

OCCUPATIONAL SORTING AND DEVELOPMENT*

Jan Eeckhout and Boyan Jovanovic

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Abstract

The rise in world trade since 1970 has raised international trade in labor services. We study the effect of such a globalization of the world's labor markets. We find that when people can choose between wage work and managerial work, a worldwide labor market raises output by more in the rich and the poor countries, and by less in the middle-income countries. This is because the middle-income countries experience the smallest change in the factor-price ratio, and where the option to choose between wage work and managerial work has the least value in the integrated economy. The same result holds qualitatively in the standard two-skill model, but it there is much smaller because that model does not allow people to switch occupations in response to globalization. We find that the occupation-switching effect is larger than the standard effect.

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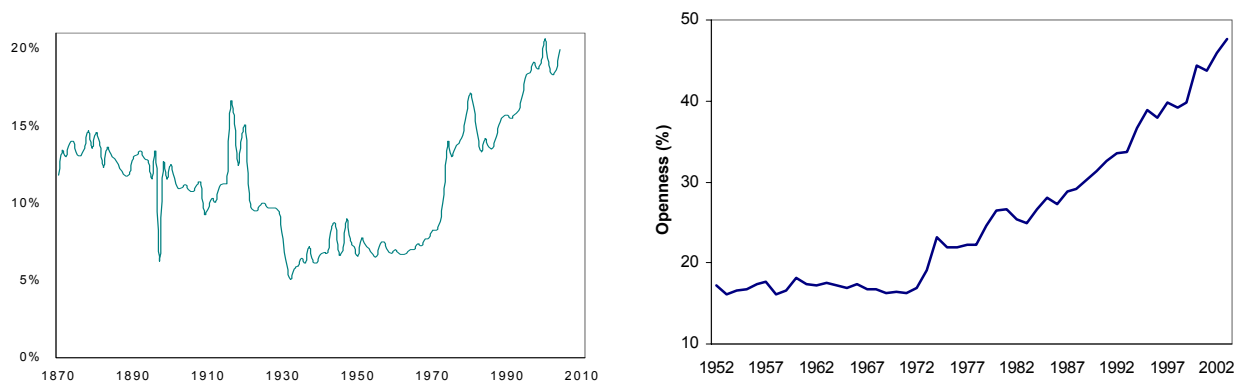
1 Introduction

We study how the integration of the world’s labor markets affects development. In recent decades, such an integration has taken place, in that people who live far away from one another can nonetheless potentially produce together. In the light of those changes, we ask which local economies will grow fastest when the world labor market opens up. Moreover, we decompose the effect into a “standard” effect in which one can take part in the global labor market but *without* changing occupations, and an occupational switching effect that arises when agents can change their occupations if they want to.

The basic premise is that in integrated labor markets, agents who are very distant from each other can nonetheless potentially produce together. The labor inputs themselves do not necessarily need to move in order for the output to be consumed, as long as there is an adequate communication or transportation technology. Examples include tax returns that are prepared remotely, call centers in India and textiles manufactured in China but designed in the West.

We find that when people can choose between wage work and managerial work, a worldwide labor market raises output by more in the rich and the poor countries, and by less in the middle-income countries. This is because the middle-income countries experience the smallest change in the factor-price ratio, and where the option to choose between wage work and managerial work has the least value in the integrated economy.

The rise in world trade since 1970.—Panel A in Figure 1 shows U.S. total trade as a percentage of GDP. The Penn World Tables (Summers-Heston) also include a measure of openness as exports plus imports (i.e., total trade) as a percentage of GDP but reports data only starting in the 1950s. In panel B we plot the population-weighted average of openness of all 58 countries in the sample that have observations for all years between 1952 and 2003. Both sets of data confirm the rise in openness in the ‘70s, with the world opening up more gradually than the U.S..



A. OPENNESS US 1870-2004: INCREASED OPENNESS FROM 1970SB. WORLD AVERAGE TOTAL TRADE/GDP (PENN WORLD TABLES)
FIGURE 1. MEASURES OF OPENNESS

U-shaped growth since 1975.—Between 1975 and 2000, the middle countries did worse than countries in the tails of the distribution of GDP per capita. Summers and Heston provide data for 154 countries on GDP per capita (ppp-adjusted and at constant prices) and population. We rank countries by GDP per capita, and weigh each country by its share in the world population. Figure 2 plots the 10-country moving average of growth rates against country rank. Evidently, poor and the rich economies grow faster than the middle economies.¹

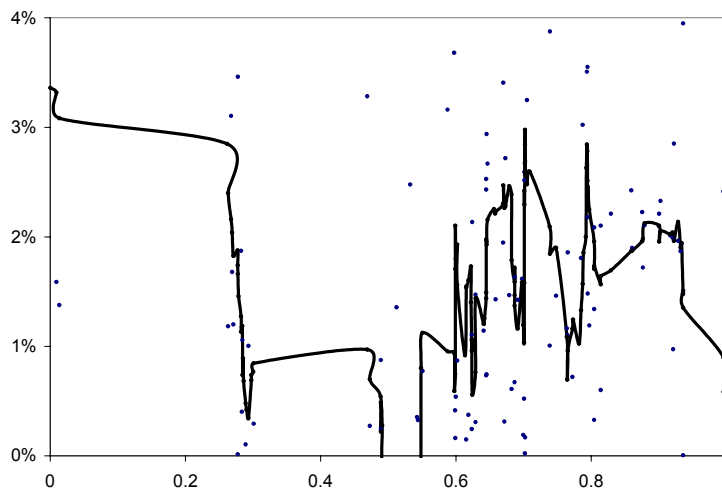


Figure 2. Annualized growth rates (data and 10-country moving average).

Robustness: Maddison Historical Data.—the argument also works in reverse: As we move from free trade to autarky, growth rates will therefore exhibit an *inverted* U shape, being highest for the middle-income countries. The period following World War I arguably such a period. Panel 1 of Figure 1 suggests it, and so we shall assume that there also was a considerable drop in effective factor mobility between the pre-WWI era and the Great Depression. To cover this period, we use as a source the Maddison (1995) historical data. More specifically, we make use of the series composed by Bourguignon and Morrisson (2002) based on Maddison (1995). To construct the entire world income distribution, this series bundles economies in 33 different groups of regions and comparable economies. It has observations for 1910 and 1929, so we calculate annual growth rates for this period.² As above for the period 1975-2000, the following Figure 3 has the annualized growth rates on the vertical and

¹AnU-shaped relation is seen at the very top of the distribution, where some very small countries have negative growth rates. Those countries are Saudi Arabia, Brunei, Kuwait, United Arab Emirates, Qatar. While all observations count, those countries are very unusual in 1975 because they are in the middle of a huge boost to their GDP due to the sharp increases in oil prices. Moreover with a 10-country moving average they are over-represented relative to their small population size. A stochastic kernel regression is likely to mitigate that effect as it will put very little weight on these tiny countries.

²We exclude data for the 1930s as people argue that the Great Depression is caused by many

the rank of GDP per capita weighted by population on the horizontal. We have both the data points together with a graph of the 5 country moving average to smooth out the relation. The plot suggests that growth rates exhibit an inverted U shape in GDP/cap. The middle economies grow faster than the small and large economies. This is consistent with our theory since that period is an era of decreasing openness.

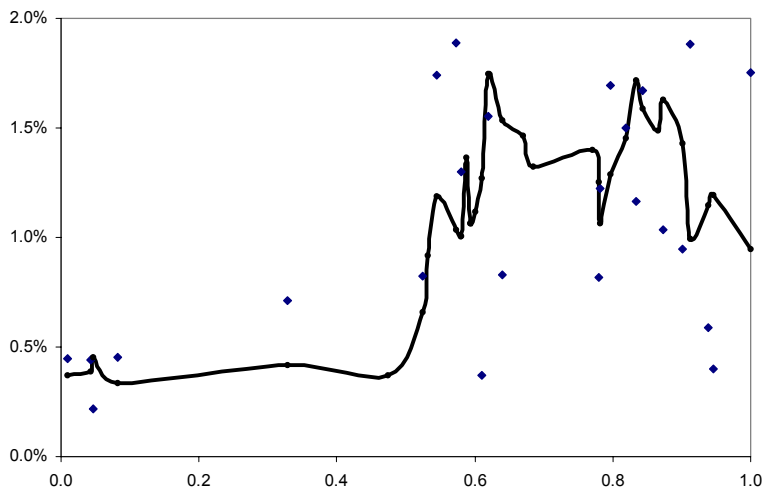


FIGURE 3. ANNUALIZED GROWTH RATES (DATA AND 5-COUNTRY MOVING AVERAGE): 1910-1929

The model.—The model is static, and distinguishes agents by a single number: Their human-capital endowment. It builds on Jovanovic (1994) who allows managers workers both to differ in their human-capital endowments, and who allows an arbitrary size of firm. We model a world without external effects and atomless agents and firms. In such a world, trade benefits all; no one is worse off. As a first approximation we interpret autarky as a situation in which agents in each country have identical skills but in which they can trade freely with agents in the same country. In that case, a conversion from autarky to a world labor market leads to benefits being highest for the high and low GDP economies, and lowest for the middle economies. There always exists a country, somewhere in the middle of the distribution, where the wage remains unchanged, and where the residents are no better off than they were under autarky.

The standard two-skill model.—The result that openness of labor markets benefits the rich and the poor more than it benefits the middle-income countries also holds in a standard two-skill model in which occupational choice is exogenous, i.e., in which there is no “sorting” of people into occupations . In such a model, free trade also raises the incomes of all countries and, again, the middle-income countries benefit the least because they have the skilled-unskilled ratio that is the most representative of the world’s average. That model, however, does not allow any occupational shift in

other factors. The next observation in this data set is 1950, which according to the openness data is already too far after the decrease in openness we aim to capture.

response to a regime shift such as the shift to free trade. Moreover, it cannot explain the rise in the gap between the skilled and unskilled workers that accompanied the rise in trade.

Contrasting our model to the standard two-skill model.—In our model, free trade raises output through two distinct channels, whereas in the standard model it works through only one channel. We shall decompose the effect of openness into (i) The standard effect in which occupation is held constant (this effect is present in both models), and (ii) The occupational sorting effect, which only our model has. Effect (i) lowers the equilibrium span of control of high-ability managers and lowers it for the low-ability managers, and implies a reallocation of existing workers among existing managers. Effect (ii) allows low-ability managers to become workers and high-ability workers to become managers, and this leads to additional output gains. The figure below plots the gains from trade for an example we work out in detail below. The blue line corresponds to the the total gains from trade (effect (i) and (ii)) and the red line to the standard effect (i) only.

