable depreciates deterministically and wealth grows stochastically.

In addition, suppose that adjustments can occur for two reasons. The consumer may, at any time, choose to leave her current house and move to a new one. In addition, moves may be required for exogenous reasons. Job changes that involve relocating to a new city and changes in family size are two possible interpretations of these exogenous moves. Assume that this shock is Poisson, with a constant arrival rate κ.

Define the stopping time T_X as the arrival of the next exogenous relocation shock, and define the stopping time T_A as the time the consumer chooses for the next adjustment in case an exogenous has not occurred. The time of the consumer's next housing adjustment is the minimum of these two, the stopping time $T' \equiv T_A \wedge T_X$.

With a transaction cost for housing, two state variables are needed, Q and H. But the consumer's value function $V(Q,H)$ is, as before, homogeneous of degree $(1-\theta)$ in the state variables, and the policy functions for C, A, and H' are homogeneous of degree one. Hence an intensive form of the problem can be written in terms of a single state variable, a ratio. It is convenient to use $q = Q/H = 1/h$, so the Bellman equation in the intensive form is

$$v(q_0) = \sup_{\{c(t),a(t)\},T_A,q'} \mathrm{E}_0 \left\{ \int_0^{T'} e^{-\eta t} \frac{u[c(t)]^{1-\theta}}{1-\theta} dt + e^{-\eta T'} \left(\frac{q(T')-\lambda}{q'} \right)^{1-\theta} v\left(q'\right) \right\} \quad (9)$$

$$\begin{aligned} \text{s.t.} \qquad dq &= \{[r+\delta+(\mu-r)\,a]\,q-(p_h+c)\}\,dt+\sigma aqdz, \\ a &\in \left[0, a_{ss}\left(1-\frac{\epsilon}{q}\right)\right], \qquad\qquad t \in [0,T'), \\ T' &= T_A \wedge T_X, \\ q' &\geq \epsilon, \end{aligned}$$

where $v(q) \equiv V(q,1)$,

$$\eta \equiv \rho + (1-\theta)\,\delta,$$

and as before $c = C/H$ and $a = A/Q$. Assumption 1 insures $\eta > 0$. A solution consists of a value function $v(q)$ defined on $\mathbf{R}_+$ satisfying (9), and policy functions $\{c(t),a(t)\}, T_A, q'$ that attain the maximum. As shown in the Appendix, under Assumptions 1 and 2 V and hence v are well defined.

Two properties of the solution are immediate from (9). First, the optimal return point q' does not depend on the state $q(T')$ when the adjustment is made. Define

$$M \equiv \max_{q'} \frac{v\left(q'\right)}{q'^{1-\theta}}, \quad (10)$$

and let S denote the maximizing value for q'. Thus, M is the optimized value for an individual with net wealth $Q=1$ when she buys a house, and S is the wealth/house ratio she chooses.

In addition, the stopping time chosen by the consumer has the form $T_A = T(b) \wedge T(B)$, where $T(\beta)$ denotes the first time the stochastic process q reaches β, and $0 \leq b < B < +\infty$ are optimally chosen thresholds. Thus, the state has an *inaction region*, the open interval (b, B). While the state remains inside this interval the consumer does not sell her house voluntarily, although the exogenous moving shock may force her to do so. The consumer immediately adjusts her housing if the state variable q is outside the interval (b, B). Hence the value function outside the inaction region has the form

$$v(q) = (q - \lambda)^{1-\theta} M, \qquad q \notin (b, B) . \tag{11}$$

After an initial transaction, if required, the state remains inside the interval (b, B).

To characterize the value function v, the critical points b, S, B, and the policy functions c and a, we can use the fact that inside the inaction region the value function satisfies the Bellman-type equation

$$\begin{aligned} (\eta + \kappa) v(q) \quad = \quad & \max_{c, a \in [0, a_{ss}(1-\epsilon/q)], q'} \left\{ \frac{u(c)^{1-\theta}}{1-\theta} + m(q) v'(q) \right. \\ & \left. + \frac{1}{2} s^2(q) v''(q) + \kappa (q - \lambda)^{1-\theta} \frac{v(q')}{(q')^{1-\theta}} \right\}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} m(q) \quad &\equiv \quad [r + \delta + (\mu - r) a] q - (p_h + c), \\ s^2(q) \quad &\equiv \quad (\sigma a q)^2, \end{aligned}$$

are the instantaneous drift and variance for q under the optimal policies $a(q)$ and $c(q)$. (See Stokey 2007, Ch. 9 for a more detailed discussion.)

The interpretation of (12) is fairly standard. The first term on the right is the current utility flow from consumption. The second and third, which come from an application of Ito's lemma, are the expected 'capital gain' from changes in the state variable. To interpret the final term, subtract $\kappa v(q)$ from both sides. Then the final term on the right, which is negative, is the expected net loss from the exogenous moving shock. The remaining term on the left side $\eta v(q)$ is the 'current return' on the value v.

The optimal policies for the portfolio and nondurable consumption are found by maximizing the term in braces in (12). Hence the optimal portfolio is

$$a(q) = \min\left\{\frac{\gamma}{-qv''/v'}, a_{ss}\left(1-\frac{\epsilon}{q}\right)\right\}. \tag{13}$$

The expression on the right is exactly analogous to the one for the problem with no transaction costs in (7) or (8). The only difference is that in the absence of a transaction cost the value function $W(Q) = w^* Q^{1-\theta}$ displays constant relative risk aversion, $-QW''/W' \equiv \theta$, while with a positive transaction cost $-qv''/v'$ varies with q.

The condition for nondurable consumption is

$$\frac{d}{dc}\left[\frac{u(c(q))^{1-\theta}}{1-\theta}\right] = v'(q). \tag{14}$$

The term in square brackets is instantaneous utility, as a function of nondurable consumption only, when housing services are fixed at unity. Nondurable consumption $c(q)$ increases with wealth, but the extent to which it varies depends on the intratemporal elasticity, $1/\zeta$, and the intertemporal elasticity, $1/\theta$. Lower elasticities imply a weaker response for nondurable consumption.

Optimal choice of the boundaries b and B requires that value matching and smooth pasting conditions hold. That is, both v and v' must be continuous. From (11) we see that this requires

$$\begin{aligned}
\lim_{q\downarrow b} v(q) &= (b-\lambda)^{1-\theta} M, \qquad (15)\\
\lim_{q\uparrow B} v(q) &= (B-\lambda)^{1-\theta} M,\\
\lim_{q\downarrow b} v'(q) &= (1-\theta)(b-\lambda)^{-\theta} M,\\
\lim_{q\uparrow B} v'(q) &= (1-\theta)(B-\lambda)^{-\theta} M,
\end{aligned}$$

where M, defined in (10), is the optimized value for a consumer with unit wealth (after the transaction cost is paid) who is buying a new house. In addition, the return point S satisfies

$$\begin{aligned} v(S) &= S^{1-\theta}M, \\ v'(S) &= (1-\theta)\,S^{-\theta}M. \end{aligned} \tag{16}$$

Although an analytic solution is not available, it is not difficult to compute solutions computed numerically, and we turn next to the simulations.

5. Calibration

The model has thirteen parameters: (μ, σ, r, a_{ss}) describing asset markets, $(\delta, m, \epsilon, \lambda, \kappa)$ for housing, and $(\rho, \theta, \zeta, \omega)$ describing preferences. Some of these, the ones about which there is better information, will be fixed throughout the analysis. For the others I will choose benchmark values and conduct sensitivity experiments.

Using U.S. data over the period 1889-2005, Mehra and Prescott (2006) calculate the average real return on a market index to be $\mu = 0.077$, and the average real return on a riskless security to be $r = 0.013$. Kocherlakota (1996) reports similar figures for 1889-1978, with a standard deviation of $\sigma = 0.1655$ on the market portfolio. I will use these values for μ and σ, and $r = 0.015$ in all the simulations.

The short sale parameter will be fixed at $a_{ss} = 1.20$ throughout. Thus, the consumer can buy risky assets on margin, but the value of these assets can never exceed 20% of her total wealth. Minimal down payments for homeowners are typically 10-15%. The upper end of this range will be used here, $\epsilon = 0.15$. In the simulations below the portfolio constraint involving these two parameters almost never binds.

Smith, Rosen, and Fallis (1988) estimate the cost of selling a house to be 8% - 10% of the value of the unit. This figure includes agents' commissions, legal fees, time cost of search and the direct cost of moving the consumer's possessions. I take the lower figure, $\lambda = 0.08$ for the benchmark and experiment with other value.

Since most people maintain their houses rather than allowing them to depreciate, I will set $\delta = 0$ throughout.

A key element in the model is the ratio of total wealth to housing wealth. The model includes only tangible wealth, while in fact the bulk of 'total wealth'—in the sense of what generates income—is intangible wealth, human capital. Capital's share in national income is about 1/3, so total wealth is about 3 times the stock of physical capital. The stock of residential structures is about 40-50% of total private fixed capital.[2] This suggests a figure of about 6.0 - 7.5 for the ratio of 'total wealth' to housing wealth. The total wealth/housing ratio in the model is sensitive to the maintenance cost m. That parameter is set at $m = 0.04$, a value that produces average wealth/housing ratios in the appropriate range. This figure for maintenance does not seem unreasonable, since it should be interpreted broadly, to include all costs of housing except mortgage payments (or foregone interest). Thus, it includes property taxes, heating, and other costs that are difficult to adjust and proportional to the value of the house.

Little direct evidence is available on the hazard rate for exogenous moves. I will use $\kappa = 0$ for the benchmark and experiment with other values.

There are four preference parameters, ρ, θ, ζ, and ω. For the rate of time preference, $\rho = 0.025$ will be used throughout. This figure is within the range used in the macro/finance literature.

The elasticity of substitution is more controversial, with values for θ in the range of $[1, 10]$ all having their advocates. With the asset returns fixed at their market values, this parameter is important in determining the average growth rate of consumption and wealth. For the U.S. over the 20th century, the average growth rate of per capita consumption was about 2%. Setting $\theta = 4$ hits this growth rate very

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[2] See Davis and Heathcote (2005, Table 7), who use NIPA data for 1948-2001.

closely, so I will use $\theta = 4$ for the benchmark and experiment with other values.

There is little consensus about the elasticity of substitution between housing and nondurables, a key parameter for the model here.

Using data from a policy experiment that involved low-income renters in two cities, Hanushek and Quigley (1980) estimated price elasticities of $\varepsilon = 0.45$ and 0.64.

Siegel (2004) obtains two estimates based on homeowners in the PSID over the period 1978-1997, using total food expenditure as a proxy for nondurable consumption and the self-reported value of the owner occupied house. Aggregating across households and using only the time series information, the estimated elasticity is $1/(1 + .907) \approx 0.53$. Alternatively, using the household level information and limiting the sample to households that own stocks, the estimated elasticity is in the range $[1.23, 1.54]$. In no case is an elasticity of unity rejected.

Flavin and Nakagawa (2005) also use data from the PSID, for 1975-1985, using a different measure of housing to sidestep the problem of price variation across cities. They use the PSID measure of the *number* of rooms, combined with data on the relationship between square footage and number of rooms, controlling for other factors like regional location, suburban, non-SMSA, renter, etc. They obtain an elasticity of substitution of $\varepsilon = 0.13$, with almost identical results for owners and renters.

