

Time-Varying Risk Aversion and Asset Prices

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Abstract

This paper uses a variant of the consumption-based representative agent model in Campbell and Cochrane (1999) to study how investors' time-varying risk aversion affects asset prices. First, we show that a countercyclical variation of risk aversion drives a procyclical conditional risk premium. Second, we show that with a small value for the volatility of the log surplus consumption ratio, a large value of risk aversion may not determine whether the equity premium and the risk-free rate puzzles can be resolved or not. Third, we show that countercyclical risk aversion may not help explain the predictability of long-horizon stock returns, the univariate mean-reversion of stock prices and the "leverage effect" in return volatility.

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1 Introduction

Our understanding of the impact of investors' risk aversion on asset prices is largely associated with using standard representative agent models to resolve asset pricing puzzles. For example, Mehra and Prescott (1985) find that in order to explain the large average risk premium of historical stock returns in a standard representative agent model with a power utility function, the representative agent must be assumed to be very risk-averse. Thus high risk aversion implies a large risk premium.

More recently, Campbell and Cochrane (1999; hereafter CC) present another example to study how investors' time-varying risk aversion affects asset prices in a consumption-based model with a representative agent whose utility function exhibits countercyclical variation in risk aversion with a large (average) value. Since the CC model is able to explain many of the empirically observed properties of aggregate stock returns, several interesting research questions arise. First, does the countercyclical variation in risk aversion in CC drive the countercyclical (conditional) risk premium? Second, does the large average value of risk aversion help resolve the equity premium and the risk-free rate puzzles? Third, does countercyclical risk aversion help explain the predictability of long-horizon stock returns, the univariate mean-reversion of stock prices and the "leverage effect" in return volatility?

In this paper, we use a variant of the CC model to study the research questions raised above. This variant has a constant value for the market price of risk and is hereafter referred to as the variant of the CC model. In a consumption-based representative agent model, such as the one in CC, there are two sources that can lead to a countercyclical risk premium. The first one is a countercyclical variation of risk aversion. When investors become more risk-averse about uncertain future dividends in recession, they require a larger risk premium. The second one is a time-varying countercyclical market price of risk, which can also yield a countercyclical (conditional) risk premium, if the stock return in recession is at least as volatile as at boom.

In CC, the two sources for a countercyclical risk premium are both active at the same time. To determine whether or not the first source induces the countercyclical variation of

the risk premium, we consider a constant value for the market price of risk and the second source is thus inactive. Then, with only the first source active, we show that countercyclical variation in risk aversion induces a procyclical risk premium. This is a surprising result, since this result is against the intuition that countercyclical risk aversion results in a countercyclical risk premium. The following is the simple intuition for this new finding. In the variant of the CC model, when investors become more risk averse in recession, they do require a higher risk premium. On the other hand, when investors become more risk averse in recession, stock returns become less volatile. This has the effect of decreasing the risk premium. Since the second effect dominates the first one, the net impact is that higher risk aversion results in a lower risk premium.

We also show that with a small value for the volatility of the log surplus consumption ratio, a large value of risk aversion may not determine whether the equity premium and the risk-free rate puzzles can be resolved or not. Finally, we calibrate the variant of the CC model to generate artificial data and demonstrate that countercyclical risk aversion may not help explain the predictability of long-horizon stock returns, the univariate mean-reversion of stock prices and the "leverage effect" in return volatility.

In this paper, we use the variant of the CC model to study how the representative agent's risk aversion affects asset prices for the following two important reasons. First, the CC model is the first rational asset pricing model that is able to explain many of the empirically observed properties of stock returns. Thus understanding how countercyclical risk aversion affects asset prices in CC improves our understanding of asset pricing theory.

Second, the CC model is also the first rational asset pricing model with the two sources active at the same time. This property provides us with a valuable opportunity to find out which source or both help explain aggregate stock market behavior, an interesting result, which is unclear in CC.

The organization of the paper is as follows. In the next section, we revisit the continuous-time and discrete-time versions of the CC model. In Section 3, we derive the closed-form solution to the equilibrium risk-free interest rate and the market price of risk. When the

market price of risk is a constant, we also derive the closed-form solution to the equilibrium stock price and return, and show that countercyclical risk aversion induces a procyclical risk premium. In Section 4, we show that with a small value for the volatility of the log surplus consumption ratio, a large value of risk aversion may not determine whether the equity premium and the risk-free rate puzzles can be resolved or not. In Section 5, we calibrate the variant of the CC model to generate artificial data and show that countercyclical risk aversion may not help explain the predictability of long-horizon stock returns, the mean-reversion of stock prices and the "leverage effect" in return volatility. Finally in Section 6, we make conclusions.