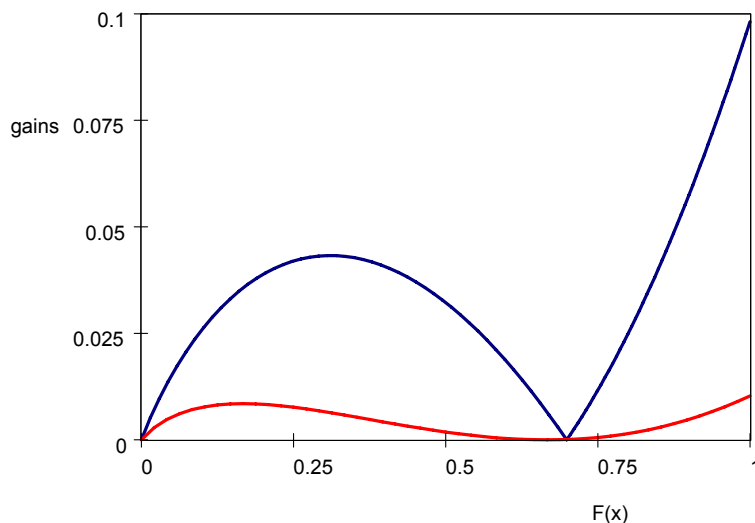


FIGURE 4. GAINS FROM TRADE: STANDARD + SORTING EFFECT

Finally our model is consistent with the finding by Gabaix and Landier (2006) that the recent rise in the level and dispersion of managerial earnings is explained by a similar rise in the level and dispersion of the resources under their control. Such a rise occurs in our model as a result of globalization, but it does not take place in the standard model.

Other related work.—Lucas (1978) has a similar model, but in it workers all have the same earnings (total wage), and so the distribution of earnings has a counterfactual spike at the lowest income. Similar spikes also exist in the models of Burstein

and Monge-Naranjo (2007) and Monge-Naranjo (2007). The former paper studies the flow of capital and management across countries and distinguishes country-specific and firm-specific effects on productivity. The latter paper endogenizes skills; we do not do that, but our results on the effects of globalization on the skill premium are highly suggestive of what would happen to skills if they were allowed to change. Kremer and Maskin (2003) study the globalization of labor markets in a two-country model in which all firms have just two employees. Gavilan (2006) adds physical capital to the model and studies its impact on the equilibrium assignment of workers to managers. Antràs, Garicano and Rossi-Hansberg (2006) argue that the information-transmission technology affects the allocation of resources among locations – they model the barriers to offshoring, whereas we simply assume that these barriers change from being insuperable, to being nonexistent. These papers are all more careful about some aspects of reality, but do not establish a relation between the facts portrayed by Figures 1, 2 and 3, and this is one part of the value added by our paper. In Section 7 we shall discuss in more detail other related papers: Gabaix and Landier (2006), and McGrattan and Prescott (2007).

2 The Model

We shall consider a world population consisting of agents endowed with a one-dimensional skill x .

Production.—Firms produce output q with the input of a manager and a set of workers. Denote the production function by

$$q = xQ(h) \tag{1}$$

where x is the manager’s skill or efficiency and h is the total number of efficiency units of labor that the firm’s workers possess. We assume $Q' > 0$, and $Q'' < 0$. The manager is the entrepreneur who owns the firm, and she hires workers at the price of w per their efficiency unit. The inputs into the production function (1) enter asymmetrically: Only one manager can perform the job, but there is substitution of quality and quantity of workers in h , and any number of workers can be hired.

The firm’s decision problem.—When facing an efficiency-units wage w , a manager of type x solves the problem

$$\pi(x, w) \equiv \max_h \{xQ(h) - wh\}, \tag{2}$$

which has the FOC

$$xQ'(h) = w. \tag{3}$$

2.1 Autarky

Under autarky, each atomless agent belongs to a local economy or country. Within that country, agents are identical, each being of type, say, x , and each can become a

worker or a manager. As a worker that person would earn wx and as a manager, he or she would earn $\pi(x, w)$.

Autarky Equilibrium is a wage w and a fraction n of people that become workers, such that they solve the pair of equations (4) and (5). In equilibrium the supply of h would be xn and per manager (the fraction of which is $1 - n$) the supply of h would be $xn/(1 - n)$. For managers to wish to employ this market-clearing quantity, it would have to satisfy (3), which then would read

$$xQ' \left(\frac{xn}{1 - n} \right) = w. \quad (4)$$

For managers and workers to all be happy in the occupation they have chosen, $\pi(x, w)$ would have to equal wx . That is,

$$xQ \left(\frac{xn}{1 - n} \right) - w \frac{xn}{1 - n} = wx. \quad (5)$$

We denote the Autarky Equilibrium by $\{w(x), n(x)\}$. It is the pair of numbers (w, n) solving (4) and (5) for the type- x autarkic economy.

These equilibrium outcomes are driven by the feasible matches. In the case of autarky, only agents of the same type can work together, implying labor income $w(x)x$ and profits $\pi(x)$ are the same. The implication is of course that wages are different in each local economy indexed by x .

Example.—Let $Q(h) = h^\alpha$. Then (4) reads $\alpha x h^{\alpha-1} = w$, and (5) reads $xh^\alpha - wh = wx$. Together, these two imply that $h = \frac{\alpha}{1-\alpha}x$. Since $h = xn/(1 - n)$, this means that

$$n(x) = \alpha \quad \text{and} \quad w(x) = (1 - \alpha)^{1-\alpha} \alpha^\alpha x^\alpha. \quad (6)$$

2.2 Worldwide labor market

Let $F(x)$ be the world distribution of $x \in R_+$, assumed atomless. Now w is a wage that prevails world wide. A type- x manager in this economy still solves (2). Denote the manager's demand function $h = g(x, w)$; it solves (3) for h .

Then the set of managers is the set $E(w) = \{x \in R_+ \mid \pi(x, w) > wx\}$. The market-clearing condition then reads

$$\int_{E(w)} g(x, w) dF(x) = \int_{R_+ - E(w)} x dF(x). \quad (7)$$

Then a *World-market Equilibrium* is a wage w that solves (7)

Denote by z the skill type that is indifferent between becoming a manager and a worker:

$$\pi(z, w) = wz. \quad (8)$$

By the envelope theorem, $\pi_x = Q(g[x, w])$, and since $g_x > 0$, $\pi_{xx} > 0$. Since $\pi(0, w) = 0$, (8) has at most two intersections. Since F is atomless, it follows that $E(w) = [z, \infty)$, i.e., employers are drawn from the top of the distribution.

Under world-wide free trade, a high-skilled agent can start a firm and hire workers on the world labor market at the world wage w (per efficiency unit). Because firms need both workers and managers, not all types can become managers. The managers are in the high skill economies and hire workers from low skill economies.

Example.—Again, let $Q(h) = h^\alpha$. As under autarky, the FOC is $w = \alpha x h^{\alpha-1}$. Using this to substitute for w in (8), we get that for entrepreneur z , factor demand is $g(z, w) = \frac{\alpha}{1-\alpha} z$ and therefore,

$$w = (1 - \alpha)^{1-\alpha} \alpha^\alpha z^\alpha,$$

which gives us w in terms of z . The second restriction on w and z is the market-clearing condition

$$\int_z^\infty g(x, w) dF(x) = \int_0^z x dF(x) \quad (9)$$

in which, for $x > z$, we have $g(x, w) = \frac{\alpha}{1-\alpha} z^{\frac{-\alpha}{1-\alpha}} x^{\frac{1}{1-\alpha}}$.

2.2.1 Globalization with no occupational switching

If a single global labor market opens, there is a single wage \tilde{w} that would clear the market. Because occupational switching is not allowed, $n(x)$ type- x agents are still workers and $1 - n(x)$ are still managers in the new regime. Manager x solves the decision problem in (2), and has a factor demand $g(x, w)$, just as before. The market clearing wage again satisfies a single condition but, instead of (9), that condition is

$$\int_0^\infty g(x, w) [1 - n(x)] dF(x) = \int_0^\infty x n(x) dF(x), \quad (10)$$

where $n(x)$ is given by the equilibrium allocation under autarky. Notice that the RHS does not depend on the wage – workers have no choice but to remain workers no matter what they are paid. There is a gain in output over autarky, but it is limited by the inability of agents to switch occupations.

The Cobb-Douglas example again.—From (6) we know that $n(x) = \alpha$, and from the FOC which reads $w = \alpha x h^{\alpha-1}$, that

$$g(x, w) = \left(\frac{\alpha x}{w}\right)^{\frac{1}{1-\alpha}}.$$

Therefore (10) reads

$$(1 - \alpha) \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \int_0^1 x^{\frac{1}{1-\alpha}} dx = \alpha \int_0^1 x dx,$$

from which we have

$$w = \alpha \left(\frac{2(1-\alpha)^2}{\alpha(2-\alpha)} \right)^{1-\alpha}$$

Now aggregate output is

$$\int_0^\infty [g(x, w)]^\alpha [1 - n(x)] dF(x) = (1 - \alpha)^2 \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}}$$

We plot the growth that is due to openness while keeping the sorting allocation constant. For the uniform distribution with $\alpha = \frac{1}{2}$, we have that $n(x) = \frac{1}{2}$ and we get $w = \frac{1}{\sqrt{6}} = 0.40825$, and $\pi(x) = 0.61237x^2$. Since there is no occupational sorting in this example, the per capita income in each country x is the weighted sum of w and $\pi(x)$ or $y(x) = 0.20413x + 0.30619x^2$. Compare to autarky where $y^A(x) = \frac{1}{2}x^{\frac{3}{2}}$.

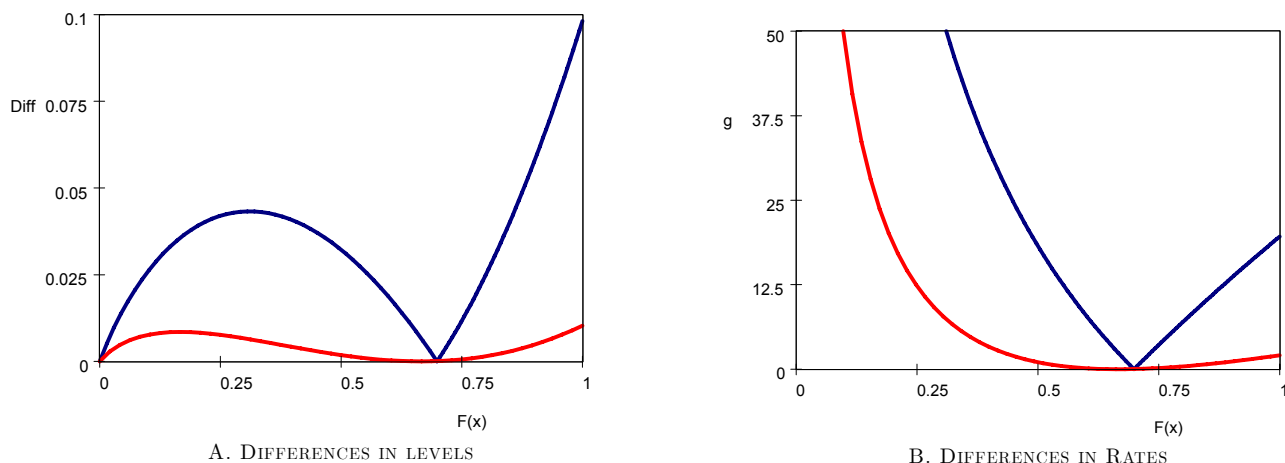


FIGURE 5. GAINS FROM TRADE WITHOUT SORTING

Keeping the occupational allocation fixed, opening up the world labor market implies that the initially identical workers now face different terms depending on their occupation. For the high x countries, because now there is a world wage that is lower than the high x wage under autarky, the entrepreneurs now earn more than the workers, even though they have the same type. In the low x countries, the opposite is true: the workers do relatively better than the entrepreneurs. Next, we plot the ratio of the highest earner by country in the economy where the allocational choice is frozen. All countries now have some degree of inequality, and it is largest at the extremes. There is one country (typically different from z) without any inequality at all.³

³The occupation-switch effect that we have emphasized becomes sizeable only when the regime shift offers a non-negligible change in earnings opportunities, such as globalizaton probably affords.