Piazzesi, et. al. (2006) estimate an elasticity of substitution using NIPA data on real rents and the aggregate expenditure share of housing over the period 1936-2001. They estimate the elasticity to be in the range $[1.05, 1.25]$.

Using CEX data for 27 cities in 2003, a simple regression of the expenditure share of housing on the relative price of housing leads to an estimated elasticity of $\varepsilon = 0.45$.[3]

I will use $\varepsilon = 0.5$ as the benchmark, and in addition try values of $\varepsilon = 0.15$, 1.0, and 1.25.

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[3] This estimate excludes Anchorage, which is an outlier.

The weight parameter ω will be calibrated using the expenditure share of housing. Aggregate data from NIPA suggest an expenditure share of about 20% over the period 1960-2005, with relatively little variation. Data from the CEX suggests a somewhat higher figure, around 33%. I will use an intermediate value, calibrating ω in each simulation so that the expenditure share of housing is 30%.

Table 1 displays the benchmark parameters.

Table 1

$\mu = 0.077$	$r = 0.015$	$\rho = 0.025$	$\zeta = 2$
$\sigma = 0.1655$	$a_{ss} = 1.20$	$\theta = 4$	$\lambda = 0.08$
$\delta = \kappa = 0$	$\epsilon = 0.15$	$m = 0.04$	$\omega = 0.184$

Another figure that can be used to check the predictions of the model (or to calibrate κ) is the average length of residence. Using Census data for 1996, Schachter and Kuenzi (2002) calculate the average length of residence for persons 15 years and older who live in owner occupied housing to be 11.3 years. For renters the figure is 3.7 years.

6. Quantitative results

a. No transaction costs.—

Figure 1 displays results for the model with no transaction costs ($\lambda = 0$) and risk aversion $\theta \in [0.75, 5.0]$. The preference parameter ω is calibrated to give housing an expenditure share of 30%, and the benchmark values are used for the other parameters.

Figure 1a shows the portfolio share in the risky asset. The short sale constraint binds for consumers who are sufficiently risk tolerant, those with $\theta < \theta^c \approx 1.9$.

Figure 1b shows the ratio of expenditures to wealth, $(p_h + c)\, h$. The curve is single peaked, as Proposition 1 predicts for $\epsilon = 0$. The solution here, for $\epsilon = 0.15$, is only slightly different, and only for $\theta < \theta^c$.

Figure 1c shows the long run growth rate for income, consumption and wealth. It declines with θ over most of the range, with a kink at θ^c. For $\theta < \theta^c$ the expenditure flow increases with θ, while the portfolio allocation is flat, constrained at $(1 - \epsilon h)\, a_{ss}$. For $\theta > \theta^c$ the ratio of expenditures to wealth decreases with θ, but there is also a portfolio reallocation toward the safe asset, which reduces the return on the portfolio. The latter effect swamps the former, so the growth rate falls. For $\theta = 4$, the growth rate is $g = 1.9\%$.

Figure 1d shows the ratio of total wealth to housing wealth. It is roughly a mirror image of the expenditure flow/wealth ratio in Figure 1b, and $Q/H = 6.0$ for $\theta = 4$.

b. Benchmark model.—

Figures 2 - 4 display results for the benchmark calibration. The portfolio constraint involving housing equity and short sales does not bind.

Figure 2 shows the value function and its first two derivatives, as well as the value function for a consumer who faces no transaction costs. Both value functions are smooth and concave, their first derivatives are smooth and convex, and their

Figure 1a: portfolio share in risky asset

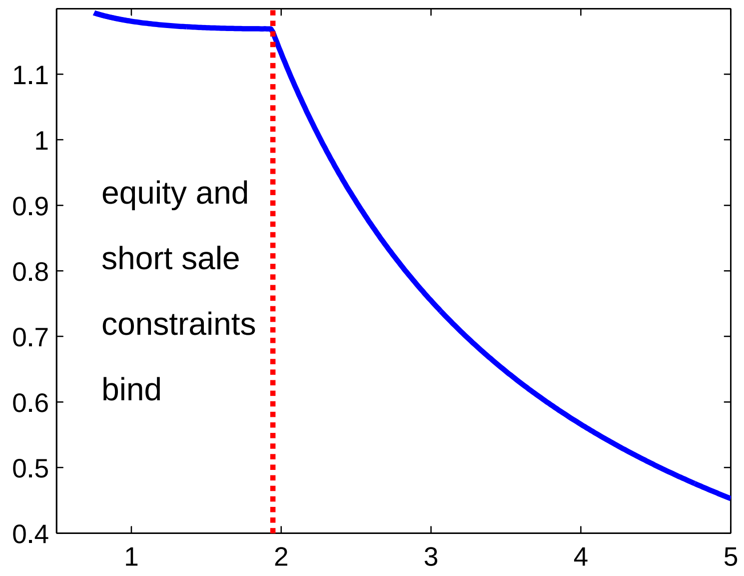


Figure 1b: expenditure flow / wealth

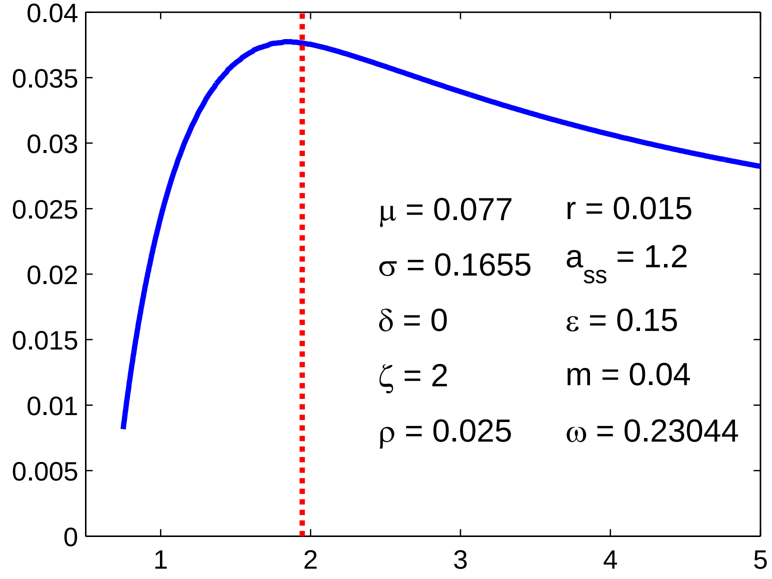


Figure 1c: expected growth rate

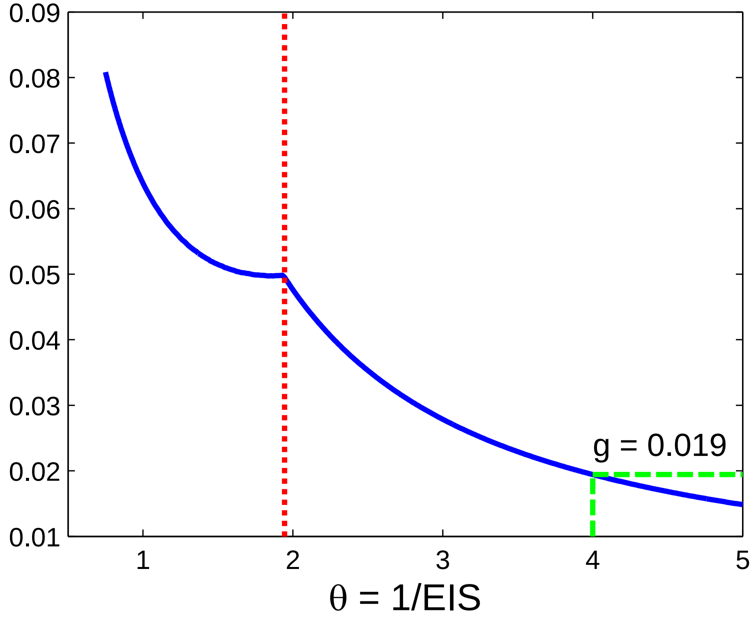


Figure 1d: wealth/house ratio

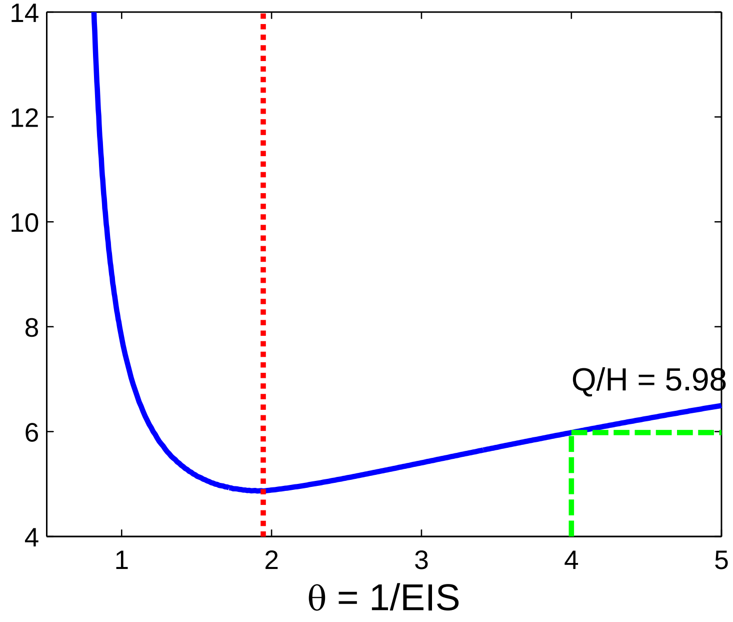
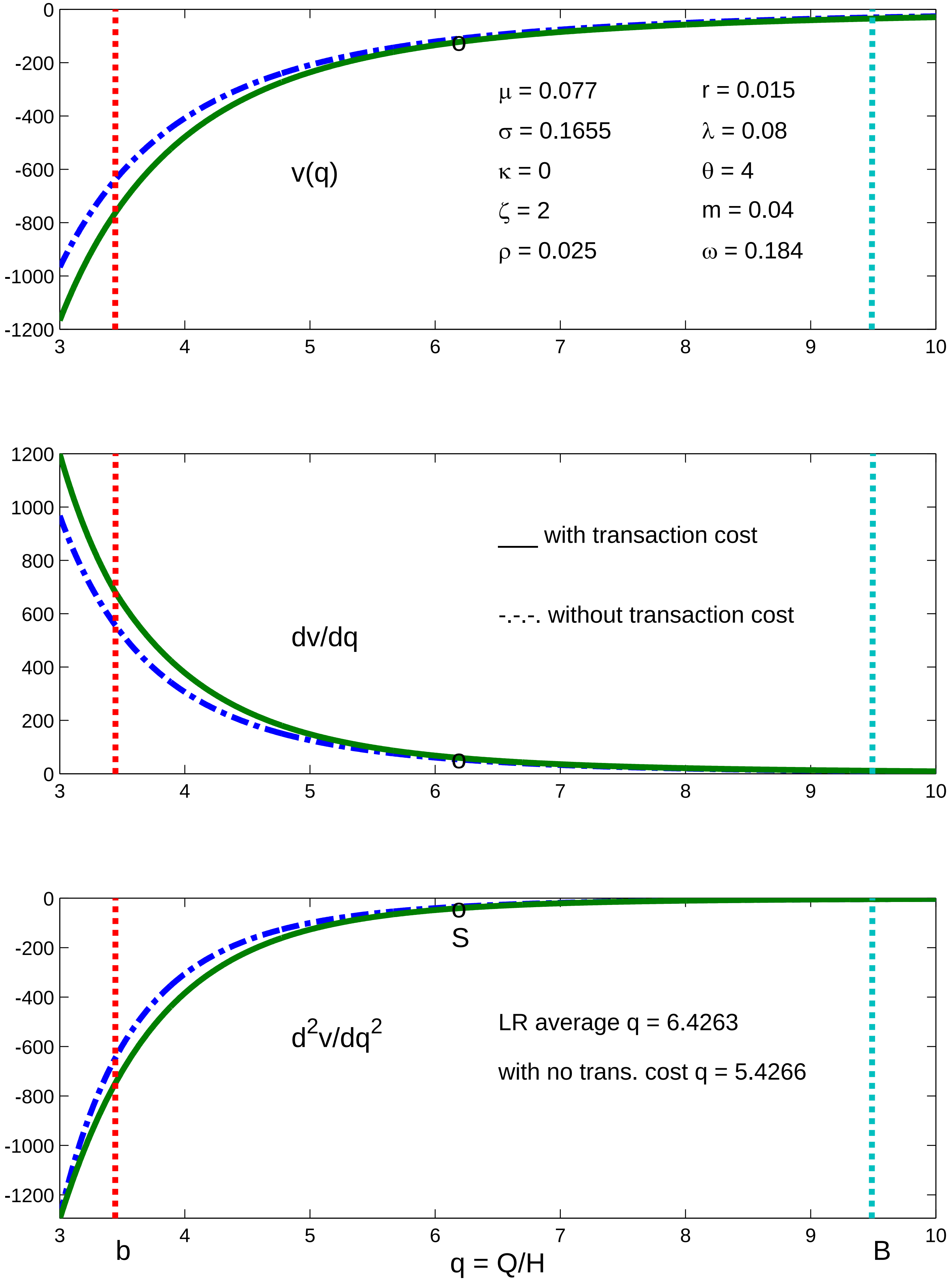