2 The CC model

In this section, we revisit the continuous-time and discrete-time versions of the CC model. The continuous-time model helps derive a closed-form solution to the equilibrium when the market price of risk is a constant, but the discrete-time one is used to generate artificial data. Consider a single-good standard pure-exchange economy with two securities: a riskless bond and a risky stock.

The stock represents a claim to the aggregate endowment flow (or total consumption) of $C(t)$, which grows according to the following process:

$$\frac{dC}{C} = gdt + \sigma dB, \quad (1)$$

where σ and g are assumed to be constants, and $B(t)$ is a standard Brownian motion.

The economy is populated by a representative agent with a utility function

$$U(c, X) = \frac{e^{-\rho t}}{1-\gamma} (c - X)^{1-\gamma},$$

where γ is the utility curvature, ρ the time impatience parameter, $c(t)$ the consumption rate at time t , and $X(t)$ the level of habit.

In CC, the consumption surplus ratio $S(t) \equiv (c(t) - X(t))/c(t)$ is used to capture the relationship between consumption and habit, where $s(t) = \ln(S(t))$ evolves according to

the following stochastic process:

$$ds = k(\bar{s} - s)dt + \pi(s)dB, \quad (2)$$

where κ is the mean-reverting rate, $\pi(s)$ the volatility of the log surplus consumption ratio and $B(t)$ the standard Brownian motion specified in equation (1). Based on the specification of the surplus ratio above, $S(t) > 1$ does not have an economic interpretation, since habit is non-negative.

Naturally, a high (low) log surplus consumption ratio, $s(t)$, can be regarded as a good (bad) state of economy. The process in equation (2) is a mean-reverting one, which is the continuous time analog of the AR(1) process. Equations (1) and (2) describe the continuous-time version of the CC model. The corresponding discrete-time version of the CC model is as follows:

$$\Delta\theta(t+1) = g - \frac{1}{2}\sigma^2 + v(t+1), \quad v(t+1) \sim i.i.d. N(0, \sigma^2), \quad (1')$$

$$s(t+1) = \bar{s}(1 - \phi) + \phi s(t) + \lambda(s(t))v(t+1), \quad (2')$$

where

$$\Delta\theta(t+1) \equiv Ln\left(\frac{C(t+1)}{C(t)}\right), \quad \phi = e^{-\kappa} \text{ and } \lambda(s(t)) = \frac{\pi(s(t))}{\sigma} \sqrt{\frac{1 - e^{-2\kappa}}{2\kappa}}.$$

Clearly $\pi(s)$ plays the role of $\lambda(s)$ ¹ in the discrete-time model. The relative risk aversion coefficient (RRA) with respect to consumption is

$$-\frac{c(t)U_{cc}(c(t), X(t))}{U_c(c(t), X(t))} = \frac{\gamma}{S(t)}.$$

Since RRA is time-varying, countercyclical with respect to the state of economy, the representative agent's risk aversion, measured by RRA, is time-varying, countercyclical. Thus the first source for a countercyclical risk premium is active.

¹Since σ and κ are constants, $\pi(s)$ has exactly the same dynamics as $\lambda(s)$ does.

The objective function of the representative agent is to choose a consumption rate $c(t)$ and a fraction of wealth invested in the stock, $\eta(t)$, to solve the following problem:

$$\sup_{\{c(t), \eta(t)\}} E_0 \left[\int_0^\infty \frac{e^{-\rho t}}{1-\gamma} (c(t) - X(t))^{1-\gamma} dt \right] \quad (3)$$

subject to

$$dW(t) = W(t) \left[\eta(t) \frac{dP(t) + C(t)dt}{P(t)} + (1 - \eta(t))r dt \right] - c(t)dt,$$

where $W(t)$ is the agent's wealth, $P(t)$ is the stock price, and $c(t)$ is the consumption rate.

An equilibrium is defined as a vector process (c, P, r, η) such that the market clears and the problem in (3) is solved optimally. In equilibrium, $\eta(t) = 1$ and $c(t) = C(t)$ (see Lucas 1978 for detail).

3 Countercyclical risk aversion and the procyclical conditional risk premium

In this section, we use the continuous-time version of the CC model defined in equations (1) and (2) to first derive the closed-form solution to the equilibrium interest rate and the market price of risk. When the market price of risk has a constant value, we also derive a closed-form solution to the equilibrium stock price and return. We show that countercyclical risk aversion drives a procyclical risk premium.

The following proposition summarizes the equilibrium interest rate and market price of risk (the Sharpe ratio of stock market returns).