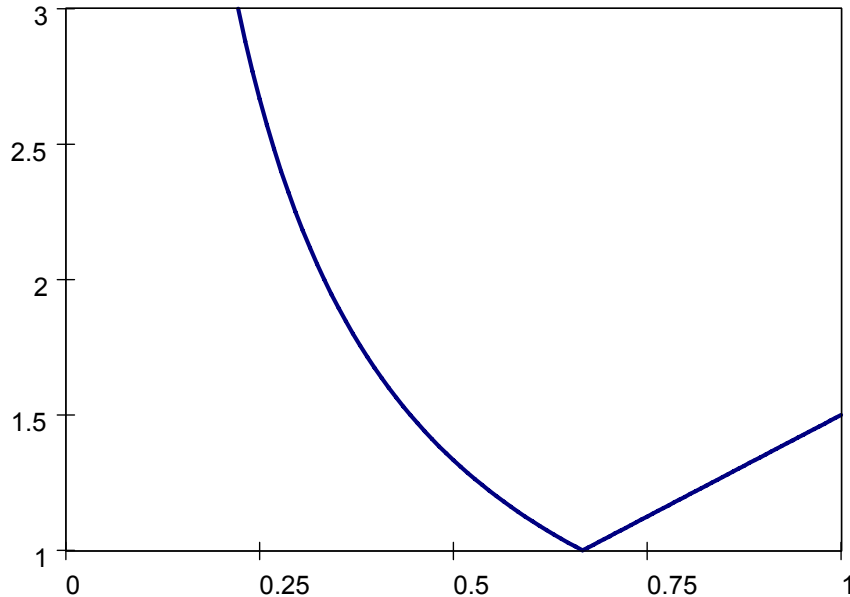


Figure 6. Ratio of high over low income without the sorting effect.

2.3 Autarky vs. the free market in an example

Now suppose that $\alpha = 1/2$ and that the skill distribution is uniform: $F(x) = x$. Then we have

The autarky solution.—For all x , profits and wage earnings are $w(x)x = \frac{1}{2}x^{\frac{3}{2}}$.

The free-market solution.—Equilibrium is $z = 0.69$, $w^F = 0.42$. Then incomes are $\max(w^F x, \pi^F(x)) = \max(0.42x, 0.59x^2)$.

Earnings under autarky in function of skills x are plotted in panel A of Figure 4. In panel B, the straight line (red) is the wage income, the constant wage times the efficiency units x . The convex function (blue) is the profit schedule. Low types are

The analog in a single agent problem is the change in utility when prices change. For instance,

$$\frac{d}{d\theta} \left\{ \max_x U(x, \theta) \right\} = \frac{\partial}{\partial \theta} U(x^*(\theta), \theta)$$

where $x^*(\theta)$ is the decision optimally taken when the environment is θ . The equality holds only for infinitesimal changes in prices; this is the envelope theorem. But for large changes, say from θ to θ' ,

$$\max_x U(x, \theta) - \max_x U(x, \theta') \geq U(x^*(\theta), \theta) - U(x^*(\theta), \theta').$$

better off in the occupation of a worker, whereas high types earn profits that are over and above the wage income. The type z is the one who is indifferent.

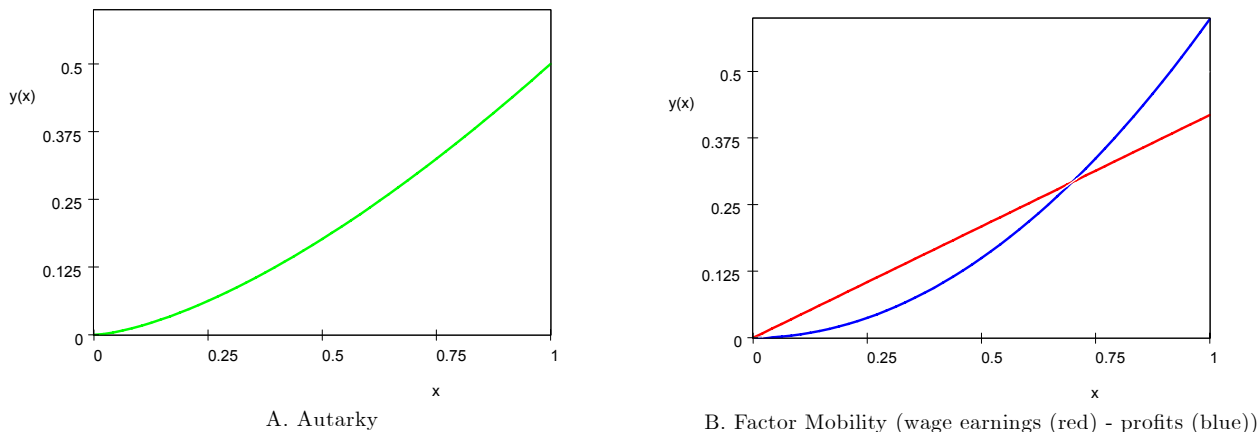


Figure 6. Income Profiles $y(x)$.by skills x .

In the case of full factor mobility, a high skilled agents start firms and their demand for labor drives up world wages. Because the lower types have a competitive advantage as workers, they prefer to be hired rather than be a manager.

3 A result for the general case: The vanishing middle

This section will show that in general, all agent types but one are strictly better off in a free market than under autarky. The one type that remains no better off than before is type z – the type that under the free market is indifferent between management and wage work. To avoid confusion, we shall use the superscript “A” for the value that a variable assumes under autarky, and the superscript “F” for its free-market value.

Proposition 1. *If (i) $F(\cdot)$ is atomless and continuous, and if (ii) $Q'(h)$ decreases continuously from $+\infty$ when $h = 0$ to 0 when $h = \infty$, an equilibrium with Factor Mobility exists at z , satisfying*

$$x_{\min} < z < x_{\max}, \tag{11}$$

and, moreover,

$$\pi(z, w^A[z]) = w^A(z)z = w^F z = \pi^F(z). \tag{12}$$

The proof starts from the premise that at z , the equilibrium allocation must satisfy the equilibrium conditions for the equilibrium with factor mobility. The proof

then shows that the exact same allocation also satisfies the equilibrium conditions for autarky. The proof consists of two lemmas:

Lemma 1. *If (z^F, w^F) is a free-market equilibrium, then w^F is the autarky wage in a country for which $x = z^F$.*

Proof. Since (z^F, w^F) is an equilibrium,

$$\pi(z^F, w^F) = w^F z^F.$$

Now in autarky in country $x = z^F$, the indifference condition is also met. I.e., (12) holds. This leaves the market-clearing condition and the FOC. This requires that there be a measure of workers n such that

$$z^F Q' \left(\frac{nz^F}{1-n} \right) = w^F \iff Q' \left(\frac{nz^F}{1-n} \right) = \frac{w^F}{z^F},$$

where $\frac{nz^F}{1-n}$ is human capital per manager. But by (ii), as the number of workers, n , rises from zero to unity, Q' declines from $+\infty$ to zero, and so a unique $n \in (0, 1)$ exists for which this equation will hold, with $1-n$ being the number of managers, so that the number of bodies adds up to unity. Finally, total human capital supplied, nz^F , equals the amount of it demanded,

$$nz^F = (1-n) \left(\frac{nz^F}{1-n} \right).$$

Thus all the conditions of an autarky equilibrium are met at (z^F, w^F) . QED.

Lemma 2. *z^F satisfies (11).*

Proof. Suppose $z^F = x_{\max}$. Then by (i) since there is no mass point at x_{\max} , demand for h would be zero, and there would be an excess supply of workers. Conversely, if $z^F = x_{\min}$ there would be an excess supply of workers. QED.

Together, Lemmas 1 and 2 imply (12) and the Proposition.

The main result establishes that the marginal type does not gain from factor mobility relative to autarky. This is illustrated for the former example where we now plot the equilibrium and profit schedules on the same graph (panel A in Figure 3): autarky (in green) intersects exactly where wage earnings (red) and profits (blue) intersect. The graph plots income $y(x) = \pi(x) = wx$.

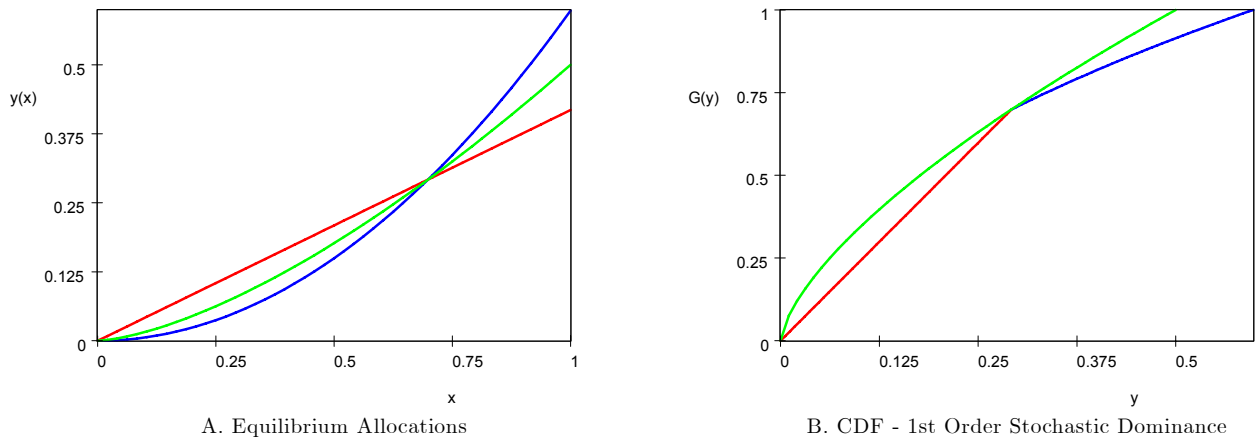


Figure 7. Income Profiles y and CDF of y .

Because under factor mobility, occupational choice effectively implies that the equilibrium allocation is the upper envelope of the wage and profit schedule, the next Proposition immediately follows:

Proposition 2. (*First order stochastic dominance*) *The distribution of earnings under Factor Mobility (weakly) stochastically dominates the distribution under Autarky.*

That this dominance is *weak* follows because z is equally well off under autarky and factor mobility.⁴ The plot for the C.D.F. under both Autarky (green) and Factor Mobility (red and blue) is in panel B of Figure 2.