Figure 2: Value function and its derivatives



second derivatives are smooth and concave. The transaction costs does not create kinks or nonconvexities.

The adjustment thresholds, indicated with dotted lines, are wealth/house ratios of $b = 3.4$ and $B = 9.5$, and the ratio chosen when a new house is purchased, indicated with a small open circle, is $S = 6.2$. Thus, an upward adjustment is made when wealth has increased by about 52% and a downward adjustment when it has fallen by 45%. The long run average, which is 6.4, is higher than the (constant) ratio of 5.4 chosen by a consumer who faces no transaction cost. The transaction cost makes housing more expensive, so the consumer who faces that cost holds less of her wealth in the form of housing.

Figures 3 and 4 describe the consumer's behavior between housing transactions. Since there is no depreciation, the consumer's flow of housing services is constant during this period. Long run averages are calculated using the density function for q following a start at $q = S$. (The density, not shown, has a fairly symmetric tent shape, with a peak at S.)

Figure 3a shows the share of her wealth that the consumer holds in risky assets. This function is U-shaped, reflecting the fact—first noted by Grossman and Laroque—that the consumer is more risk tolerant when she is close to the adjustment thresholds, and more risk averse in the middle of the inaction region. The fairly high risk aversion coefficient used here, $\theta = 4$, means that the consumer puts only 53% - 64% of her wealth in the risky asset. The long run average is 55%, a little lower than the 57% chosen by a consumer facing no transaction cost.

Figure 3b shows nondurable consumption relative to housing wealth. It moves linearly with total wealth, rising 69% or falling 51% relative to its level just after the most recent housing adjustment. Its long run average is 13%, which is a little higher than the 10% for a consumer who faces no transaction costs. The transaction cost induces the consumer to shift her consumption mix toward nondurables.