Proposition 1 *The equilibrium interest rate $r_f(t)$ and the market price of risk (Sharpe ratio of market stock returns) $q(t)$ are*

$$r_f(t) = \rho + \gamma g + \gamma \kappa (\bar{s} - s) - \frac{1}{2} \gamma^2 (\sigma + \pi(s))^2 - \frac{1}{2} \gamma \sigma^2,$$

$$q(t) = \gamma (\sigma + \pi(s)),$$

where $\pi(s)$ is the volatility of the log surplus consumption ratio defined in equation (2).

Clearly, the equilibrium interest rate and the market price of risk are both affected by the volatility of the log surplus consumption ratio, $\pi(s)$. In particular, since γ and σ are constants, there is a one-to-one relationship between the dynamics of the market price of risk $q(t)$ and the one of $\pi(s)$. Like $\lambda(s)$ in the discrete-time version of the CC model, $\pi(s)$ is countercyclical with respect to $s(t)$. Thus $q(t)$ is countercyclical and the second source for a countercyclical risk premium is also active.

On the other hand, risk aversion, which is measured by $\gamma/S(t)$ (RRA), is countercyclical but not affected by whether the volatility of the log surplus consumption ratio, $\pi(s)$, is countercyclical or not. Thus there is no interaction between the two sources for a countercyclical risk premium. This independence between the two sources is important, since it enables us to separate the impacts of the two sources on asset prices. For example, if we let $\pi(s)$ be a constant, then the second source is inactive, but the first source is still active and we can examine how the first source affects stock returns. Another important reason that we can pinpoint the impact of risk aversion on asset prices is that the consideration of a constant value for the volatility of the log surplus consumption ratio doesn't change how risk aversion affects asset prices.

Before proceeding to show the impact of the first source on asset prices, we briefly discuss why the volatility of the log surplus consumption ratio, $\pi(s)$, determines the dynamics of the equilibrium market price of risk.

From the definition of the log surplus consumption ratio in the previous section, surplus consumption, Θ , can be written as

$$\Theta \equiv C(t) - X(t) \equiv S(t)C(t) \equiv C(t)e^{s(t)}.$$

Using Ito's lemma, we have

$$\frac{d\Theta}{\Theta} = \left[\kappa(\bar{s} - s) + \frac{1}{2}\pi^2(s) + g + \pi(s)\sigma \right] dt + (\sigma + \pi(s)) dB,$$

where $d\Theta/\Theta$ is (instantaneous) surplus consumption growth.

Since σ is a constant, it is clear that surplus consumption growth in CC has countercyclical volatility, since $\pi(s)$ is countercyclical. As in the standard representative agent model

(see Mehra and Prescott 1985), where a constant value of volatility in consumption growth drives a constant market price of risk², a countercyclical variation of volatility in surplus consumption growth in CC induces a countercyclical conditional market price of risk, since the product of surplus consumption and consumption in CC plays the role of consumption in the standard representative agent model with a power utility function.

In the following, to examine how the first source affects stock returns, we let the second source be inactive by considering a constant value for the volatility of the log surplus consumption ratio. In the rest of paper, we use π and $\pi(s)$ to denote a constant value of volatility and time-varying volatility, respectively, for the log surplus consumption ratio. It should be emphasized here that the consideration of a constant value of π doesn't aim to modify the CC model but to illustrate how investors' risk aversion affects asset prices.

When the second source is inactive, the equilibrium stock price and return are characterized in the following lemma and proposition.

Lemma 1 *If the market price of risk has a constant value, $\gamma(\sigma + \pi)$, then the equilibrium stock price is $P(t, s) = C(t)p(t, s)$, where*

$$p(t, s) = \int_t^\infty \exp(\Pi(t, y, s)) dy,$$

$$\Pi(t, y, s) = \Phi_1(y - t) + \gamma\kappa\beta(t, y)(s(t) - \bar{s}) + \frac{1}{2}\gamma^2\pi^2\beta_2(t, y) + \gamma(\gamma - 1)\sigma\pi,$$

$$\Phi_1 = - \left[\rho + (\gamma - 1)g - \frac{1}{2}\gamma(\gamma - 1)\sigma^2 \right],$$

$$\beta(t, y) = \frac{1 - \exp(-\kappa(y - t))}{\kappa}, \quad \beta_2(t, y) = \frac{1 - \exp(-2\kappa(y - t))}{2\kappa}.$$

In Lemma 1, $p(t, s)$ is the time-varying price-dividend ratio, since it is a function of the log surplus consumption ratio, $s(t)$. In order to make sure that the stock price is finite,

²In a standard representative agent model with a power utility function (Mehra and Prescott 1985), if the volatility of consumption growth is σ , the market price of risk is $\gamma\sigma$, where γ is the curvature of the power utility function.