In light of the competitive equilibrium we are solving, the (weak) stochastic dominance is consistent with the properties of general equilibrium economies. We are comparing an economy with missing markets (high skilled types are not able to hire in low skilled countries) to one with complete markets. Since there is no market power or external effect in the model, from the first theorem of welfare economics, it follows that the outcome of opening markets is Pareto efficient. Therefore it must be Pareto improving relative to the outcome without opening up those markets.

Proposition 3. (*Free vs. Partially-free trade*). *Suppose F is atomless on the interval $[x_{\min}, x_{\max}]$. Then there is a partially-free trade allocation that is not weakly Pareto dominated by free trade.*

⁴That country z is no better or worse off under free trade is a result that has a counterpart in the standard two-skill model with a continuum of countries but with no occupational choice. Let factor endowments differ. There always would be one country in which the skill premium under autarky is the same as the world skill premium under free trade. That country would then be no better off under free trade than under autarky.

Proof. Since z^F is on the interior of the support of F , consider two free-trade zones, $[x_{\min}, z^F]$ and $(z^F, x_{\max}]$ with C.D.F.s $\frac{F(x)}{F(z^F)}$ and $\frac{F(x)-F(z^F)}{1-F(z^F)}$ respectively. Let us focus on the first free-trade zone, $[x_{\min}, z^F]$, which we call zone 1. Atomlessness of F and market clearing imply that the indifferent agent in zone 1, call him z_1 , satisfies $z_1 < z^F$. Now refer to income of agent x in zone 1 by $y^1(x)$. But then the reasoning leading up to (12) implies that $y^1(z_1) = w^A(z_1)z_1$, and $y^1(x) = w^A(x)x$ for all $x \in (z_1, z^F]$. In particular, $y^1(z^F) > w^A(z^F)z^F = w^F z^F$, the second equality following from (12). This is illustrated in Figure ???. But then it follows that there is an entire interval (x^*, z^F) in which agents in zone 1 are strictly better off than they would be under free trade. QED.

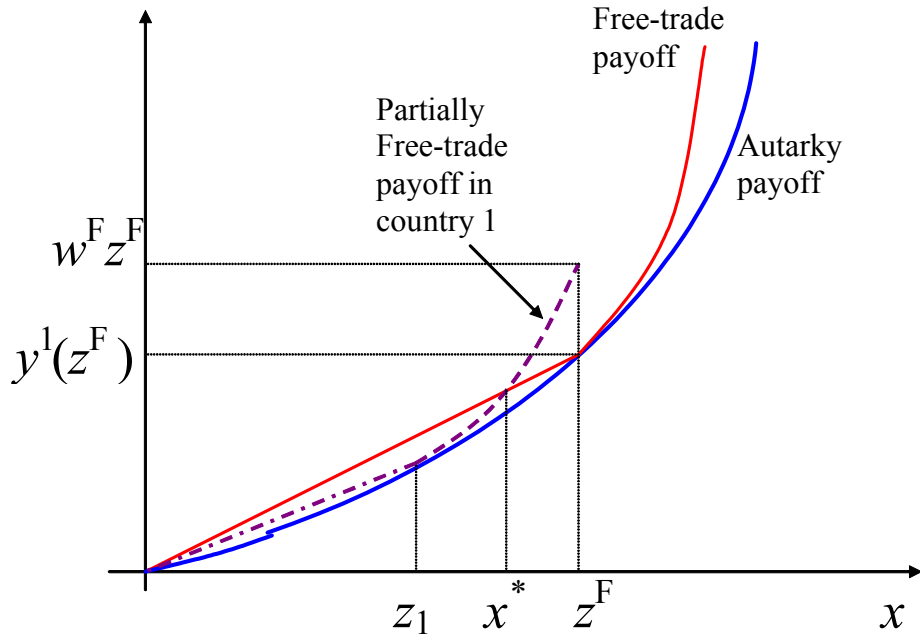


Figure 8. Illustration of Proposition 3

Comparing Free vs. Partially free trade, the question arises which trade regimes are in the core. A trade regime is in the core, if the distribution of incomes it induces has the “core property”. We say that a distribution of incomes induced by a trade regime has the core property if there is no coalition $S \subset [x_{\min}, x_{\max}]$ that can improve upon the income in that distribution.

Proposition 4. *Autarky is not in the core.*

Proof. Under Factor Mobility, all agents except the indifferent type z are strictly better off. As a result, the autarky equilibrium is dominated. QED

The Implications for Growth. By Proposition 2, there are gains from factor mobility. However, from Proposition 1 those gains are not distributed equally over

all types. At least one type is no better off. In the next figure, we plot the gains from factor mobility by rank of the distribution⁵. Panel A has the absolute differences and panel B has the growth rates.

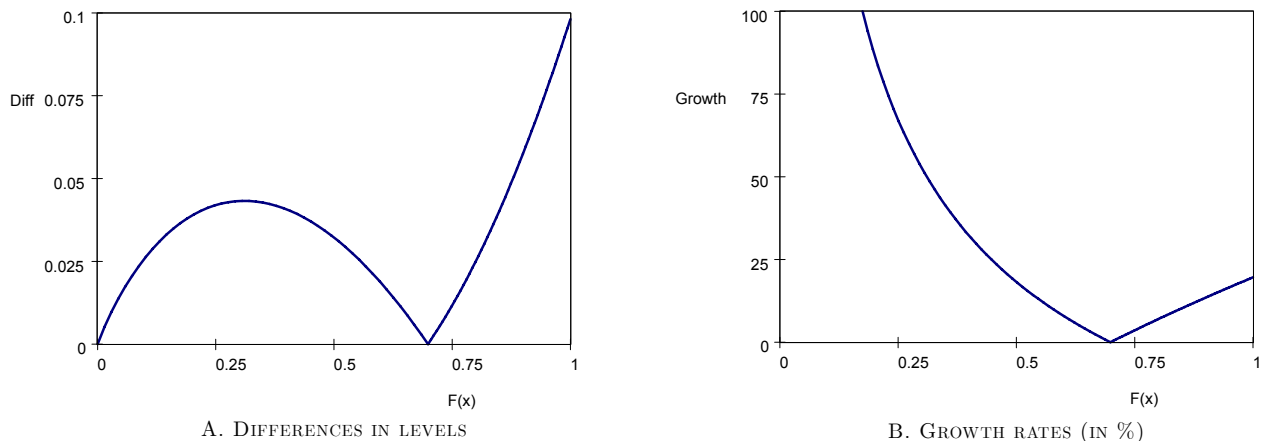


FIGURE 9. GAINS IN EARNINGS Y FROM FACTOR MOBILITY

Panel A shows that in absolute terms, the biggest winners are in the right tail: The high types who own the firms and become managers gain most from factor mobility. The type who is indifferent does not gain, and... workers gain throughout, except for the lowest type. This is because there is no lower bound on ability bounded away from zero. In our example with the uniform distribution, the lower type does not gain anything from factor mobility because output is zero before and after.

Panel B shows that relative to their initial position, the biggest winners are in the *left* tail. Growth rates exhibit a U shape. The extremes of the distribution gain most from factor mobility. To see this, consider the lowest types, who under autarky work with low productivity managers and earn very low wages. After opening up to factor mobility, their labor is demanded from all over the world and their wage is determined in the world labor market. This results in a huge increase in earnings.

The high types do grow and the growth rate is increasing in type, i.e. the top of distribution gains proportionally more the higher up in the distribution. At the bottom of the distribution (below the no-gaining middle income group) in growth rates now there is monotonicity. While worker salaries went up everywhere in the lower part, they went up proportionally more for the lower types. Their output therefore grows more the lower the type. That nonetheless does not translate into any income differences as the lowest types still produce zero output; hence the non-monotonicity in income differences.

While the U-shaped pattern of growth plotted above was only for an example, the conjecture is that this is true for any generic economy described above.

⁵Because there is no rank-reversal in our model, we could as well use ability x and in our uniform distribution example the scale does not even change, $x = F(x)$.

Conjecture 1. (*U-shaped Growth Pattern*) Growth rates are monotonically decreasing in x for $x < z$ and they are monotonically increasing in x for $x > z$.

Proof. First, $x < z$. Then the growth factor is

$$\gamma = \frac{w^F}{w(x)}$$

where $w^F = w(z)$. Then

$$\frac{d\gamma}{dx} = -w^F \frac{w'(x)}{w(x)^2} < 0.$$

provided $w'(x) > 0$. Now $w(x)$ solves

$$wx - xQ(h) + wh = 0$$

$$(1 - n)h - nx = 0$$

$$xQ'(h) - w = 0$$

and from the implicit function theorem:

$$\begin{aligned} \frac{\partial w}{\partial x} &= \frac{\begin{vmatrix} -w + Q(h) & -x_2Q' + w & 0 \\ n & 1 - n & -h - x \\ 0 & xQ'' & 0 \end{vmatrix}}{\begin{vmatrix} x & -xQ' + w & 0 \\ 0 & 1 - n & -h - x \\ -1 & xQ'' & 0 \end{vmatrix}} \\ &= \frac{-(h+x)xQ''(-w+Q(h))}{-(h+x)(x^2Q''+xQ'-w)} \\ &> 0 \end{aligned}$$

The denominator is positive since $Q'' < 0$ (concavity) and $xQ' - w = 0$ (first order condition). The numerator is positive since $Q - w > 0$ (profits are positive; under autarky $\pi = xQ - wx > 0$).

Second, $x > z$. Then

$$\gamma = \frac{\pi(x)}{w(x)x}.$$

Provided $\pi' > w'x + w$, $\frac{d\gamma}{dx} > 0$. TBC.

QED.

Corollary. *Under Factor Mobility, there is a unique z .*

Example: $Q(h) = h^\alpha$ so that total output is $q = xQ(h)$. First, $x < z$. Then the growth factor is

$$\gamma = \frac{w^F x}{w(x)x} = \frac{(1-\alpha)^{1-\alpha} \alpha^\alpha z^\alpha x}{(1-\alpha)^{1-\alpha} \alpha^\alpha x^{1+\alpha}} = \left(\frac{z}{x}\right)^\alpha$$

and it immediately follows that

$$\frac{d\gamma}{dx} = -\alpha \left(\frac{z}{x}\right)^{\alpha-1} \frac{z}{x^2} < 0.$$

Second, $x > z$ so that

$$\gamma = \frac{\pi(x)}{w(x)x} = \frac{\alpha^{1-\alpha}}{(1-\alpha)^{2-\alpha}} z^{\frac{-\alpha}{1-\alpha}} x^{\frac{\alpha^2}{1-\alpha}}$$

and

$$\frac{d\gamma}{dx} = \frac{\alpha^{3-\alpha}}{(1-\alpha)^{3-\alpha}} z^{\frac{-\alpha}{1-\alpha}} x^{\frac{\alpha^2}{1-\alpha}-1} > 0$$

4 The standard two-skill model

A criticism of the trade-based explanations for the rise in the U.S. skill premium has been the failure to find a terms-of-trade effect that would have reduced the wages of the U.S. unskilled through a cheapening (induced by foreign competition) of the goods that they produce (Krugman 2000). Our model circumvents this criticism because the effects work directly through the labor market rather than indirectly through the goods that workers of different skills produce.