Figure 3a: portfolio share of risky asset

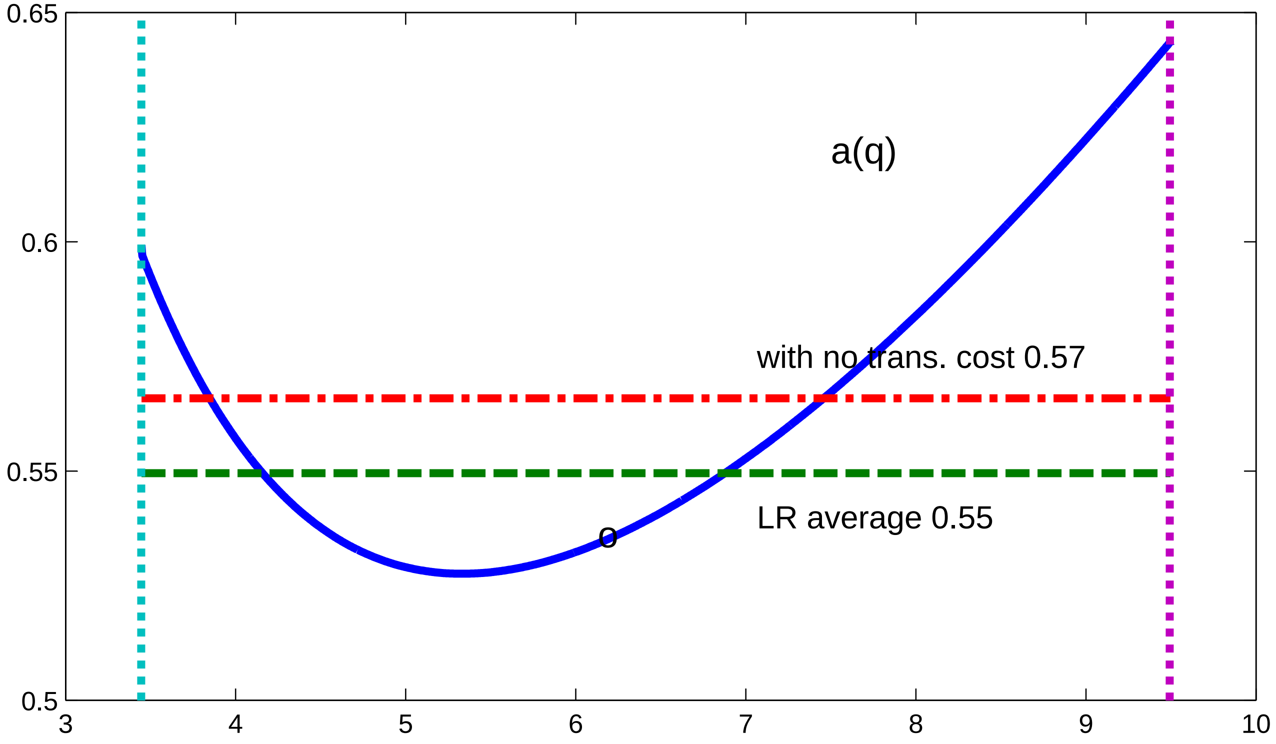


Figure 3b: nondurable consumption/housing wealth

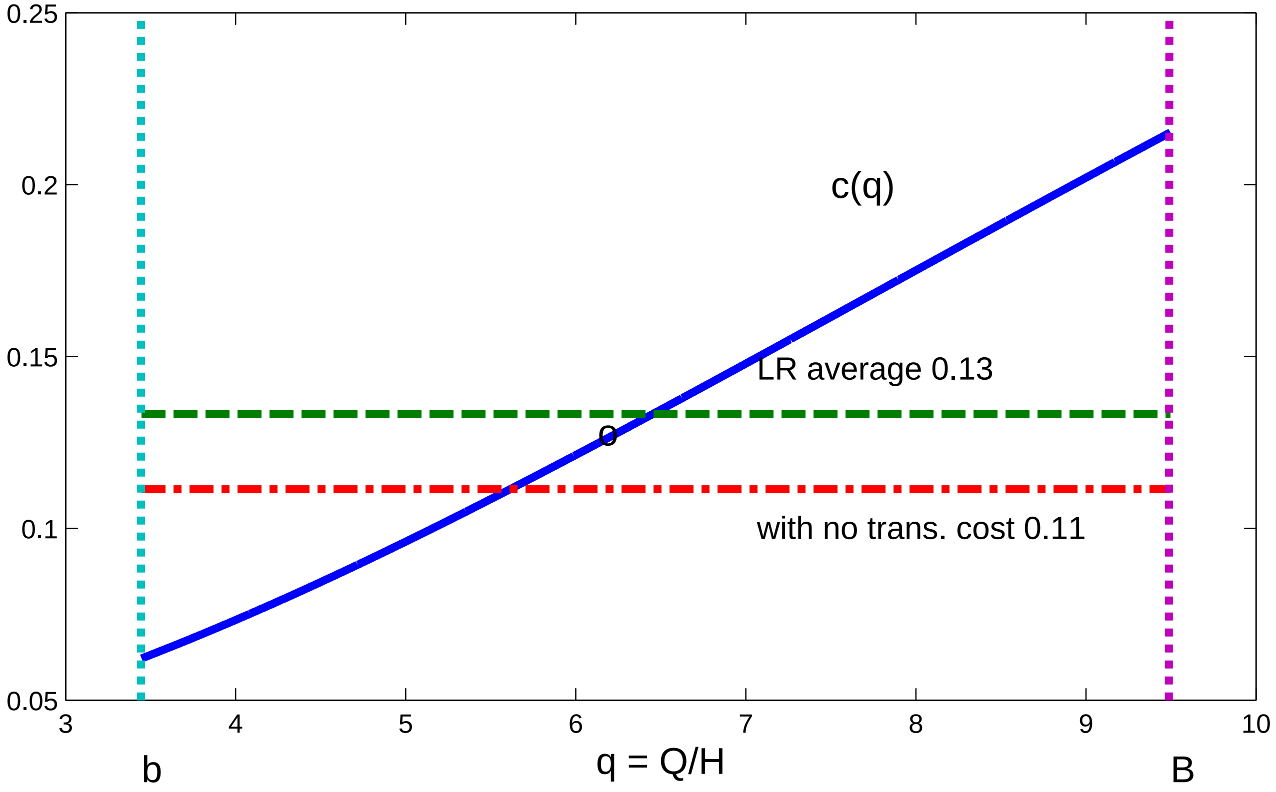


Figure 4a: expenditure share of housing

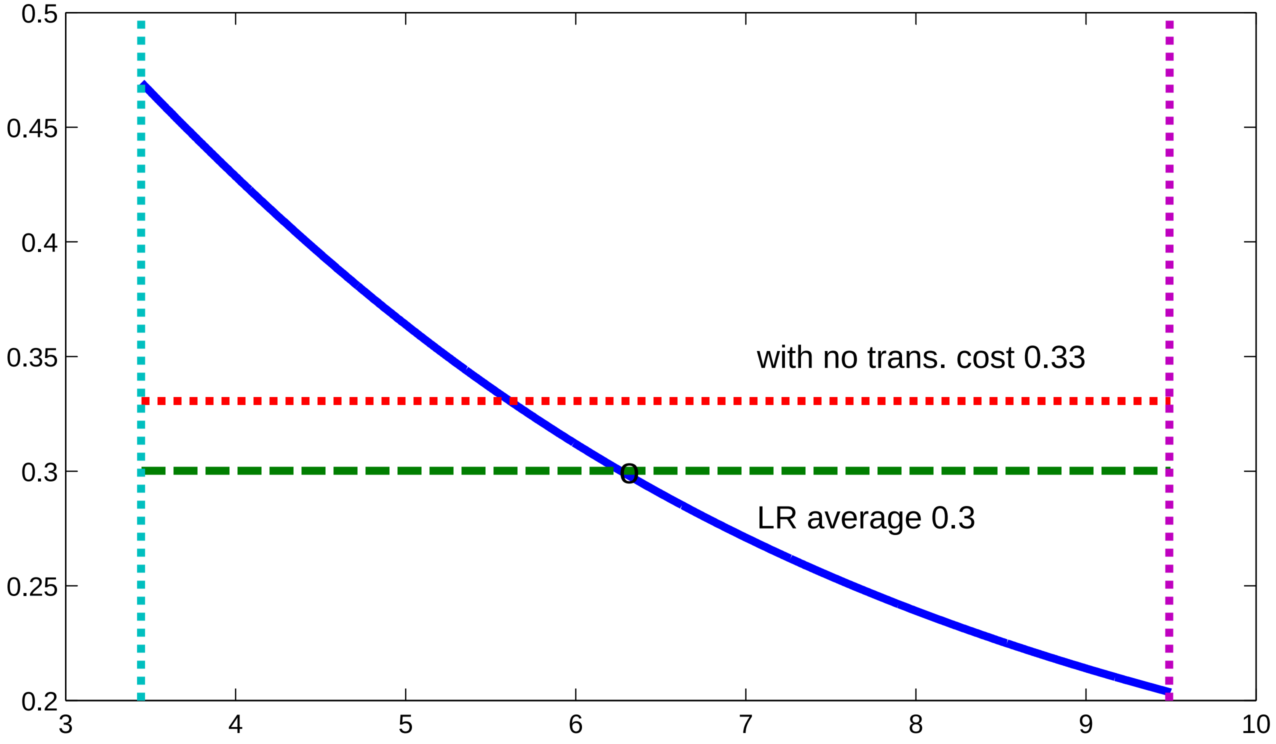
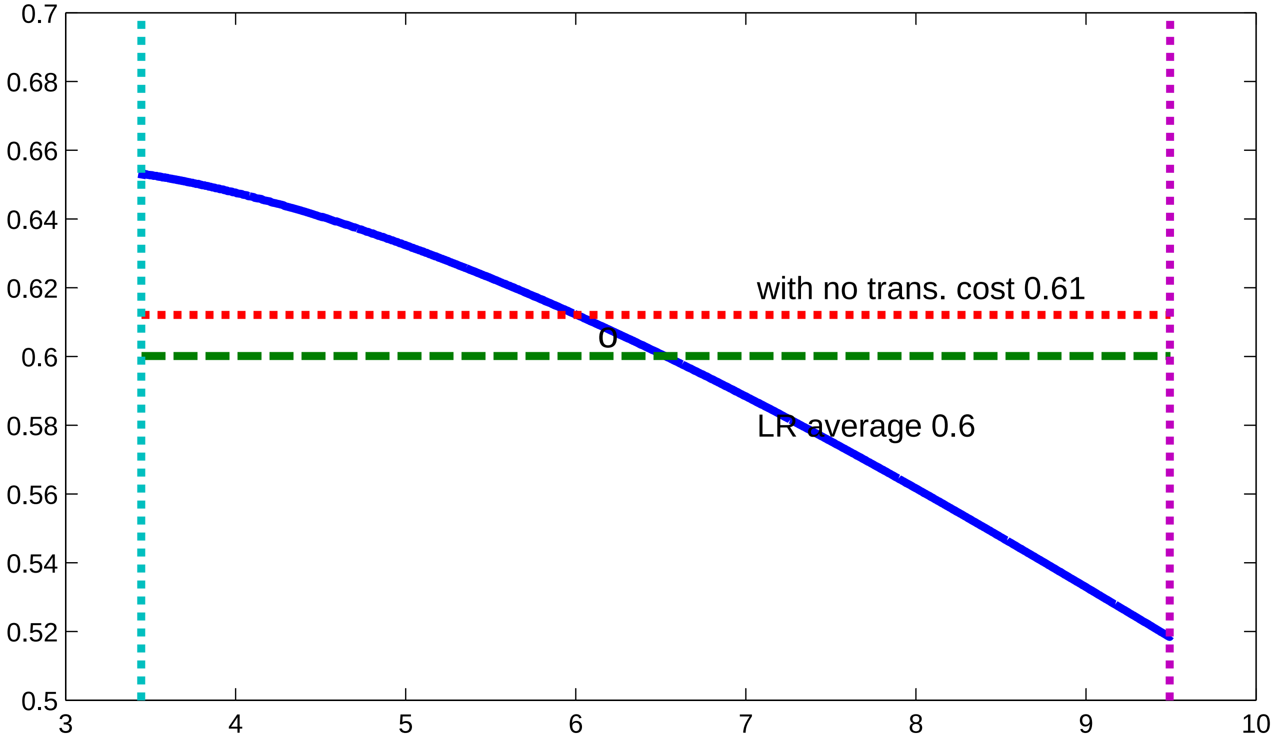


Figure 4b: consumption expenditures/income,



As wealth increases, expenditures on nondurables increase, but housing expenditures are constant. Figure 4a shows the housing share of total expenditure, which falls from about 47% at the lower threshold to about 20% at the upper threshold. The long run average value of 30% is slightly lower than the 33% for a consumer with the same preferences who faces no transaction costs.

Figure 4b shows the ratio of total consumption expenditures to expected income, the expected return on her portfolio. This ratio declines with wealth, as expected. Income increases roughly in proportion to wealth—although not quite, because of the portfolio shifts. Nondurable expenditures increase linearly with wealth, as shown in Figure 3b, but housing expenditures are constant. Averaging over different wealth levels, the consumer spends about 60% of her income, slightly less than a consumer who faces no transaction costs.

The average return on the portfolio is 4.9%, and the average growth rate of consumption, income and wealth (they are all the same) is 2.0%.

For a consumer who has just transacted, with $q = S$, the expected time to the next adjustment is 20.8 years. Table 2 describes those transactions. The probability that the next adjustment is downward is only 0.09. Since the consumer's wealth grows, on average, only a (relatively rare) sequence of bad portfolio returns drives her to downsize her house. Thus, in the long run 91% of transactions are upward.

Table 2

	at b	at B
probability of adjustment ($q = S$)	0.09	0.91
new/old house	0.54	1.51
new/old nondurable consumption	1.11	0.89
change in portfolio share	−0.06	−0.11

For transactions at the lower threshold the value of the new house is about 54% of the value of the one being sold, nondurable consumption rises by about 11%, and the portfolio share in the risky asset falls by 6 percentage points. For transactions at the upper threshold, the value of the new house is about 51% higher than the value of the one being sold, nondurable consumption falls by about 11%, and the portfolio share in the risky asset falls by 11 percentage points.

Thus, the presence of a transaction costs implies that the consumer's mix of durables and housing and her portfolio allocation change with the ratio of her total wealth to housing wealth. For the fairly low elasticity of substitution used here, $\varepsilon = 1/\zeta = 1/2$, the swings in nondurable consumption and the portfolio are fairly wide.

c. Sensitivity analysis: ζ.—

As noted above, there is conflicting evidence about the elasticity of substitution between housing and nondurables. To explore the effect of this parameter, we will next compare results for elasticities of $\varepsilon = 1/\zeta = 0.15$, 0.50, 1.0, and 1.25. In each case ω is adjusted to keep the average expenditure share for housing at 30%.

A higher elasticity allows the consumer to substitute more easily into nondurables as her wealth increases and hence reduces the incentive to pay the transaction cost associated with a change in housing services. Thus, as shown in Table 3, the inaction region gets wider as the elasticity of substitution increases—b falls and B rises.

Table 3

ε	b	B	$\mathrm{E}[T]$	g	$\Pr(B)$
0.15	3.9	9.0	14.9	1.98	0.87
0.50	3.4	9.5	20.8	1.97	0.91
1.00	3.1	10.0	26.6	1.96	0.93
1.25	2.9	10.2	28.7	1.96	0.94

The wider inaction region leads to longer expected times between adjustments, with the expected duration rising from 14.9 years to 28.7 years. The long run growth rate g does not change much as the substitution elasticity varies, remaining at about 2.0%. The probability that the next adjustment is at the upper threshold increases slightly with the elasticity, rising from 0.87 to 0.94. As ε increases, the widening of the inaction region reduces the probability of a sequence of low returns sufficiently long and severe to induce a downward housing adjustment.

Figure 5a shows the portfolio allocations. All four portfolio policies have a U shape, but the U is flatter for higher elasticities. Portfolio behavior is also remarkably similar for the higher three elasticities. For the lowest elasticity, $\varepsilon = 0.15$, it displays much sharper fluctuations.

Figure 5b shows nondurable consumption. In all cases it increases with wealth, but a higher elasticity leads to larger adjustments, a steeper slope. The behavior of nondurable consumption is remarkably similar for the three higher elasticities. Only for $\varepsilon = 0.15$ is the curve significantly flatter..

Next consider the changes when a transaction is made. We will focus on the upper threshold, since that is where most transactions occur. These are displayed in Table 4.

Table 4

Transactions at B

ε	$\hat{H}/H$	$\hat{C}_{ND}/C_{ND}$	$a(S) - a(B)$
0.15	1.4	1.09	−0.17
0.50	1.5	0.90	−0.11
1.00	1.6	0.84	−0.07
1.25	1.7	0.83	−0.06

The ratio of new to old house values after an adjustment, $\hat{H}/H$, increases slightly with the elasticity of substitution, from 1.4 to 1.7. This pattern is a straightforward result of the widening of the inaction region.