Φ_1 must be negative. If ρ is assumed to be positive, for standard parameter values, Φ_1 is indeed negative and thus the equilibrium stock price is finite.

Proposition 2 *Let dR be the equilibrium instantaneous excess return, which is defined as $dR = (dP + Cdt)/P - r_f dt$. If the market price of risk has a constant value, $\gamma(\sigma + \pi)$, then dR evolves according to the following stochastic differential equation:*

$$dR = \mu_R dt + \sigma_R dB,$$

where

$$\begin{aligned} \mu_R &= \left[\sigma + \gamma \kappa \frac{p_s}{p} \pi \right] \gamma (\pi + \sigma), \\ \sigma_R &= \sigma + \gamma \kappa \frac{p_s}{p} \pi, \end{aligned}$$

p is the price-dividend ratio, p_s/p is the partial derivative of the log price-dividend ratio with respect to $s(t)$, that is, $p_s/p \equiv \partial \ln(p)/\partial s$.

Since σ , γ , π and κ are constants, countercyclical risk aversion doesn't directly affect the risk premium and return volatility. However, it affects the value of p_s/p , the partial derivative of the log price-dividend ratio with respect to $s(t)$, and thus indirectly the stock return. In the following lemma, we show that p_s/p is procyclical with respect to $s(t)$.

Lemma 2 *Let $p(t, s)$ be the price-dividend ratio as defined in Lemma 1. Then the partial derivative of the log price-dividend ratio is increasing with the state of economy. That is,*

$$\frac{\partial(p_s/p)}{\partial s} > 0.$$

Thus, the log price-dividend ratio is convex with respect to the state of the economy, $s(t)$. Since $p_s > 0$, the price-dividend ratio is procyclical. Intuitively, when $s(t)$ increases, the risk-free rate, based on Proposition 1, will decrease. This tends to decrease the expected return or the discount rate³. Thus, the price-dividend ratio increases, since the stock price

³When $s(t)$ increases, the risk-free rate will decrease and this tends to decrease the discount rate. On the other hand, when $s(t)$ increases, the risk premium will increase and this tends to increase the discount rate. Since the former effect dominates the latter one, the net effect is that when $s(t)$ increases, the discount rate decreases.

is just the discounted present values of future dividends.

From Proposition 2 and Lemma 2, both the risk premium and return volatility are clearly procyclical. This is a surprising result, since this result is against the intuition that countercyclical variation in risk aversion results in a countercyclical risk premium and return volatility.

Why is return volatility procyclical? The simple intuition is as follows. When $s(t)$ is large, the risk-free rate is low and the discount rate tends to be small. Then when $s(t)$ changes, the stock price will become more volatile than when $s(t)$ is small and the discount rate is high, since the stock price is just the discounted present value of future dividends. Now we are in a position to understand the procyclical risk premium. When $s(t)$ decreases, investors become more risk averse and thus require a higher risk premium. On the other hand, when $s(t)$ decreases, stock returns become less volatile. This has the effect of decreasing the risk premium. When the market price of risk is a constant, the second effect dominates the first one. Thus countercyclical risk aversion results in a procyclical risk premium⁴.

If we closely examine the equilibrium stock return in Proposition 2, two interesting observations emerge. First, if σ , γ and κ are constants, the only way to make the conditional risk premium and return volatility countercyclical is a countercyclical market price of risk, which implies countercyclical volatility for the log surplus consumption ratio, $\pi(s)$. Thus, in CC, it is the second source (the countercyclical market price of risk) that induces the countercyclical risk premium and return volatility. In other words, if countercyclical risk aversion drives the countercyclical risk premium and return volatility in CC, then this countercyclical risk aversion should induce similar results for stock returns when the market price of risk becomes a constant, since a consideration of a constant market price of risk doesn't change how risk aversion affects asset prices. Thus this result indirectly shows that it is not countercyclical risk aversion but the countercyclical market price of risk that

⁴We are very grateful to one of the two anonymous referees for this explanation of the procyclical risk premium and return volatility.

induces the countercyclical variation of the risk premium and return volatility in CC.

Second, the value of π affects the size of the risk premium and the risk-free rate, and thus determines whether or not the CC model can be used to explain the large average risk premium and the low average risk-free rate in historical data.

In the next section, we demonstrate that with a small value of π , a large value of risk aversion may not determine whether the risk-free rate and the equity premium puzzles can be resolved or not.