But there is also a model simpler than ours that does not involve a managerial task which also can be used to analyze trade-induced changes in the skill premium. We shall refer to this alternative model as the “standard two-skill model.” Having shown that our model produces a quantitatively larger effect of free trade on development, we shall now contrast our model’s implications for the skill premium with those of a standard two-skill model. We shall find that under a free-trade labor market the skill premium would, in both models, rise in the rich countries and fall in the poor countries. In the standard model, the absolute income gap between the skilled and the unskilled also rises in the rich countries and falls in the poor ones. What also sets our model apart from the standard model is its implication that in the poorest countries, the absolute income gap between the skilled and the unskilled should rise, and on this score our model fits the evidence better.

4.1 The skill premium

The standard model of the skill premium is one in which there are two kinds of skill – labor is skilled or unskilled. Let u and s be the numbers of unskilled and skilled workers in a particular country. Let $G(u, s)$ be the world distribution of (u, s) pairs over countries. We continue to assume that there is only one good and that openness refers to openness of labor markets.

Autarky: Under autarky, output is $y = F(u, s)$. The skill premium and the absolute earnings gap or “gap” for short, are

$$R^A \equiv \frac{F_s(u, s)}{F_u(u, s)} \quad \text{and} \quad \gamma^A \equiv F_s(u, s) - F_u(u, s)$$

and world output under autarky is $Y^A = \int F(u, s) dG(u, s)$.

Free trade: Under free trade, world output is

$$Y^F = F(\bar{u}, \bar{s}).$$

where

$$\bar{u} = \int u dG(u, s) \quad \text{and} \quad \bar{s} = \int s dG(u, s)$$

and the skill premia are

$$R^F \equiv \frac{F_s(\bar{u}, \bar{s})}{F_u(\bar{u}, \bar{s})} \quad \text{and} \quad \gamma^F \equiv F_s(\bar{u}, \bar{s}) - F_u(\bar{u}, \bar{s})$$

4.2 Gains from trade in the standard two-skill model

We assume F is CRS in which case $F_s(u, s) = \phi_s\left(\frac{s}{u}\right)$ and $F_u(u, s) = \phi_u\left(\frac{s}{u}\right)$, i.e., marginal products are functions only of the ratios of the two inputs. Let $\tau = \frac{s}{u}$ and $\bar{\tau} = \frac{\bar{s}}{\bar{u}}$. It is seen that for a country with $\tau = \bar{\tau}$ there are no gains from trade: Upon opening up, total income change is

$$\begin{aligned} \Delta &\equiv [F_u(\bar{u}, \bar{s}) - F_u(u, s)]u + [F_s(\bar{u}, \bar{s}) - F_s(u, s)]s \\ &= u(\phi_u(\bar{\tau}) - \phi_u(\tau)) + [\phi_s(\bar{\tau}) - \phi_s(\tau)]\tau \end{aligned}$$

and this expression is zero when $\tau = \bar{\tau}$. Thus the country that has the same skill composition as the world does not gain from opening its markets.

It remains to be shown that income rises for every country for which $\tau \neq \bar{\tau}$.

Proposition 1

$$\tau \neq \bar{\tau} \implies \Delta > 0$$

Proof. If u is on the horizontal axis and s on the vertical, the slope of the isoquant is

$$-\frac{F_u}{F_s} = -\frac{\phi_u(\tau)}{\phi_s(\tau)}$$

Because F concave, the set $\{(u, s) \mid F(u, s) \geq F_0\}$ is convex, which means that the isoquant’s slope becomes less negative as we raise τ :

$$0 < \frac{d}{d\tau} \left(-\frac{\phi_u(\tau)}{\phi_s(\tau)} \right)$$

TBC. ■

4.3 Cobb-Douglas + Uniform example

We shall now show by example that in the standard model, the effect of free trade is to raise both R and γ in the rich countries, and to lower both R and γ in the poor countries. We assume three things:

1. The production function is $F(u, s) = u^{1-\beta} s^\beta$,
2. In all countries, u is normalized to unity
3. s is uniformly distributed on $[0, 1]$ over countries. The higher is s , the richer is the country, and we can use s as the country index

Autarky: Autarky output in country s is $F(1, s) = s^\beta$ and the world's output under autarky is

$$Y^A = \int_0^1 s^\beta ds = \frac{1}{1 + \beta}.$$

The marginal products in country s are $F_u(1, s) = (1 - \beta) s^\beta$ and $F_s(1, s) = \beta s^{\beta-1}$ so that the premium and the gap are

$$R^A \equiv \frac{\beta}{(1 - \beta) s} \quad \text{and} \quad \gamma^A \equiv \beta s^{\beta-1} - (1 - \beta) s^\beta,$$

respectively.

Free trade: Under free trade, world output is

$$Y^F = \left(\int_0^1 s ds \right)^\beta = \left(\frac{1}{2} \right)^\beta$$

and the premium and the gap are

$$R^F \equiv \frac{\beta}{(1 - \beta) \bar{s}} = \frac{2\beta}{1 - \beta}, \quad \text{and} \quad \gamma^F \equiv \beta (\bar{s})^{\beta-1} - (1 - \beta) (\bar{s})^\beta = \beta 2^{1-\beta} - (1 - \beta) 2^{-\beta},$$

respectively.

Comparison of Autarky and free trade in the Cobb-Douglas-Uniform example: First the gains to trade. When $u = 1$, $\tau = s$ and $\bar{\tau} = \bar{s} = \frac{1}{2}$, and so the gain to country s is

$$\Delta \equiv (1 - \beta) 2^{-\beta} - (1 - \beta) s^\beta + (\beta 2^{1-\beta} - \beta s^{\beta-1}) s.$$

Figure 10 plot Δ as a function of s for $\beta \in \left\{ \frac{1}{10}, \frac{1}{2}, \frac{99}{100} \right\}$.

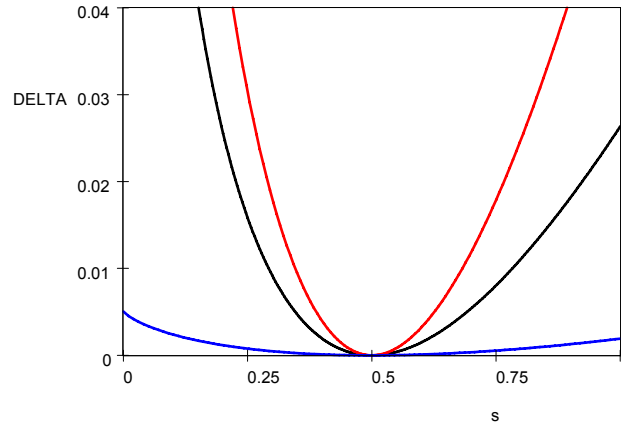


FIGURE 10. GAINS TO TRADE, Δ , FOR $\beta = \frac{1}{10}$ (BLACK), $\beta = \frac{1}{2}$ (RED) AND $\beta = \frac{99}{100}$ (BLUE)

The gains are not large. The total output gain for the world in Figure 11 shows autarky output, Y^A in red and free-trade output, Y^F , in blue. There are gains to trade as long as both factors have strictly positive marginal products.

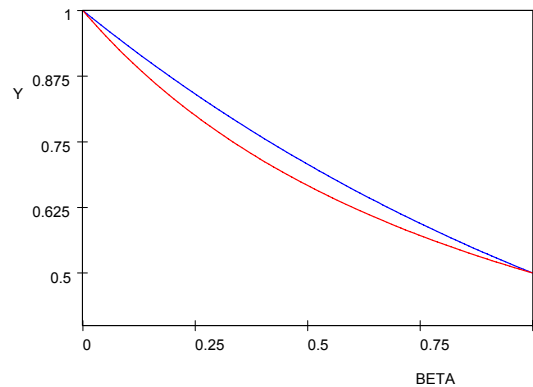
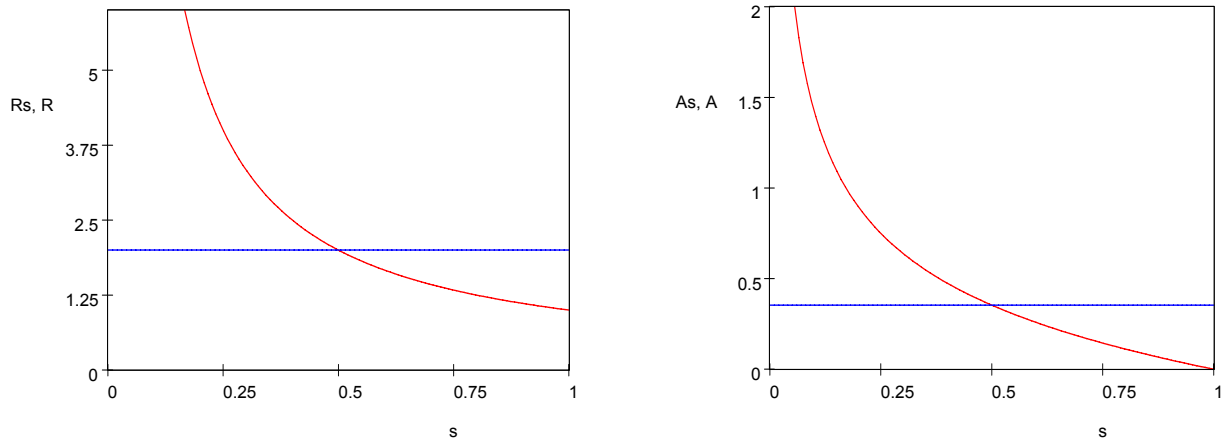


FIGURE 11. THE STANDARD TWO-SKILL MODEL: WORLD OUTPUT AUTARKY (RED) - FREE TRADE (BLUE)

Figures 12A and B plot R and γ in the two regimes. We see that under autarky, R^A and γ^A are both downward sloping (the red lines in Figures 12A and B), but that under free trade they are both flat. Therefore free trade raises both the premium and the gap in rich countries and lowers them both in poor countries.



A. SKILL PREMIUM: R^A (RED) AND R^F (BLUE).

B. GAP: γ^A (RED) AND γ^F (BLUE).

FIGURE 12. SKILL PREMIUM IN STANDARD MODEL

4.4 Sorting

Skill premia have risen in rich and poor countries alike. This presents a problem for the standard model which, according to Figures 12A and B, implies that skill premia should fall in the poor world as free trade raises the earnings of the low skilled in the low-skill-abundant South.

In our model, by contrast, Free Trade raises the absolute skill premium in both tails. The absolute skill premium is $y'(x)$, and the skill premium is $y'(x)/y(x)$.