Figure 5a: Portfolio shares, various subst. elasticities

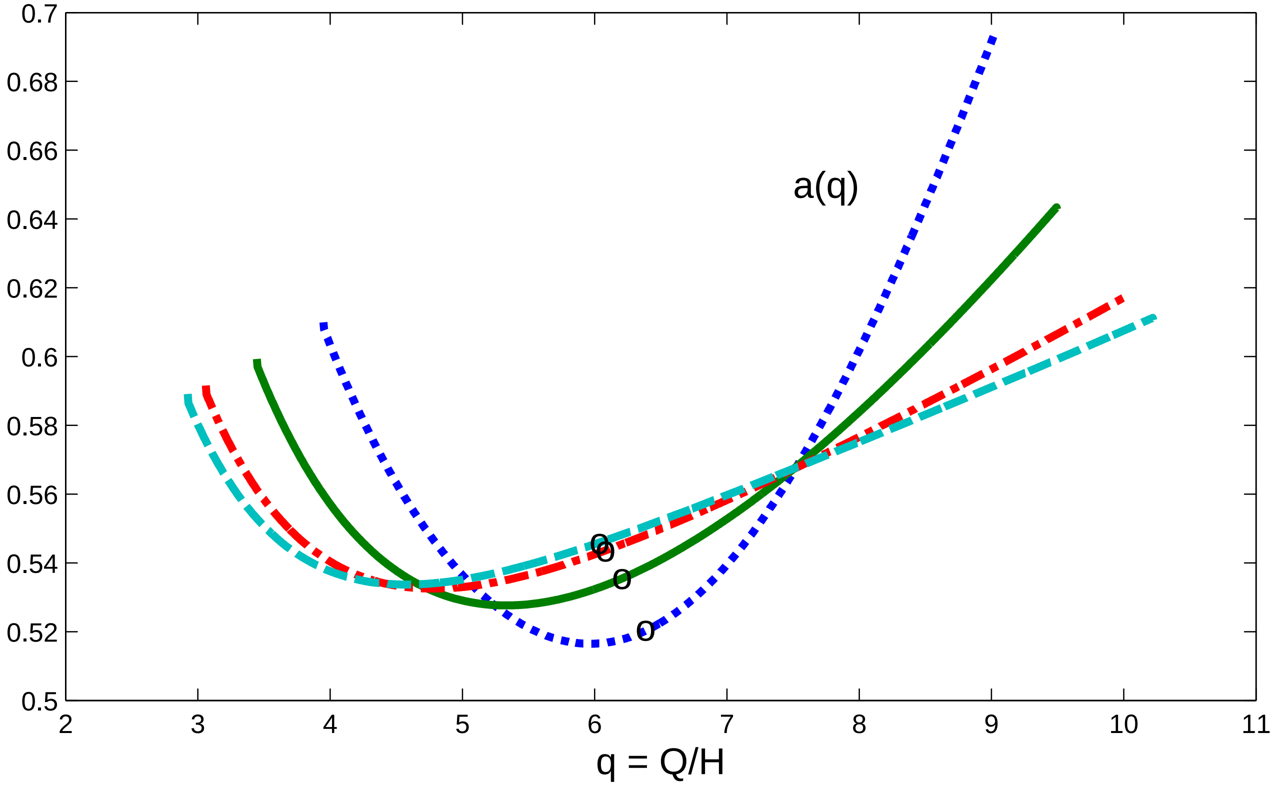
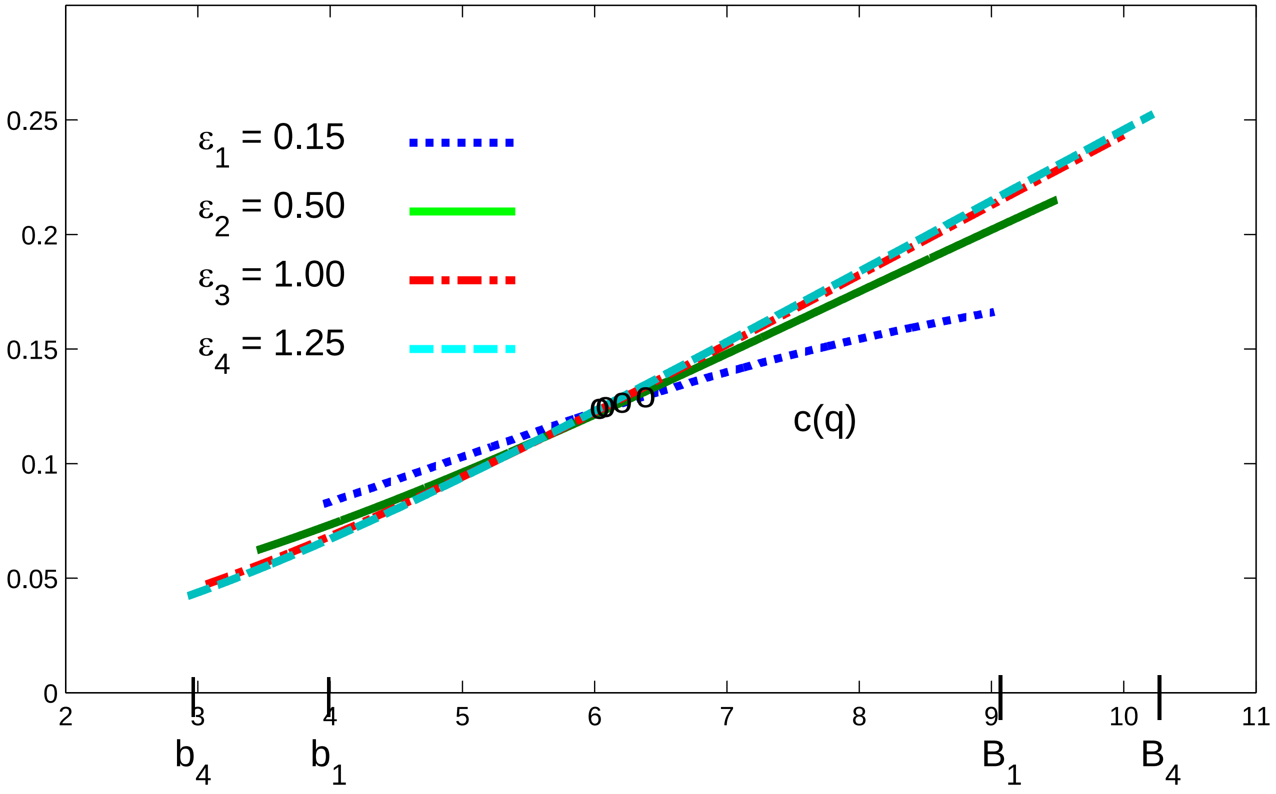


Figure 5b: Nondurable cons., various subst. elasticities



The ratio of new/old nondurable consumption, $\hat{C}_{ND}/C_{ND}$, falls with the elasticity. Recall that since $1/\theta = 0.25$, for $\varepsilon = 0.25$ (not shown), preferences are additively separable between housing and nondurables. In this case nondurable consumption is unchanged after a housing transaction, $\hat{C}_{ND}/C_{ND} = 1$. For $\varepsilon = 0.15$, the ratio exceeds one: in this case the consumer increases her nondurable consumption as well when she purchases a larger house. For the higher elasticities the ratio is less than one: the consumer reduces her nondurable consumption after purchasing a larger house, with the size of the reduction increasing with the elasticity. Thus, for elasticities exceeding $1/\theta$ the consumer behaves like someone who is 'house poor,' although she is not liquidity constrained, as that term suggests.

When a transaction is made at the upper threshold, the consumer reduces the share of her portfolio in the risky asset. This shift simply reverses, in a single jump, the increases that occurred gradually as the consumer's wealth accumulated since the last housing transaction. The jump is larger for lower elasticities. For $\varepsilon = 0.15$, the consumer reduces her risky asset holdings by 17 percentage points. For $\varepsilon = 1.25$, the reduction is only 6 percentage points.

d. Sensitivity analysis: $\lambda, m, \theta, \kappa$.—

A higher transaction cost has two effects: it increases the incentive to avoid a housing adjustment and it increases the cost of housing. The first effect tends to widen the inaction region, and the second tends to shift it to the right. Thus, the two effects work in opposite directions at the lower threshold of the inaction region, and in the same direction at the upper threshold. As shown in Figure 6, an increase in λ widens the inaction region, but the effect is greater at the upper end. Increasing λ also increases the ratio S of total wealth to housing chosen when a new house is purchased and raises the average ratio of total wealth to housing. A higher transaction cost has little effect on the expenditure share of housing, however. Here ω is not recalibrated,

Figure 6a: Portfolio shares, various transaction costs

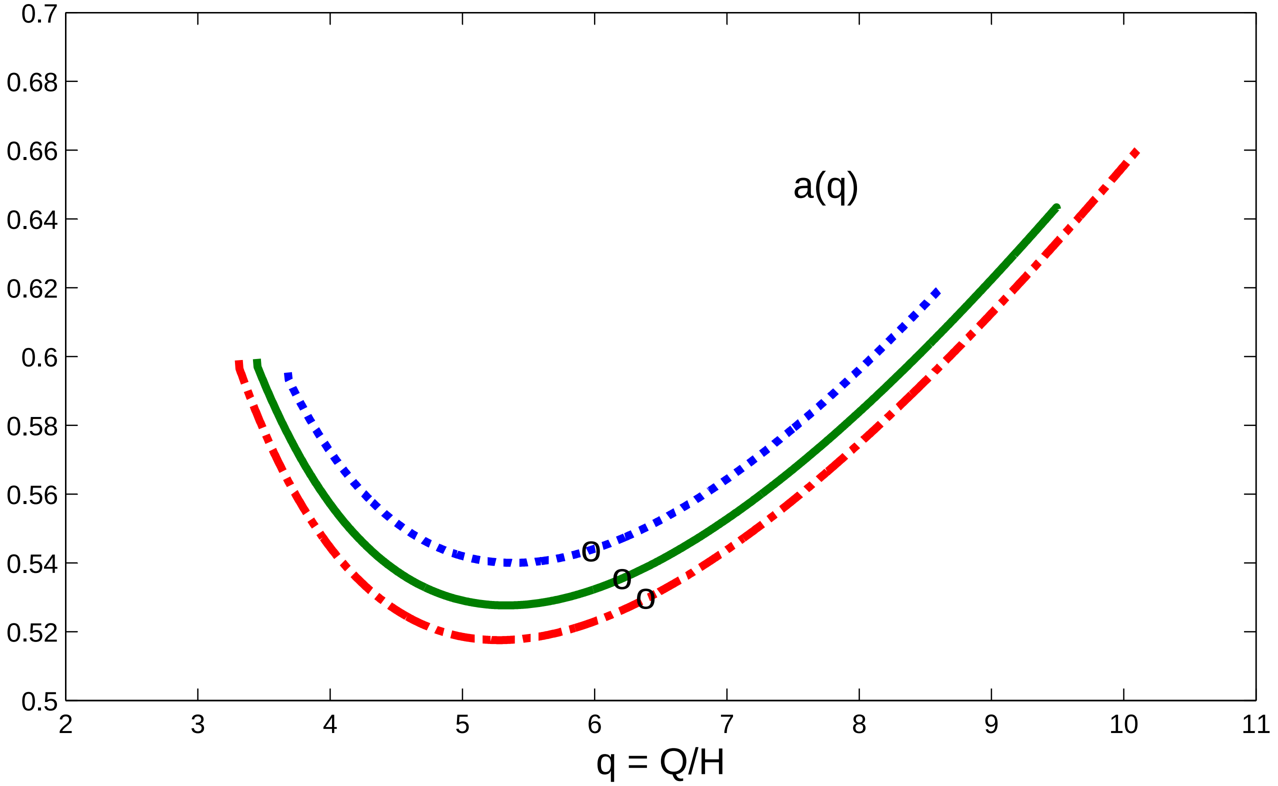
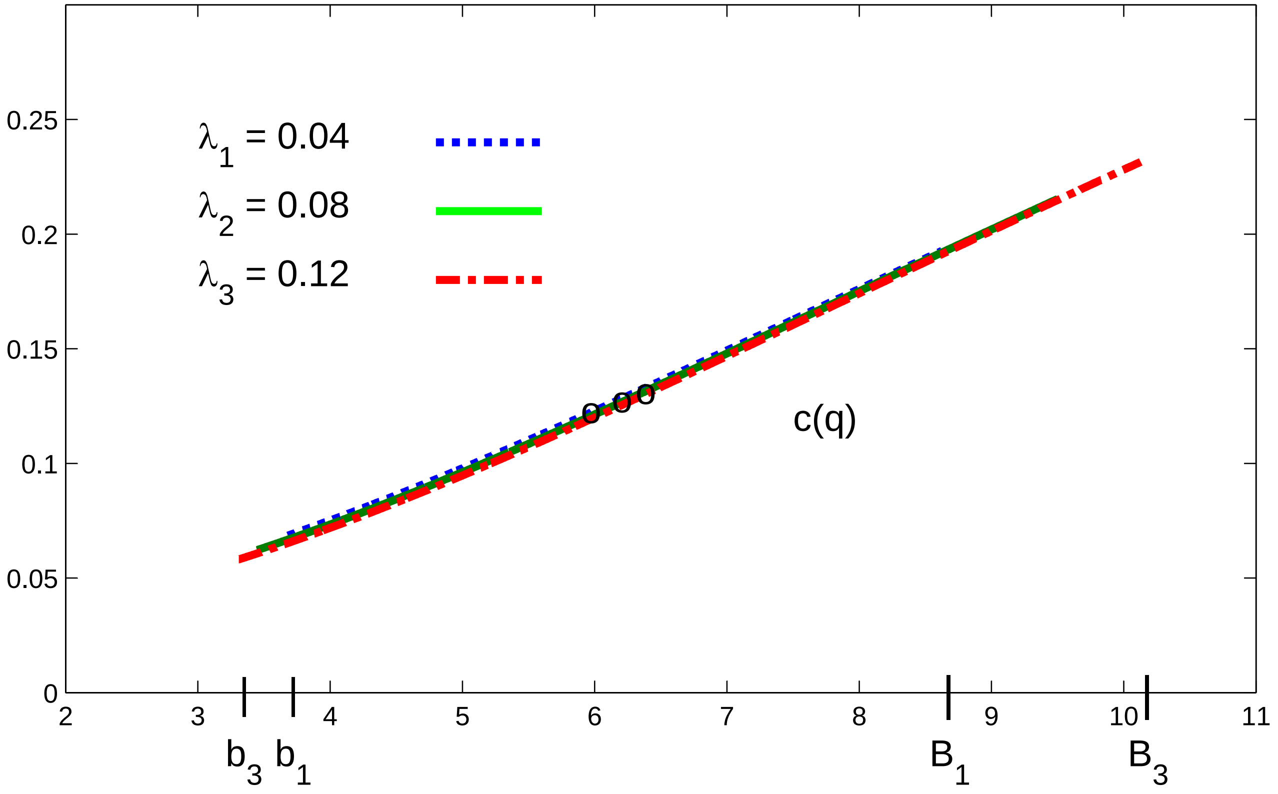