4 The risk-free rate and the equity premium puzzles

The CC model is able to explain the equity premium and the risk-free rate puzzles with a large value of RRA, for example, 80 in the steady state. Intuitively, this high risk aversion is regarded as responsible for resolving these two well-known puzzles. In this section, we use a simple numerical example to study whether or not with a small value of π , high risk aversion can help resolve the two puzzles.

Suppose π has a small value, for example, the value of σ . If we want to match the average risk free rate of 0.94% and the average risk premium of 6.69% in the postwar data set, then $\pi = \sigma = 1.22\%$ and $g = 1.89\%$ (see CC for detail). We let $\bar{S} = 0.001$ and $\kappa = 0.15$. Next we study whether we can find reasonable parameter values for ρ and γ to resolve the risk-free rate and equity premium puzzles.

From Proposition 1, when the log surplus consumption ratio has a constant value of volatility, π , the risk-free interest rate becomes

$$r_f(t) = \rho + \gamma g + \gamma \kappa (\bar{s} - s) - \frac{1}{2} \gamma^2 (\sigma + \pi)^2 - \frac{1}{2} \gamma \sigma^2.$$

From equation (2), we have $s(t) = \bar{s} + (s(0) - \bar{s})e^{-\kappa t} + \pi \int_0^t e^{-k(t-\tau)} dB(\tau)$. When t is very

large, the unconditional expectation of the risk-free rate at time t , $E_0 [r_f(t)]$, is close to⁵

$$\begin{aligned} & \rho + \gamma g - \frac{1}{2}\gamma^2 (\sigma + \pi)^2 - \frac{1}{2}\gamma\sigma^2 \\ = & \rho + \gamma g - 2\gamma^2\sigma^2 - \frac{1}{2}\gamma\sigma^2. \end{aligned}$$

To have an average risk-free rate of 0.94%, γ must be less than 1, if ρ is assumed to have a value between 0 and 1.

From Proposition 2, the unconditional expectation of the risk premium at time t , $E_0 [\mu_R(t)]$, is

$$\gamma(\sigma + \sigma)(\sigma + \gamma\kappa E_0 \left[\frac{p_s}{p} \right] \sigma) < \gamma(\sigma + \sigma)(\sigma + \gamma\sigma) = 2r(r + 1)\sigma^2,$$

since $p_s/p < 1/\kappa$. To have an average risk premium of 6.69%, γ must be larger than 14.5.

Thus a tension between a high risk premium and a low risk-free rate arises, and we now cannot resolve the Mehra and Prescott's (1985) equity premium puzzle and Weil's (1989) risk-free rate puzzle with a positive time impatience parameter. So, if π has a small value, for example, the value of σ , we cannot use a large value of risk aversion to resolve these two puzzles at the same time unless we have a negative time impatience parameter, which is highly undesirable, as pointed out by CC and other researchers.

Two observations are ready from this simple exercise. First, when π has a small value, a large value of risk aversion may not help resolve the two puzzles. On the other hand, a large value of π is the key to resolving these two puzzles.

So far, we focus on a constant value of π . When the volatility of the log surplus consumption ratio is time-varying, as in CC, its average value must be much larger than the value of σ to help resolve the risk-free rate and the equity premium puzzles. Readers may have observed that the constant risk-free rate defined in equation (12) in CC seems to have nothing to do with the average value of the volatility of the log surplus consumption ratio, since $\lambda(s)$ is not there. However, from the definition of $\lambda(s)$ in equation (10) in CC, it

⁵For example, since $\kappa = 0.15$, when $t = 100$, $E_0 [r_f(t)]$ is very close to $\rho + rg - \frac{1}{2}\gamma^2 (\sigma_C + \pi)^2 - \frac{1}{2}\gamma\sigma_C^2$. In numerical exercises, if we generate more than 10,000 years of data for the risk-free interest rate, the sample mean will be close to $E_0 [r_f(t)]$.

is not difficult to find that the average value of $\lambda(s)$ is affected by the value of \bar{S} , which affects the risk-free rate. Thus the average value of $\lambda(s)$ indirectly plays an important role in resolving the risk-free rate and equity premium puzzles there.

Second, since \bar{S} ⁶ is assumed to be 0.001, even a small value of γ , such as 2, can result in a very large value of 2,000 (2/0.001) for RRA in the steady state, but the risk premium is very small, less than 0.18%. Thus, when the value of π is small, high risk aversion doesn't imply a large risk premium, a result that is totally different from the one in Mehra and Prescott (1985) that a large value of risk aversion leads to a large risk premium.

The results above are based on a small value of π . However, there is no economic reason for π to be restricted to be small. Thus, our simple exercise just illustrates one interesting aspect in resolving the equity premium and the risk free rate puzzles in CC.