Example:

Autarky: $w(x) = \frac{1}{2}x^{\frac{3}{2}}$ therefore $w'(x) = \frac{3}{4}x^{\frac{1}{2}}$ and $s^A(x) = \frac{3}{2}\frac{1}{x}$

Free trade

$$y'(x) = \begin{cases} 0.41796 & \text{if } p < 0.69 \\ 1.1963x & \text{if } p \geq 0.69 \end{cases}$$

$$\frac{y'(x)}{y(x)} = \begin{cases} \frac{1}{x} & \text{if } p < 0.69 \\ \frac{3}{2} & \text{if } p \geq 0.69 \end{cases}$$

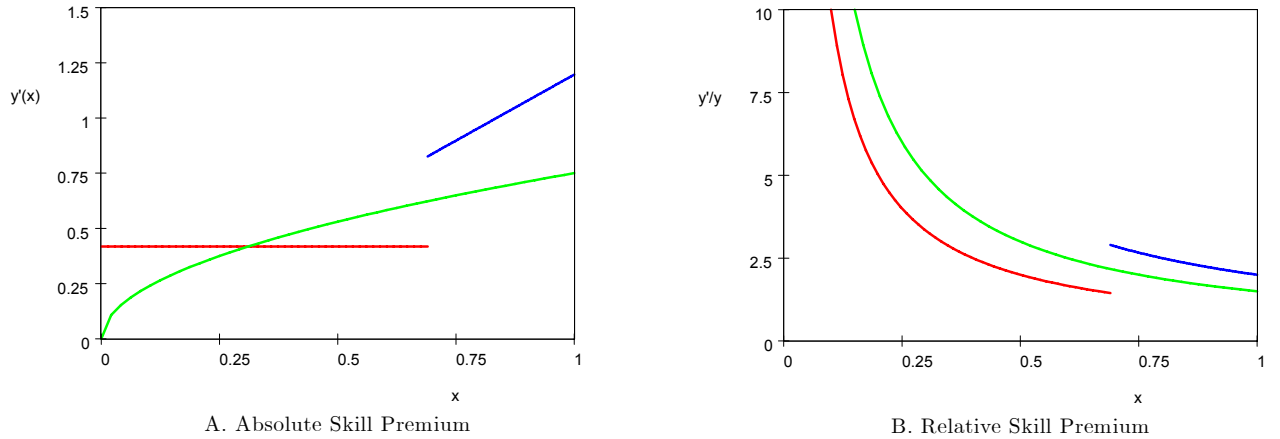


Figure 13. Skill Premium

Notes on the comparison of the two models.—Although both models used the uniform distribution for x and s , respectively, the example has homogeneity of the endowment of u , which we needed for simplicity. Beyond this, and important difference remains between the two sets of assumptions: The skill endowments under “autarky” are different for the two models. In our model “autarky” means homogeneity of skill. Such an assumption does not make sense in the standard two-skill model, however, hence we have gone with the simplest alternative.

4.5 Change in Measures of Inequality

One way to measure the degree of inequality is to measure the ratio of income say the 90-th and the 10-th percentile, or in general, that of a the $(1 - p)$ 100-th and p -th percentile. We plot $\frac{y(1-p)}{y(p)}$. For our example to which inequality changes as the

In the case of autarky:

$$\frac{y(1-p)}{y(p)} = \frac{(1-p)^{\frac{3}{2}}}{p^{\frac{3}{2}}}$$

and in the case of factor mobility:

$$\frac{y(1-p)}{y(p)} = \begin{cases} \frac{0.59814(1-p)^2}{0.41796p} & \text{if } p < 0.31 \\ \frac{1-p}{p} & \text{if } p \geq 0.31 \end{cases}$$

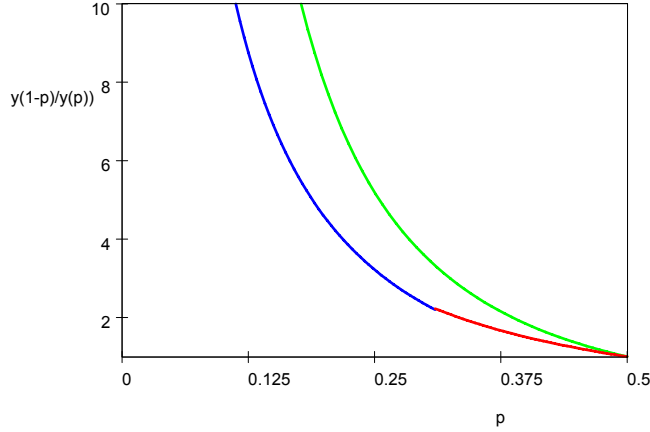


Figure 14. The skill premium

There is a non-differentiability in the profile under factor mobility at $p = 1 - z = 0.31$.

World Income Inequality. We know there is first order stochastic dominance of the income distribution under factor mobility $G^F(y)$ relative to autarky $G^A(y)$. Can we say something about inequality, i.e., a mean-adjusted concept of stochastic dominance. The Gini coefficient for example is a summary statistic calculated based on mean-adjusted stochastic dominance. Bourguignon and Morrisson (2002) calculate the Gini coefficient and several other measures of inequality for the world income distribution over nearly two centuries. They find that world income inequality peaked in the 1950s after a century of continuous divergence and stabilized in the second half of the nineteenth century. They argue that any remaining increases in inequality after the 1950s are minor compared with the dramatic evolution before. There continues to be a debate around the fact whether inequality in the last 50 years has gone up or not. What does our theory predict in that respect?

Numerous mean-adjusted measures of stochastic dominance, including the Gini coefficient, are derived from the Lorenz curve. The Lorenz curve calculates for each percentile p of the CDF of income $F(y)$ the average income of the fraction of individuals with an income below y , relative to the average income over the entire distribution. The Lorenz curve for any distribution $F(y) = p$ is therefore defined by

$$L(p) = \frac{\int_0^p y(\tilde{p})d\tilde{p}}{\int_0^1 y(\tilde{p})d\tilde{p}}$$

for $y(F)$ the inverse of F . In the case of autarky $F^A(y) = (2y)^{\frac{2}{3}}$ and therefore $y^A(p) = \frac{1}{2}p^{\frac{3}{2}}$ and $L^A(p) = p^{\frac{5}{2}}$. For the case of factor mobility, we integrate piecewise since G is discontinuous at $y(z)$. That is from $G^F(y)$ we obtain $y^F(p)$ as the inverse of $G^F(y)$, and as a result

$$L^F(p) = \begin{cases} 0.89540p^2 & \text{if } p \leq 0.69876 \\ 0.85427p^3 + 0.14573 & \text{if } p > 0.69876 \end{cases} .$$

Below there is a plot of both Lorenz curves $L^A(p)$ (green) and $L^F(p)$ (red-blue).

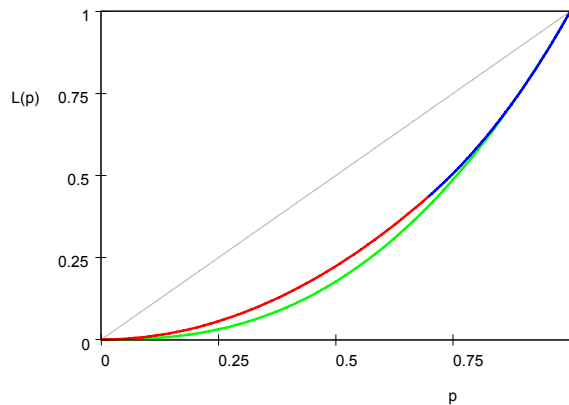


Figure 15. Lorenz curves: Autarky (green) – Factor Mobility (red-blue).

In this example, the Factor Mobility distribution seems to dominate the Autarky distribution for the most part. The closer the Lorenz curve is to the 45-degree line, the more equal is the economy as long as we are comparing non-intersecting Lorenz curves. The more equal economy then also has a smaller Gini coefficient. Most likely, the Gini coefficient in this example will be larger under Autarky, suggesting there is a reduction in inequality due to factor mobility. However, Bourguignon and Morrisson (2002) argue that this is counterfactual. They find (weakly) increasing Gini coefficients from the seventies onwards. While this plot is merely an example, and therefore does not shed any light on the validity of the theory with respect to relative inequality, the example is nonetheless illustrative. Closer inspection reveals that for this example there is no Lorenz dominance of the Factor Mobility distribution. There is an intersection of both curves in the upper tail of the distribution at $p = 0.901$. With a strong increase of income at the top of the distribution, the upper decile is more unequal in the Lorenz sense. While more work is needed here, this may be indicative that Gini coefficients under factor mobility need not be smaller than under autarky.

5 A market for management

Even though returns are not constant, we can decentralize the equilibrium using markets for both labor and management. We start with autarky which is much simpler.

Autarky.—Under autarky, it follows immediately that zero-profit firms would replicate the free-market equilibrium. A firm would hire N workers, and assign a fraction n of them to be managers, and a fraction $1 - n$ to be workers. Let p be the

wage per worker. The firm would solve the problem

$$\max_{n,N} \left\{ \left[(1-n) x Q \left(\frac{nx}{1-n} \right) - p \right] N \right\}$$

Proposition 2 *Under autarky, the introduction of a managerial market and zero-profit firms leaves the equilibrium unchanged, and*

$$p = w(x) x.$$

Proof. The firm must make zero profits, i.e., $Q = \frac{p}{(1-n)x}$, or

$$Q = \frac{w}{1-n}, \tag{13}$$

so N drops out of the problem. Moreover, since (5) can be written as $Q - w \frac{n}{1-n} = w$, it is equivalent to (13). It remains to show that (5) holds too. Upon dividing by x , the firm solves

$$\max_n \left\{ \left[(1-n) Q \left(\frac{nx}{1-n} \right) - w \right] \right\}.$$

The FOC is

$$\begin{aligned} 0 &= -Q + \left(1 - \frac{n}{1-n} \right) x Q' \\ &= Q - \frac{x}{1-n} Q' \left(\frac{1-n}{n} x \right) \end{aligned}$$

Substituting from (13) and multiplying by n , we get $xQ'(h) = w$, i.e., (4). ■

Free trade.—In this decentralization, each person has a price, $p(x)$ that depends on his or her skill. Taking the prices as given, firms hire people and assign them to be either managers or workers. Markets are complete in the sense that for each x there is a price. For each x , a representative firm hires $n(x) = f(x)$ people of type x and uses $n_m(x)$ of them as managers, and the rest as workers.⁶ It allocates $h(x)$ efficiency units to managers of type x . It chooses these things to solve the following problem

$$V = \max_{n(\cdot), n_m(\cdot), h(\cdot)} \left\{ \int x Q(h[x]) n_m(x) dx - \int p(x) n(x) dx \right\}$$

subject to the (single) constraint that the number of efficiency units employed in wage work not exceed the number available among the non-managerial workers of the firm:

$$\int h(x) n_m(x) dx \leq \int x [n(x) - n_m(x)] dx.$$

⁶We use the more intuitive $n(x)$

and to the constraint (one for each x) that each type is divided between management and

$$0 \leq n_m(x) \leq n(x).$$

Finally, free entry of firms requires that profits be zero

$$V = 0.$$

The Lagrangean (we ignore the constraint $n(x) \geq 0$) is

$$\int xQ(h_x)n_{m,x}dx - \int p_x n_x dx - \lambda \left(\int h_x n_{m,x} dx - \int x(n_x - n_{m,x}) dx \right) + \theta(x)(n_x - n_{m,x}) + \mu(x)n_{m,x}.$$