Figure 6b: Nondurable cons., various transaction costs



and housing's share rises to 30.8% for $\lambda = 0.04$ and falls to 29.4% for $\lambda = 0.12$.

An increase in λ also increases the expected duration between adjustments, which here rises from 16.0 years for $\lambda = 0.04$ to 20.8 and 23.8 at the higher values. Finally, as shown in Figure 6, a higher transaction cost makes the consumer more risk averse, but has virtually no effect on the consumption mix chosen for a given ratio of total wealth to housing.

An increase in the maintenance cost m makes housing more expensive, and hence shifts the consumption mix toward nondurables. As shown in Figure 7, an increase in m shifts the critical values b, S, B upward, to higher wealth/housing ratios. (Here ω is re-calibrated to keep housing's share at 30%.)

Figure 8 shows the effects of changing the risk aversion parameter θ. The most dramatic effect is, as expected, on the portfolio chosen, with more risk tolerant consumers putting higher fractions of their wealth in the risky asset. For $\theta = 2$ the short sale constraint comes into play, constraining the consumer when she is near either transaction threshold. Since the elasticity of intertemporal substitution is $1/\theta$, a reduction in θ also increases the consumer's willingness to substitute intertemporally. This is evidently the reason that the lower threshold b shifts to the left as θ falls. Here ω is not recalibrated, but housing's share changes very little, rising to 30.6% for $\theta = 2$ and falling to 29.8% for $\theta = 5$.

A positive hazard rate for exogenous moves means that the consumer is sometimes forced to sell her house and pay the transaction cost. This makes housing less attractive and moves more frequent. Figure 9 shows the effect a positive hazard rate, $\kappa = 0.10$. With the benchmark values for the other parameters the critical values b, S and B increase to 3.7, 6.7, and 10.3. The average ratio of total wealth to housing wealth rises to 7.0, and the expected duration between adjustments falls to 19.6 years. The average housing share in expenditures remains at 30%. The growth rate rises to 2.2%, reflecting the smaller fraction of total wealth tied up in housing.

Figure 7a: Portfolio shares, various maintenance costs

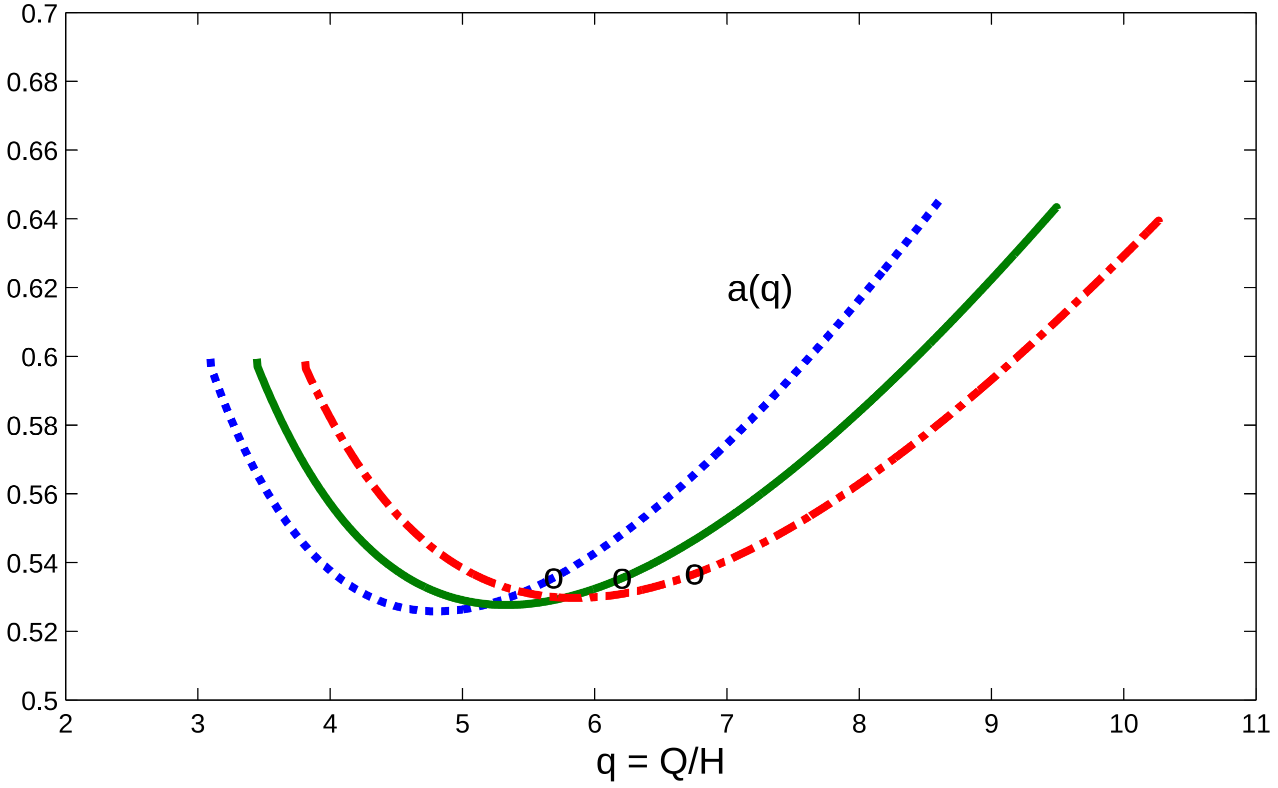
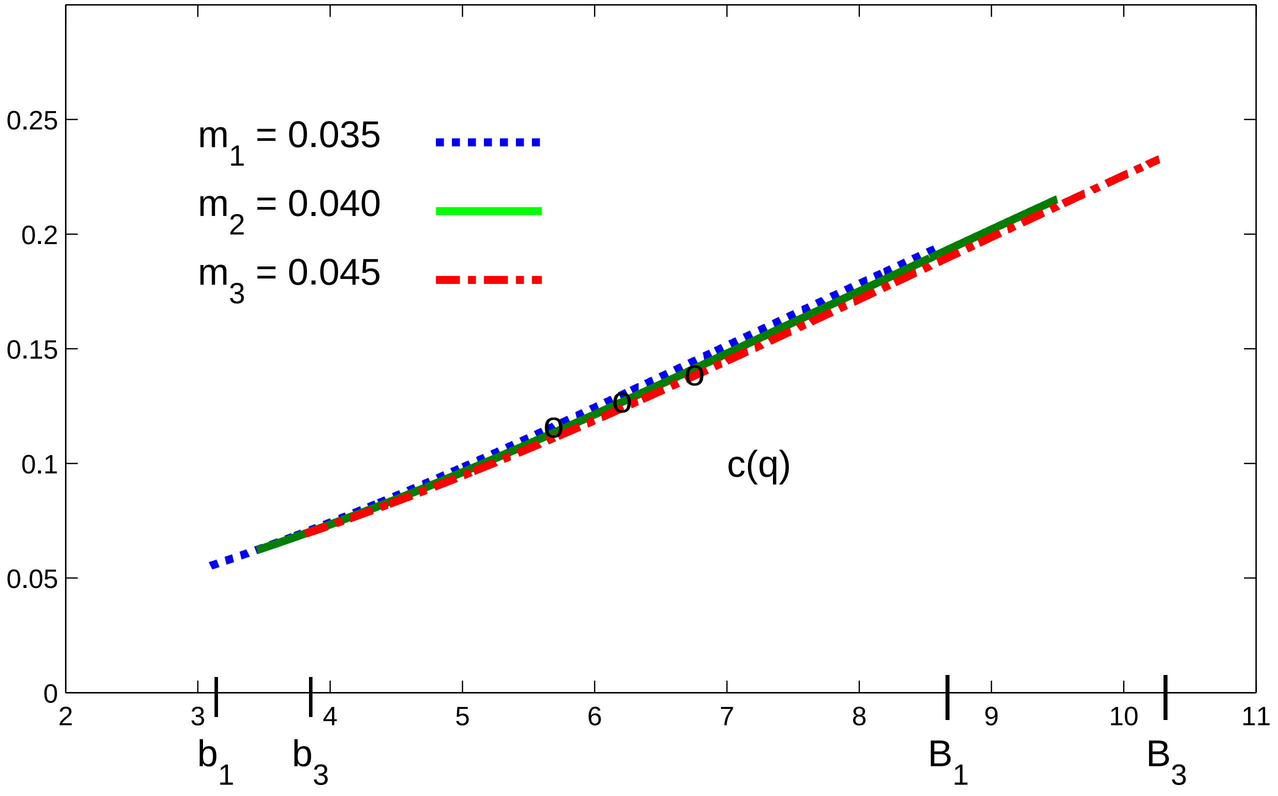
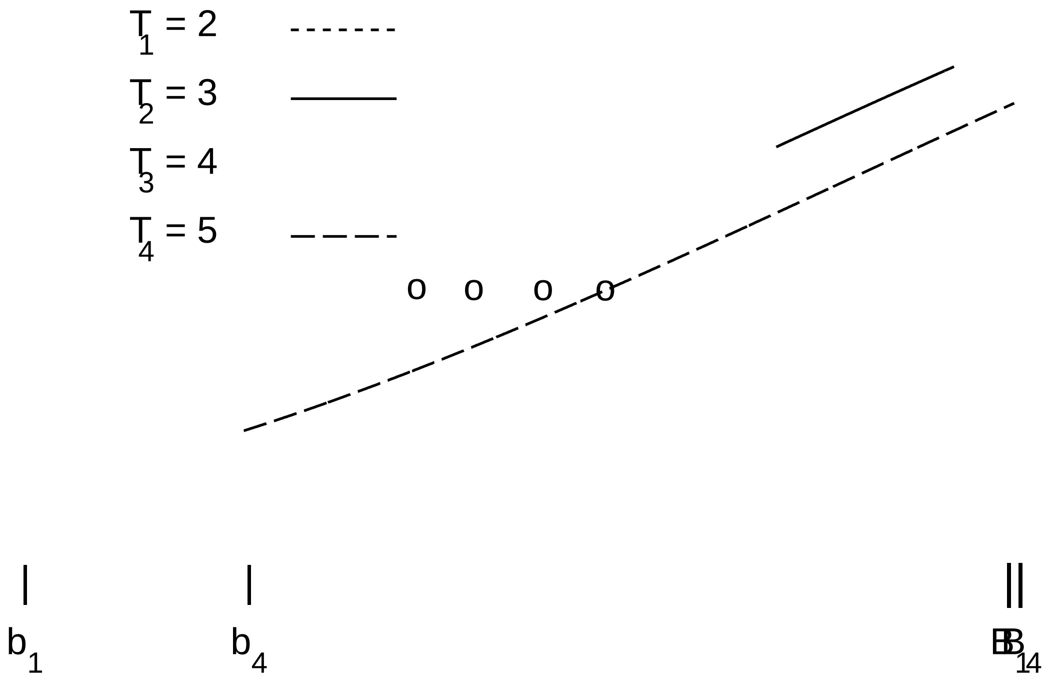