Next, we calibrate the variant of the CC model to generate artificial data and show that countercyclical risk aversion may not help explain many of the empirically observed dynamic properties of stock returns, e.g., the predictability of long horizon stock returns, the univariate mean-reversion of stock prices, and the "leverage effect" in return volatility.

5 Return predictability, price reversion and the "leverage effect" in return volatility

In the last section, we have demonstrated that when π has a small value, such as the value of σ , it seems impossible to resolve the risk-free rate and the equity premium puzzles with a positive time impatience parameter. As pointed out in the last section, there is no economic reason for π to have a small value. In this section, we let π be a free parameter and thus calibrate the variant of the CC model to match some moments in the postwar data set (1947-1995). This data set (see CC) includes value-weighted New York Stock Exchange stock index returns from CRSP, three-month treasury bill rate, and per capita nondurables and services consumption. The moments we try to match are the mean and

⁶In our numerical exercise, the value of \bar{S} is found to have little impact on the risk premium.

standard deviation of per capita consumption growth, the mean of the risk-free rate and the mean of the risk premium. We use the calibrated model to generate artificial data and study whether the artificial data exhibit the predictability of long horizon stock returns, the mean-reversion of stock prices and the "leverage effect" in return volatility, which are documented in empirical studies. As argued before, if countercyclical risk aversion drives these empirical properties in CC, our artificial data should show these properties.

We choose the mean and standard deviation of consumption growth, g and σ , to match the corresponding values of per capita aggregate consumption growth; we choose the time impatience parameter ρ , the mean-reverting speed parameter κ , the volatility of the log surplus consumption ratio π , the long-run mean of the log surplus consumption ratio \bar{S} , and the curvature of the power utility function γ to match the average risk-free interest rate and the average risk premium. Table 1 summarizes our parameter choices⁷ and Figure 1 draws the conditional risk premium, return volatility and the Sharpe ratio as a function of the surplus consumption ratio. As shown in the figure, countercyclical risk aversion leads to counterfactual properties of stock returns: procyclical variation in the risk premium and return volatility.

Table 1 yields one interesting observation. The value of π is almost seven times the value of σ . This is consistent with our argument that a large value of π is the key to resolving the risk-free rate and the equity premium puzzles.

To be consistent with CC, we also consider another century-long data set that includes S&P 500 index and commercial paper returns (1871-1993) and per capita consumption (1889-1992) from Campbell (1999). We use the calibrated model to generate 50,000 years of artificial data and then calculate a variety of statistics. We compare statistics from our artificial data to the corresponding ones in the two data sets.

Table 2 reports the means and standard deviations in the artificial data, and the corre-

⁷If ρ is positive, σ and g have the values of the standard deviation and mean, respectively, of historical consumption growth, then Φ_1 in Lemma 1 is always negative for reasonably large values of γ and thus the stock price is finite. For example, for the values of ρ , σ and g in Table 1, in order to make Φ_1 positive, γ must have a value of at least 255, which is unreasonably large.

sponding statistics for the two data sets. The volatility of the risk-free rate in the artificial data is slightly larger than the corresponding statistics in two data sets. Return volatility and the Sharpe ratio in the artificial data are close to the corresponding statistics in the postwar data set. The average price-dividend ratio is slightly below but its standard deviation is close to the corresponding statistics in the two data sets. When the second source is also active, CC reports similar results.

Table 3 reports the autocorrelations for the risk premium, the partial sum of autocorrelation coefficients for the risk premium, and the cross-correlations between the log price dividend ratio and subsequent absolute risk premia or subsequent risk premia.

In panel 1, the autocorrelations for the risk premium are positive. Thus excess stock returns don't exhibit the univariate mean-reversion of stock prices documented by Fama and French (1988a), and Poterba and Summers (1988). This finding implies that countercyclical risk aversion may not be the driver of the univariate mean-reversion of stock prices in CC; To take care of the poorly measured individual small coefficients, we aggregate individual autocorrelations into partial sums, which are reported in panel 2. As shown there, the calibrated model with the second source inactive doesn't generate the pattern and the magnitude of the partial sums in the two data sets.

In panel 3, the cross-correlation between the price-dividend ratio and subsequent absolute excess returns in the artificial data are positive and different from the corresponding statistics in the two data sets. Thus the price-dividend ratio does not forecast the "leverage effect" in return volatility that is documented by Black (1976), Schwert (1989), Campbell and Hentschel (1992) and others. So the "leverage effect" in return volatility in CC may not be induced by countercyclical risk aversion.