The FOCs are

$$\begin{aligned} h : \quad & xQ'(h_x) - \lambda = 0 \\ n_{m,x} : \quad & xQ(h_x) - \lambda h_x + \lambda x - \theta(x) + \mu(x) = 0 \\ n_x : \quad & -p_x + \lambda x + \theta(x) = 0 \end{aligned}$$

Proposition 3 *Let*

$$p(x) = \begin{cases} w^F x & \text{for } x < z^F \\ \pi(x, w^F) & \text{for } x \geq z^F \end{cases} \quad (14)$$

Proof. The proof will show that when (14) holds, occupational selection is the same, and allocation of efficiency units to managers is the same. Substitute from (14) into the FOCs along with $\lambda = w^F$ and show that they hold. Doing this the FOCs read

$$\begin{aligned} h : \quad & xQ'(h_x) - w^F = 0 \\ n_{m,x} : \quad & xQ(h_x) - w^F h_x + w^F x - \theta(x) + \mu(x) = 0 \\ n_x : \quad & w^F x + \theta(x) - \begin{cases} w^F x & \text{for } x < z^F \\ \pi(x, w^F) & \text{for } x \geq z^F \end{cases} = 0. \end{aligned}$$

Using the third to solve for

$$\theta(x) = \begin{cases} 0 & \text{for } x < z^F \\ \pi(x, w^F) - w^F x & \text{for } x \geq z^F \end{cases}.$$

Thus, the constraint $n_m(x) \leq n(x)$ is slack exactly for the same set of workers that choose the wage-work option in equilibrium. Now substitute for $\theta(x)$ into the second FOC to get:

$$xQ(h[x]) - w^F h(x) + \begin{cases} w^F x & \text{for } x < z^F \\ \pi(x, w^F) & \text{for } x \geq z^F \end{cases} + \mu(x) = 0.$$

Now from the definition of $\pi(x, w^F)$ we have the following solution for $\mu(x)$:

$$\mu(x) = \begin{cases} w^F(h(x) - x) - xQ(h[x]) & \text{for } x < z^F \\ 0 & \text{for } x \geq z^F \end{cases}.$$

Thus, the constraint $n_m(x) \geq 0$ is slack exactly for the same set of workers that choose the self-employment option in equilibrium. ■

6 Within-country Inequality

The notion of a country so far is very stylized. In particular, each country is populated with agents of one skill type only and as a result, both under autarky and under free trade we have a representative agent economy. We extend the model to allow for within-country inequality. First, we consider multiple skill characteristics, then we look at the class of autarkic economies where skills are lognormally distributed.

6.1 Multiple Skill Characteristics

Following Jovanovic (1993) we endow agents with a pair $\{x, y\}$, where x represents the skill level as a manager and y is the skill level of the worker, distributed according to $F(x, y)$. Assume x and y are independently distributed. The conditional distribution of x is denoted as before by $F(x)$ and the conditional distribution of y is $G(y)$. Note that before in our model, x and y were perfectly correlated and $F(x) = G(y)$. Now w is a wage that prevails in the economy for each efficiency unit y and $h(\cdot)$ is the total amount of efficiency units hired by the firm. The FOC is as before

$$xQ'(h) = w,$$

the solution of which solves the manager's demand function $h = g(y, w)$. Then the set of managers is the set $E(w) = \{x, y \in \mathbb{R}_+^2 \mid \pi(x, w) > wy\}$. The market-clearing condition then reads

$$\int_{E(w)} g(y, w) dF(x, y) = \int_{\mathbb{R}_+^2 - E(w)} x dF(x, y).$$

Now the skill type \bar{y} that is indifferent between becoming a manager and a worker:

$$\pi(x, w) = w\bar{y}.$$

Under autarky, for each country the solution for \bar{y} solves for all x

$$\int_0^{\bar{y}} g(\bar{y}, w) dG(y) = \int_{\bar{y}}^{\infty} y dG(y).$$

Since output is multiplicatively separable in x and $Q(h)$, the equilibrium allocation is independent of x and therefore \bar{y} is the same for all countries with types x . This is shown in Jovanovic (1993) in the context of technological change. He considers the impact of Hicks-neutral technical change, which is a Solow residual-type shift term s on $Q(h)$, $sQ(h)$, and proves that the allocation remains unaltered.

For the case of free trade, and with x and y independently distributed, market clearing solves

$$\int_0^{\infty} \int_0^{\bar{y}(x)} g(\bar{y}, w) dG(y) dF(x) = \int_0^{\infty} \int_{\bar{y}(x)}^{\infty} y dG(y) dF(x).$$

Since the wage is determined world wide, the following result follows:

Proposition 4 *Under free trade, $\bar{y}(x)$ is strictly increasing in x .*

Proof. From indifference, we know that $xQ(g) - wg = w\bar{y}$ and therefore

$$\frac{d\bar{y}}{dx} = \frac{1}{w} [Q(g) + xQ'(g)g' - wg'] = \frac{Q(g)}{w} > 0$$

where the second equality follows from FOC $xQ'(g) = w$. ■

This is illustrated in the unit-elasticity example. Let $Q(h) = h^\alpha$. The FOC is $w = \alpha x h^{\alpha-1}$. Using this to substitute for w in the indifference condition for worker type \bar{y} : $xh^\alpha - wh = w\bar{y}$. We get that for worker \bar{y} , factor demand is $g(\bar{y}, w) = \frac{\alpha}{1-\alpha}\bar{y}$ and therefore,

$$w = (1 - \alpha)^{1-\alpha} \alpha^\alpha x \bar{y}^{\alpha-1}$$

which gives us w in terms of x and \bar{y} . Observe that because all types have the same x , g is a constant. The second restriction on w and z is the market-clearing condition which with G uniform gives:

$$\int_0^{\bar{y}} \frac{\alpha}{1-\alpha} \bar{y} dy = \int_{\bar{y}}^1 y dy$$

and \bar{y} is independent of x .

$$\bar{y} = \sqrt{\frac{1-\alpha}{3-\alpha}}.$$

Under free trade, for the example we have again that $g(\bar{y}, w) = \frac{\alpha}{1-\alpha}\bar{y}$ and therefore that $\bar{y} = \frac{1-\alpha}{\alpha} \left(\frac{\alpha x}{w}\right)^{\frac{1}{1-\alpha}}$. Now market clearing with the uniform distributions solves

$$\int_0^1 \int_0^{\bar{y}(x)} g(\bar{y}, w) dy dx = \int_0^1 \int_{\bar{y}(x)}^1 y dy dx$$

which gives $w = \left[(1 + \alpha) (1 - \alpha)^2 \alpha^{\frac{2\alpha}{1-\alpha}} \frac{1}{3-\alpha} \right]^{\frac{1-\alpha}{2}}$. This allows us to calculate $\bar{y}(x)$

$$\bar{y} = \frac{1-\alpha}{\alpha} (\alpha x)^{\frac{1}{1-\alpha}} \left[\frac{1}{(1 + \alpha) (1 - \alpha)^2 \alpha^{\frac{2\alpha}{1-\alpha}} \frac{1}{3-\alpha}} \right]^{\frac{1}{2}}$$

For example, for $\alpha = \frac{1}{2}$ we get $\bar{y}(x) = 1.291x^2$, increasing in x , whereas under autarky we get $\bar{y}(x) = 0.447$ and constant.

6.2 An Example calibrated to Lognormal skill distribution

For the unit-elasticity case, we can solve explicitly including for distributions of skill $F(x)$ different from the uniform. Here we calibrate the example to the lognormal distribution of types. The market clearing condition is

$$\frac{\alpha}{1-\alpha} z^{\frac{-\alpha}{1-\alpha}} \int_z^\infty x^{\frac{1}{1-\alpha}} dF(x) = \int_0^z x dF(x)$$

or

$$\frac{\alpha}{1-\alpha} z^{\frac{-\alpha}{1-\alpha}} \underline{m} \left(\frac{1}{1-\alpha}, z \right) = \overline{m}(1, z)$$

where $\underline{m} \left(\frac{1}{1-\alpha}, z \right)$ is the $\frac{1}{1-\alpha}$ -th moment of the lower truncated distribution and $\overline{m}(1, z)$ is the first moment of the upper truncated distribution.

Then using the following parameter values: $\alpha = 0.75 (N = 4), \mu = 0, \sigma = 0.125$ the type distribution is quite close to symmetric (though slightly skewed). Solving for the market clearing condition we get $z = 1.1237$. The equilibrium wage then is $w = 0.62197$ and profits are $\pi(x) = 0.43835x^4$, plotted below. Incomes in the case of autarky are $wx = 0.56988x^{1.75}$.

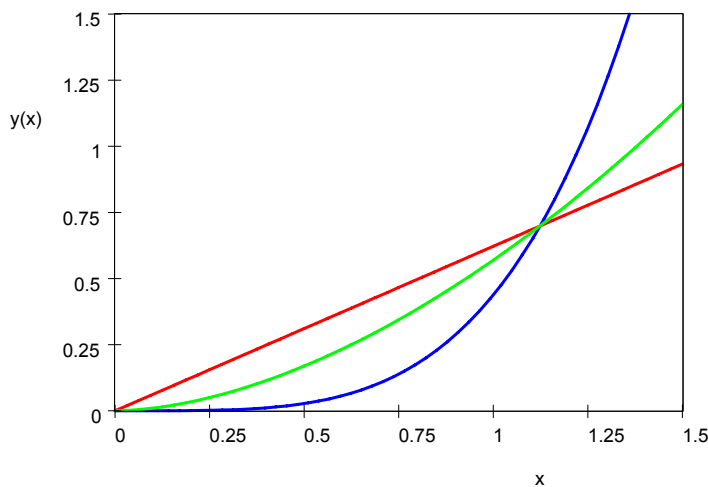


Figure 16. Income Profiles $y(x)$: Factor Mobility (red/blue) and Autarky (green)

At z , the income is $y(z) = 0.69891$. The wage distribution can be decomposed in two parts:

$$g(y) = \begin{cases} \frac{1}{0.62197} f(y) & \text{if } y \leq 0.69891 \\ \frac{1}{(0.43835)^{\frac{1}{4}}} f(y^{\frac{1}{4}}) & \text{if } y > 0.69891 \end{cases}$$

where $f(y)$ is the lognormal (μ, σ) . A plot of the pdf $g(y)$ is below, with the dashed

curve representing the original lognormal skill distribution $f(x)$.