Figure 7b: Nondurable cons., various maintenance costs




$T_1 = 2$
$T_2 = 3$
$T_3 = 4$
$T_4 = 5$
o o o o
b_1
b_4
B_1
B_4

7. The equity premium puzzle

A standard exercise in finance[4] is to use the Euler equation

$$\mathrm{E}_t \left[\frac{U'\left(X_{t+s}\right)}{U'\left(X_t\right)} e^{-\rho s} \left(1 + r_{t+s}^j\right) \right] = 1, \tag{17}$$

which hold for any asset j, to conclude that

$$\left(\mu^j - r\right) = \theta \mathrm{Cov}\left(\frac{dX_t}{X_t}, r_t^j \right), \tag{18}$$

where X_t is total consumption, r_{t+s}^j is the instantaneous return on asset j, μ^j is the expected return on asset j, r is the risk-free rate, and θ is the coefficient of relative risk aversion. This is the equation commonly used to back out an estimate of the risk aversion parameter θ, using data on consumption growth and asset returns.

The equity premium is a puzzle because the covariance of consumption growth with asset returns is low. Thus, a large value of θ is needed to justify the excess return on the left side of (18).

But the relationship in (17) is derived in a frictionless model. If some components of consumption expenditures are costly to adjust, then they will be smoother than is predicted by that model. Consequently, the covariance of consumption growth and asset returns will be lower than predicted by the frictionless model. The model here, which is quantitative, allows us to calculate the magnitude of the error an econometrician would make if using a misspecified model.

Let $r_t^a = \mu dt + \sigma dz_t$ denote the instantaneous return on our model's risky asset. First note that with no transaction cost, as in section 3, consumption growth tracks growth in wealth, so

$$\frac{dX_t}{X_t} = \frac{dQ_t}{Q_t} = \left[\left(1 - a^*\right) r + \mu a^* - x^*\right] dt + \sigma a^* dz_t,$$

---

[4] See Mankiw and Zeldes (1991) for more detail.

where x^* is the (constant) ratio of consumption expenditures to wealth and a^* is the (constant) portfolio share in the risky asset. The term in square brackets is not stochastic. Hence

$$\begin{aligned} \mathrm{Cov}\left(\frac{dX_t}{X_t}, r_t^a\right) &= \mathrm{E}\left[(\sigma a^* dz_t)(\sigma dz_t)\right] \\ &= a^* \sigma^2 dt, \end{aligned}$$

so the econometrician using (18) would obtain

$$\begin{aligned} \hat{\theta} &= \frac{\mu - r}{\sigma^2}\frac{1}{a^*} \\ &= \theta, \end{aligned}$$

where the second line uses the optimal portfolio rule $a^* = \gamma/\theta$ and the definition of γ. In the absence of a transaction cost the model is correctly specified, and the estimate of θ is correct.

Now suppose we add the transaction cost. Consider a consumer who is using the thresholds b, S, B and the policy functions $c(q)$ and $a(q)$. To compute the covariance that would be obtained using long time series, we must average over q values inside the interval (b, B) using the stationary density, and also add terms for the discrete adjustments.

First consider consumption growth inside the inaction region. If the consumer's wealth at t is $q(t)$, then her consumption expenditure (per unit time) is

$$X(t) = p_h + c(q),$$

and the increment to her wealth is

$$dq = h(q)dt + a(q)q\sigma dz,$$

where

$$h(q) \equiv r\left[1 - a(q)\right]q + \mu a(q)q - \left[p_h + c(q)\right]$$

is the expected return on her portfolio less consumption expenditures. Thus, inside the inaction region growth in consumption expenditure is

$$\left[\frac{dX}{X}\middle| q \neq b, B\right] = \frac{c'(q)}{p_h + c(q)}\left[h(q)dt + a(q)q\sigma dz\right].$$

The jumps in consumption expenditure for adjustments at B and b are

$$\begin{aligned} J(B) &\equiv \left(\frac{B-\lambda}{S}\right)\frac{p_h + c(S)}{p_h + c(B)} - 1 > 0, \\ J(b) &\equiv \left(\frac{b-\lambda}{S}\right)\frac{p_h + c(S)}{p_h + c(b)} - 1 < 0. \end{aligned}$$

Let R_B and R_b denote the exit rates at B and b. Note that $R_B + R_b = 1/\mathrm{E}[T]$, and the proportions are $\Pr(B)$ and $\Pr(b)$. Also that jumps at B occur only if $dz > 0$ and jumps at b only if $dz < 0$.

Thus, averaging over positions inside the inaction region and jumps, consumption growth is

$$\begin{aligned} \frac{dX}{X} &= \int_b^B \frac{c'(q)}{p_h + c(q)}\left[h(q)dt + a(q)q\sigma dz\right]\psi(q)dq \\ &\quad + \frac{1}{\mathrm{E}[T]}\begin{cases} \Pr(B)J(B)dz, & \text{if } dz > 0, \\ \Pr(b)J(b)dz, & \text{if } dz < 0, \end{cases} \end{aligned} \tag{19}$$

where $\psi(q)$ is the stationary density for q on (b, B).

Hence the covariance between consumption growth and the return on the risky asset is

$$\begin{aligned} \mathrm{Cov}\left(\frac{dX}{X}, r^m\right) &= \sigma^2 \int_b^B \frac{c'(q)}{p_h + c(q)} a(q)q\psi(q)dq \\ &\quad + \frac{\sigma}{\mathrm{E}[T]}\left[\Pr(B)J(B) + \Pr(b)J(b)\right], \end{aligned} \tag{20}$$

where we have used (19) and the fact that only the terms involving dz enter.

Using (20) we can calculate the value $\hat{\theta}$ that an econometrician would arrive at (neglecting sampling error) if he estimated $\hat{\theta}$ using (18). For the benchmark calibration the estimated value would be $\hat{\theta} = 4.64$, while the true value is $\theta = 4.0$. A lower elasticity of substitution between housing and nondurables increases the magnitude of the error. But even for the very low elasticity of $\varepsilon = 0.15$, the estimated value would be only $\hat{\theta} = 6.2$ Thus, while the effect is in the right direction, it is far too small to explain much of the equity premium puzzle.

8. Conclusions

To be added.

REFERENCES

Aten, Bettina H. 2005. Report on interarea price levels, working paper 2005-11, Bureau of Economic Analysis, November.

Campbell, John Y. and Joao R. Cocco. 2005. How do house prices affect consumption? Evidence from micro data, NBER working paper 11534.

Chetty, Raj and Adam Szeidl. 2004. Consumption commitments: neoclassical foundations for habit formation, working paper, U.C.-Berkeley.

Cocco, J., 2005. Portfolio choice in the presence of housing, *Review of Financial Studies* 18, 535-567.

Davis, Morris, and Jonathan Heathcote. 2005. Housing and the business cycle, *International Economic Review*, 46: 751-784.

Dunn, K., Singleton, K., 1986. Modeling the term structure of interest rates under non-separable utility and durability of goods, *Journal of Financial Economics* 17, 27-55.

Eichenbaum, Martin S. and Lars P. Hansen. 1990. Estimating models with intertemporal substitution using aggregate time series data. *Journal of Business and Economic Statistics* 8, 53-69.

Eichenbaum, Martin S., Lars P. Hansen, and Kenneth J. Singleton. 1988. A time series analysis of representative agent models of consumption and leisure choice under uncertainty, *Quarterly Journal of Economics* 103: 51-78.

Fillat, Jose L. Housing as a measure for long-run risk in asset pricing, working paper, University of Chicago.

Flavin, Marjorie and Shinobu Nakagawa. 2004. A model of housing in the presence of adjustment costs: a structural interpretation of habit persistence, NBER working paper 10458, April 2004.

Flavin, Marjorie and Takashi Yamashita, 2002. Owner-occupied housing and the composition of the household portfolio, *American Economic Review* 92, 345-362.

Fukushima, Kenichi. 2005. Asset pricing implications of precommitted consumption, working paper, U. of Minnesota.

Grossman, Sanford and Guy Laroque. 1990. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption good, *Econometrica* 58: 25-51.

Hanushek, Eric A. and John M. Quigley. 1980. What is the price elasticity of housing demand? *Review of Economics and Statistics* 62: 449-454.