In panel 4, the cross-correlations between the price-dividend ratio and subsequent excess returns are totally different in sign from the corresponding statistics in the two data sets. This finding implies that countercyclical risk aversion may not drive the predictability of long-horizon stock returns; otherwise there should be negative cross-correlations between the price-dividend ratio and subsequent excess returns in the artificial data.

Table 4 presents the long-horizon regression of log excess stock returns on the log price-dividend ratio. Clearly, the regression coefficients and R^2 in the artificial data have different patterns from those in historical data. This finding further confirms that countercyclical risk aversion may not induce the predictability of long-horizon stock returns, an empirical evidence that is documented by Campbell and Shiller (1988b) and Fama and French (1988b). The finding in Table 4 also indicates that the equity premium is procyclical but economically small.

Thus, countercyclical risk aversion may not help explain the predictability of long-horizon stock returns, the mean-reversion of stock prices, and the "leverage" effect in return volatility in CC. Since the results above may not be robust, this numerical exercise can be regarded as a step stone toward further study of the impact of countercyclical risk aversion on aggregate stock market return behavior.

6 Conclusion

In this paper, we have used the variant of the CC model to examine how the representative investor's risk aversion affects asset prices. We obtain several results on the impact of risk aversion on asset prices that seem counterintuitive at first. First, we show that countercyclical risk aversion induces a procyclical risk premium and return volatility. Second, we show that with a small value for the volatility of the log surplus consumption ratio, a large value of risk aversion may not determine whether the equity premium and the risk-free rate puzzles can be resolved or not. Finally, we show that countercyclical risk aversion may not help explain the predictability of long-horizon stock returns, the univariate mean-reversion of stock prices and the "leverage effect" in return volatility.

The findings in this paper indicate that time-varying countercyclical risk aversion with a large average value may not play a big role in explaining aggregate stock market return behavior; rather that the time-varying countercyclical market price of risk may help understand many of the empirical properties of the stock market return. Thus the future research topic is to develop a consumption-based representative agent model with a small,

constant value of risk aversion and a time-varying countercyclical market price of risk to explain aggregate stock behavior.

Appendix The Proof of the Results in the paper

Proof of Proposition 1

Let $\Lambda(t)$ be the marginal utility process. Then $\Lambda(t) = e^{-\rho t} U_c(C - X) = C^{-\gamma}(t) e^{-\rho t - \gamma s(t)}$. Clearly, this marginal utility process defines the state-price density or the pricing kernel, whose dynamic follows:

$$\frac{d\Lambda}{\Lambda} = -r_f(t)dt - q(t)dB,$$

where $r_f(t)$ is the risk-free interest rate to be decided endogenously, and $q(t)$ is the market price of risk. According to Ito's lemma, we have

$$\begin{aligned} r_f(t) &= \rho + \gamma g + \kappa \gamma (\bar{s} - s) - \frac{1}{2} \gamma^2 (\pi(s) + \sigma)^2 - \frac{1}{2} \gamma \sigma^2, \\ q(t) &= \gamma (\sigma + \pi(s)). \end{aligned}$$

See Basak and Cuoco (1998) for using the same approach to derive the interest rate and the market price of risk. ■

Proof of Lemma 1

The first order condition for the investor's optimal portfolio problem yields

$$P(t, s) = E_t \left[\int_t^\infty \frac{\Lambda(y)}{\Lambda(t)} c(y) dy \right]$$

where $\Lambda(t)$ is the marginal utility mentioned above.

Since the market clears, $c(\tau) = C(\tau)$ for all τ . Thus,

$$\begin{aligned} P(t, s) &= E_t \left[\int_t^\infty \frac{\Lambda(y)}{\Lambda(t)} C(y) dy \right] \\ &= C(t) E_t \left[\int_t^\infty e^{-[\rho + (\gamma - 1)(g - \frac{1}{2}\sigma^2)](y-t) + \gamma(1 - e^{-(y-t)\kappa})(s(t) - \bar{s}) - \int_t^y [r\pi e^{-(y-\tau)\kappa} + (\gamma - 1)\sigma] dB(\tau)} dy \right]. \end{aligned}$$

Using Fubini's theorem, we then have

$$P(t, s) = C(t) \int_t^\infty \exp(\Pi(t, y, s)) dy,$$

where

$$\Pi(t, y, s) = \Phi_1(y - t) + \gamma \kappa \beta(t, y)(s(t) - \bar{s}) + \frac{1}{2} \gamma^2 \pi^2 \beta_2(t, y) + \gamma(\gamma - 1)\sigma \pi \beta(t, y),$$

$$\Phi_1 = - \left[\rho + (\gamma - 1)g - \frac{1}{2}\gamma(\gamma - 1)\sigma^2 \right],$$

$$\beta(t, y) = \frac{1 - \exp(-\kappa(y - t))}{\kappa} \text{ and } \beta_2(t, y) = \frac{1 - \exp(-2\kappa(y - t))}{2\kappa}. \blacksquare$$