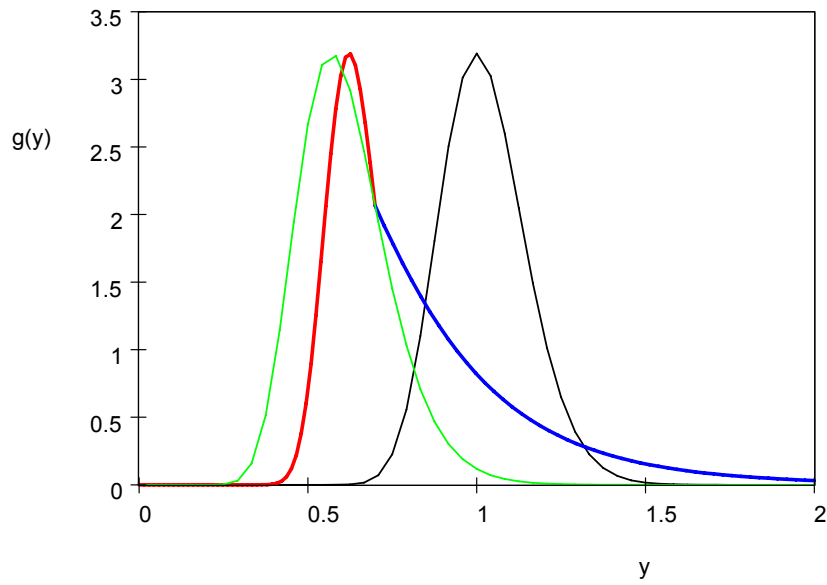
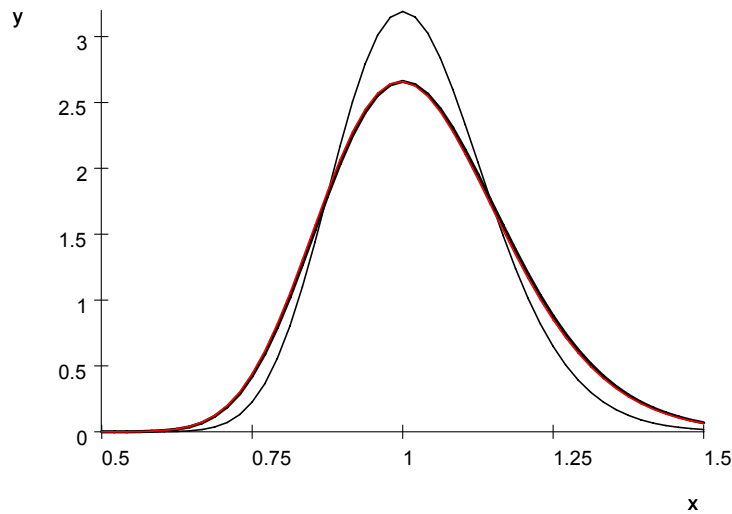


Figure 16. Income Distribution $g(y)$: Factor Mobility (red/blue) and Autarky (green)

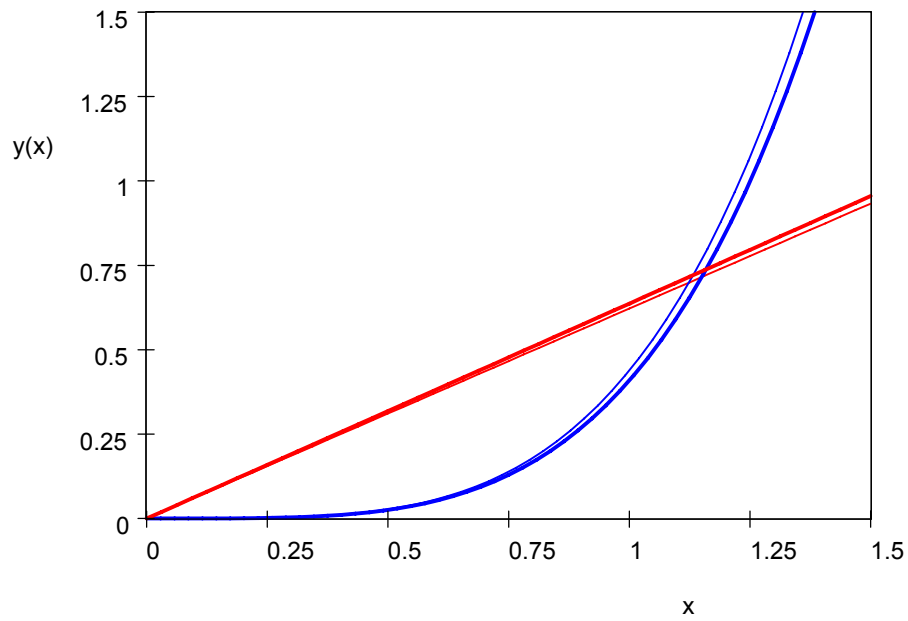
Modelling Factor Mobility: The effect of increasing variance

Now consider the solution to same problem but with $\sigma = 0.15$:

$$\text{NormalDen}(\ln(u); 0, .15)$$



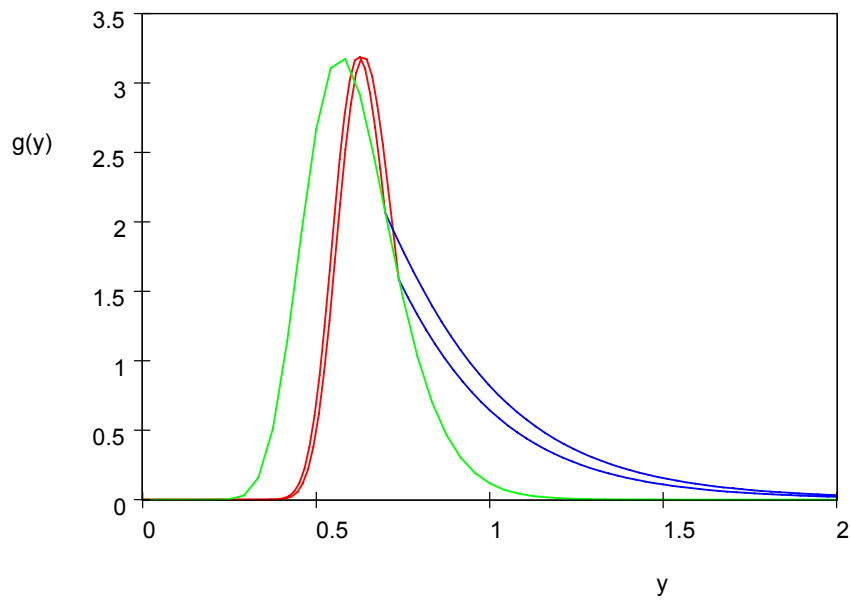
z goes up from 1.1237 to 1.1592. Then $w = 0.63665$ (up from 0.62197) and profits are down $\pi(x) == 0.40872x^4$



Factor Mobility (wage earnings (red) - profits (blue))
 Figure. Income Profiles $y(x)$.by skills x .

Write income as

$$y = \begin{cases} 0.63665x & \text{if } x \leq 1.1592 \\ 0.40872x^4 & \text{if } x > 1.1592 \end{cases}$$



7 More discussion of related papers

McGrattan and Prescott (2007, henceforth “MP”).—When there is globalization our model allows management to reach across borders. MP have a related notion of the gains to trade: The worldwide application of ideas. Ideas they call technology capital. Paradoxically, they have no span of control problem internationally but they have it locally (see their eq. (2)), and an idea can be used everywhere, it can be replicated, using some local inputs. A firm that owns an idea that it applies in another country draws rents on that idea, and pays local factors just as a manager does in our model when he hires workers abroad. MP could have called their x “Management Capital” – the only other possible identifier of x is the way it is produced, and it is in their model created by using goods alone.

MP do not develop the distributional implications of their model, but aside from this, MP put the limits to firm size at the plant level but not at all at the firm level, and this affects their quantitative conclusions. Let N be the number of locations in which a firm operates, or the number of its “plants” (a concept we have not introduced in the model). We may write our production function as

$$Y = xQ \left(\sum_{i=1}^N h_i \right). \quad (15)$$

Under autarky, all N plants would have to be in the country where the manager resides, whereas in a global labor market these plants could be anywhere on the planet. In our model, however, N is not determined, but $\sum_{i=1}^N h_i$ is. This is because in our model returns to scale diminish at the firm level, not at the plant level.

The MP model assumes the opposite. In contrast to (15), the MP model says that

$$Y = x \sum_{i=1}^N Q(h_i), \quad (16)$$

where $Q'' < 0$. Therefore returns to scale diminish at the plant level, not at the firm level. The firm can operate as many plants as it wishes to, anywhere on the planet, and this lies behind MP’s large estimates of the gains to openness.⁷

Production-function estimates favor (15) over (16). That is, constant returns at the level of the plant and diminishing returns at the level of the firm fit the data better. People have estimated plant-level production functions in several industries, and usually they find roughly constant returns, at least over the range of scales observed in the data. Clearly, capacity constraints will eventually bind in the short run for any plant, and there has to be some fixed cost that makes it inefficient to have

⁷When there are no span of control limits at the global level, by opening up a country attracts ideas from all over, and with these ideas comes the inflow physical capital. This inflow of capital and ideas raises the income of the liberalizing country.

very small plants. But the average cost curve appears to have a very flat and very wide bottom: Olley and Pakes (1996), Levinsohn and Petrin (2003), and Syverson (2004) find roughly constant returns for particular industries. Nguyen and Lee (2002) find constant returns using data from several industries. Less evidence is available about firm-level technologies. If we depart from perfect competition and assume that the global demand for the firm’s product is downward sloping, (15) would be the appropriate formulation, and not (16).

Evidence on firm’s locations does not favor MP’s model in its current form. MP implies that if $x < x'$, any location that firm x is in, firm x' would also be in. Therefore a location to which the lowest- x firm exports should be the most popular, and every firm with a higher x should also export there. The most popular destination country for French exports is Belgium, and MP’s model implies that 100 percent of French exporters should be exporting to Belgium. But Eaton, Kortum and Kramarz (2007) find that only 40 percent of French exporters exported to Belgium.

Gabaix and Landier (2006, henceforth “GL”).—GL’s paper explains managerial earnings as a competitive outcome in a market in which firms of different efficiency match with managers of different ability. Given the distribution of efficiencies and abilities, the rents are divided between managers and shareholders competitively. GL explain the rise in managerial earnings by an improvement in the efficiency and value of the firms, in particular the largest firms. Our results complement those of GL; we explain why the distribution of value has changed, namely the globalization of the labor market. We assume that the manager collects all the profits, but our model would similar implications to theirs if we assume that the manager collects a fixed fraction of $\pi(x, w)$, say θ , leaving the rest to the shareholders. Then z would be determined by the equation $\theta\pi(z, w) = wz$.

8 Conclusion

We have argued that the integration of labor markets reallocates existing workers among existing managers, and that it prompts people to switch occupations so that the set of managers and workers changes. As in the standard model, a worldwide labor market raises output by more in the rich and the poor countries than in the middle-income countries. But we have also found that occupational sorting adds substantially to the output and welfare gains to free trade in labor services. We also found that when sorting is allowed, the model fits better the evolution of skill premia in the poor countries.

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