Kocherlakota, Narayana R. 1996. The equity premium: it's still a puzzle, *Journal of Economic Literature*, 34: 42-71.

Kullman, Cornelia, and Stephan Siegel. 2004. Real estate and its role in household portfolio choice, Working paper, UBC.

Lustig, H., Van Nieuwerburgh, S., 2006. Housing collateral, consumption insurance and risk premia, working paper, UCLA..

Mankiw, N. Gregory and Stephen P. Zeldes. 1991. The consumption of stockholders and nonstockholders, *Journal of Financial Economics* 29: 97-112.

Marshall, David A. and Nayan G. Parekh. 1999. Can costs of consumption adjustment explain asset pricing puzzles? *Journal of Finance* 54: 623-654.

Martin, Robert F. 2003 Consumption, durable goods, and transaction costs, International Finance Discussion Paper No. 756, Federal Reserve Board, Washington, DC.

Mehra, Raj and Edward C. Prescott. 2006.

Ogaki, M., Reinhart, C.M., 1998. Measuring intertemporal substitution: the role of durable goods, *Journal of Political Economy* 106, 1078-1098.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel. 2006. Housing, consumption and asset pricing, *Journal of Financial Economics*,

Schachter, Jason P. and Jeffrey J. Kuenzi. 2002. Seasonality of moves and the duration and tenure of residence: 1996, Population Division working paper 69, U.S.

Census Bureau, Washington, DC, December.

Siegel, Stephan. 2004. Consumption-based asset pricing: durable goods, adjustment costs, and aggregation, working paper, Graduate School of Business, Columbia University.

Smith, L.B., K.T. Rosen, and G. Fallis. 1988. Recent developments in economic models of housing markets, *Journal of Economic Literature* XXVI, 29-64.

Stokey, Nancy L. 2007. *Brownian Models in Economics*, Princeton University Press, forthcoming.

APPENDIX

Proof of Proposition 1: First we must show that (6) has a finite maximum. If $0 < \theta < 1$, then we need to restrict the parameters so that $\rho > \Gamma$ for all feasible (c, h, a), so that utility does not diverge to $+\infty$ along any feasible path. If $\theta > 1$, then we need to insure that $\rho > \Gamma$ for at least one feasible (c, h, a), so that expected discounted utility does not diverge to $-\infty$ along every feasible path.

To this end we will first show that if

$$\rho > \Gamma(c, h, a; \theta), \begin{cases} \text{all feasible } (c, h, a), & \text{if } 0 < \theta < 1, \\ \text{some feasible } (c, h, a), & \text{if } \theta > 1, \end{cases} \tag{21}$$

then the optimal portfolio $\alpha(h; \theta)$ is

$$\alpha(h; \theta) = \min \left\{ \frac{\gamma}{\theta}, a_{ss} (1 - \epsilon h) \right\}. \tag{22}$$

Then we will show that under Assumptions 1 and 2, (21) holds.

(i) Suppose (21) holds. Since a appears in (6) only as an argument of Γ, the optimal portfolio solves

$$\max_{a \in [0, a_{ss}]} \frac{1}{1-\theta} \frac{1}{\rho - \Gamma(c, h, a; \theta)}.$$

Hence the objective is to maximize Γ if $0 < \theta < 1$ and to minimize Γ if $\theta > 1$. Note that

$$\begin{aligned} \Gamma_a(c, h, a) &= (1-\theta) \left[(\mu - r) - \theta a \sigma^2 \right] \\ &= (1-\theta) \sigma^2 (\gamma - \theta a). \end{aligned}$$

If $0 < \theta < 1$, then $\Gamma_{aa} < 0$, so Γ is concave. Since $\Gamma_a(c, h, 0) > 0$, there cannot be an optimum at $a = 0$. If $\gamma/\theta > a_{ss} (1 - \epsilon h)$, then

$$\Gamma_a(c, h, a_{ss} (1 - \epsilon h); \theta) = (1-\theta) \sigma^2 \left[\gamma - \theta a_{ss} (1 - \epsilon h) \right] > 0,$$

so the solution is at a corner, $\alpha(h; \theta) = a_{ss} (1 - \epsilon h)$. Otherwise the solution is interior and satisfies $\Gamma_a = 0$. Hence the optimal portfolio is as in (22).

If $\theta > 1$, the objective is to minimize $\Gamma(c, h, a)$. In this case Γ is convex, and the preceding argument holds with a sign change. Hence (22) also holds for $\theta > 1$.

(ii) Next we will show that Assumptions 1 and 2 insure (21) holds. Suppose $0 < \theta < 1$. Then

$$\begin{aligned} \frac{d}{dh}\Gamma(c,h,\alpha(h;\theta);\theta) &= \Gamma_h + \Gamma_a \alpha'(h;\theta) \\ &= -(1-\theta)(p_h + c) + (1-\theta)\sigma^2\left[\gamma - \theta\alpha(h;\theta)\right]\alpha'(h;\theta) \\ &\leq -(1-\theta)(p_h + c) \\ &< 0, \end{aligned} \tag{23}$$

where the second line uses the fact that $\gamma/\theta \geq \alpha(h;\theta)$ and $\alpha'(h;\theta) \leq 0$. Hence for any $\epsilon \geq 0$,

$$\rho > \Gamma(0,0,\alpha(0;\theta);\theta) \geq \Gamma(c,h,\alpha(h;\theta);\theta) \geq \Gamma(c,h,a;\theta), \quad \text{all feasible } (c,h,a),$$

where the first inequality uses Assumption 2, the second uses (23) and the fact that $\partial\Gamma/\partial c < 0$, and the third uses the fact that $\alpha(h;\theta,\epsilon)$ maximizes $\Gamma(c,h,a;\theta)$.

If $\theta > 1$, then since $r > 0$, for $a = 0$ and all c, h sufficiently small,

$$\rho > 0 > (1-\theta)\left[r - (p_h + c)h\right] = \Gamma(c,h,0).$$

Proof of part (a): Suppose $\epsilon = 0$. The optimal portfolio a_R is as in (22). To characterize c_R, note that for any fixed expenditure flow E on housing and nondurables, the optimal consumption mix (c, h) solves

$$\max_{c,h}\ u(c)h \qquad \text{s.t. } (p_h + c)h = E,$$

so the ratio of c of nondurables to housing solves

$$\max_c \frac{u(c)}{p_h + c}.$$

For the CES preferences here, the solution is (7).

Then since a_R does not involve h, maximizing (6) with respect to h gives the FOC

$$\begin{aligned} 0 &= \frac{1-\theta}{h_R} + \frac{\Gamma_h}{\rho - \Gamma} \\ &= (1-\theta)\left(\frac{1}{h_R} - \frac{p_h + c_R}{\rho - \Gamma}\right), \end{aligned}$$

or

$$(p_h + c_R)\, h_R = \rho - \Gamma\left[0, 0, \alpha_R(\theta); \theta\right] + (1-\theta)\,(p_h + c_R)\, h_R,$$

or

$$\theta\,(p_h + c_R)\, h_R = \rho - (1-\theta)\left[r + (\mu - r)\, a_R(\theta) - \theta \frac{1}{2}\sigma^2 a_R^2(\theta)\right],$$

as claimed.

Then

$$(p_h + c_R)\, h_R = \frac{\rho - r}{\theta} + r - (1-\theta)\left(\frac{1}{\theta}\gamma\sigma^2 a_R - \frac{1}{2}\sigma^2 a_R^2\right)$$

so

$$\begin{aligned} (p_h + c_R)\, h_R'(\theta) &= -\frac{1}{\theta^2}\left(\rho - r - \gamma\sigma^2 a_R\right) - \frac{1}{2}\sigma^2\alpha_R^2 - (1-\theta)\left(\frac{1}{\theta}\gamma - \alpha_R\right)\sigma^2 a_R' \\ &= \frac{1}{\theta^2}\left[r - \rho + \left(\gamma - \frac{1}{2}\theta^2\alpha_R\right)\sigma^2 a_R\right], \end{aligned}$$

establishing the last claim.

Proof of part (b): The solution in (7) solves the problem with $\epsilon > 0$ if and only if a_R, h_R satisfies the tighter portfolio constraint. Otherwise the tighter portfolio constraint binds, so $h_B(\theta, \epsilon) < h_R(\theta)$. Using (22), the consumer's problem is as in (8). The conditions for a maximum are

$$\begin{aligned} 0 &= \frac{(1-\theta)\, u'(c)}{u(c)} + \frac{\Gamma_c}{\rho - \Gamma}, \\ 0 &= \frac{1-\theta}{h} + \frac{\Gamma_h - \Gamma_a \epsilon a_{ss}}{\rho - \Gamma}, \end{aligned}$$

or

$$\begin{aligned}
\frac{u'(c)}{u(c)} &= \frac{h}{\rho - \Gamma}, \\
\frac{1}{h} &= \frac{c + p_h + \left[\gamma - \theta a_{ss}\left(1 - \epsilon h\right)\right]\sigma^2 \epsilon a_{ss}}{\rho - \Gamma}.
\end{aligned}$$

Combining these two gives gives

$$\begin{aligned}
\frac{1}{c + p_h + \left[\gamma/\theta - a_{ss}\left(1 - \epsilon h\right)\right]\theta\sigma^2 \epsilon a_{ss}} &= \frac{u'(c)}{u(c)} \\
&= \frac{\omega}{\omega c + \left(1 - \omega\right)c^{\zeta}},
\end{aligned}$$

so

$$c_B(\theta, \epsilon) = \left[\frac{\omega}{1 - \omega}\left\{p_h + \left[\gamma/\theta - a_{ss}\left(1 - \epsilon h\right)\right]\theta\sigma^2 \epsilon a_{ss}\right\}\right]^{1/\zeta}.$$

For a renter the term on the right in square brackets is zero, while for a constrained buyer it is positive. Hence $c_B(\theta, \epsilon) > c_R$. ■

PROOF: That in the model with a transaction cost, for any fixed initial condition (Q_0, H_0) with $Q_0 > \lambda H_0$, the value $V\left(Q_0, H_0\right)$ is finite. The transaction cost cannot raise utility, so clearly $V(Q_0, H_0)$ is bounded above by $Q_0^{1-\theta}w^*$. In addition, under Assumption 1 it is possible to choose a feasible strategy for which expected utility is bounded below. For example, set $A \equiv 0$, so the risky asset is not held; let $C = cH$, where $c > 0$ is small; set $H = hQ$ when a transaction is made, where $h > 0$ is small; choose a long period T between transactions; and make the first transaction at date 0. During intervals when no transaction is made wealth grows at a constant rate

$$\frac{dQ}{Q} = \left[r - \left(p_h + c\right)h\right]dt.$$

For c and h sufficiently small, this growth rate is positive. Hence $W_{n+1} > W_n$, all n. where W_n is wealth after the n^{th} housing transaction. Over each interval with no transaction, utility is $vW_n^{1-\theta}$, where v is a constant. Hence lifetime utility under this strategy is

$$\sum_{n=0}^{\infty} e^{-\rho n T} v W_n^{1-\theta} \geq v \left(Q_0 - \lambda H_0\right)^{1-\theta} \frac{1}{1 - e^{\rho T}},$$

where the right side is finite.