Proof of Proposition 2

According to Ito's lemma, we have

$$dP = \frac{\partial P}{\partial t} dt + \frac{\partial P}{\partial C} dC + \frac{\partial P}{\partial s} ds + \frac{1}{2} \frac{\partial^2 P}{\partial s^2} (ds)^2 + \frac{\partial^2 P}{\partial s \partial C} ds dC.$$

Then calculating each partial derivative and collecting terms yield the results. \blacksquare

Proof of Lemma 2

First

$$\frac{\partial (p_s/p)}{\partial s} = \frac{p_{ss}p - (p_s)^2}{p^2}.$$

To show

$$\frac{\partial (p_s/p)}{\partial s} > 0,$$

we need to show

$$p_{ss}p - (p_s)^2 > 0.$$

From the expression for p and p_s ,

$$p_{ss}p - (p_s)^2 = \int_t^\infty e^{\Pi} dy \int_t^\infty e^{\Pi} \psi^2 dy - \left(\int_t^\infty e^{\Pi} \psi dy \right)^2,$$

where $\psi = \gamma\kappa\beta(t, y)$.

From Schwartz's inequality, we have

$$\int_t^\infty e^{\Pi} dy \int_t^\infty e^{\Pi} \psi^2 dy > \left(\int_t^\infty e^{\Pi} \psi dy \right)^2.$$

This shows the result. \blacksquare

References

- [1] Abel, A.B., 1990. Asset prices under habit formation and catching up with the Joneses. *American Economic Review* 80, 38-42.
- [2] Abel, A.B., 1999. Risk premia and term premia in general equilibrium. *Journal of Monetary Economics* 43, 3-33.
- [3] Basak, S., Cuoco, D., 1998. An Equilibrium Model with Restricted Stock Market Participation. *Review of Financial Studies* 11, 309-341.
- [4] Black, F., 1976. Studies of Stock price Volatility Changes. Proceedings of the 1976 Meetings of the Business and Economics Statistics Section, American Statistical Association, 177-181.
- [5] Campbell, J.Y., 1999. Asset prices, consumption, and the business cycle, *Handbook of Macroeconomics*, volume 1.
- [6] Campbell, J.Y., Cochrane, J.H., 1999. By force of habit: consumption-based explanation of aggregate stock market behavior. *Journal of political Economy* 107, 205-251.
- [7] Campbell, J.H., Hentschel, L., 1992. No News is Good News: An Asymmetric Model of Changing Volatility in Stock Return. *Journal of Financial Economics* 31, 281-318.
- [8] Campbell, J.Y., Shiller, R.J., 1988a. The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *Review of Financial Studies* 3, 195-227.
- [9] Campbell, J.Y., Shiller, R.J., 1988b. Stock Prices, Earnings, and Expected Dividends. *Journal of Finance* 43, 661-676.
- [10] Cochrane, J.H., 2001. *Asset Pricing*, Princeton University Press.
- [11] Cochrane, J.H., 1991a. Volatility Tests and Efficiency Markets: A Review Essay. *Journal of Monetary Economics* 127, 463-485.

- [12] Cochrane, J.H., 1991b. Explaining the Variance of Price-Dividend Ratios. *Review of Financial Studies* 15, 243-280.
- [13] Constantinides, G.M., 1990. Habit Formation: A Resolution of the Equity Premium Puzzle. *Journal of Political Economy* 98, 519-543.
- [14] Fama, E.F., French, K.F., 1988a. Permanent and Temporary Components of Stock Prices. *Journal of Political Economy* 96, 246-273.
- [15] Fama, E.F., French, K.R., 1988b. Dividend Yields and Expected Stock Returns. *Journal of Financial Economics* 18, 127-146.
- [16] Ferson, W.E., Constantinides, G.M., 1991. Habit Persistence and Durability in Aggregate Consumption: Empirical Tests. *Journal of Financial Economics* 29, 199-240.
- [17] Goldstein, R., Zapatero, F., 1996. General equilibrium with constant relative risk relation and Vasicek interest rates. *Mathematical Finance* 6, 331-340.
- [18] Karatzas, I., Shreve, S.E., 1991. *Brownian Motion and Stochastic Calculus*. Second Edition, Springer-Verlag, New York.
- [19] Lucas, R., 1978. Asset Prices in an Exchange Economy. *Econometrica* 46, 1429-1445
- [20] Mehra, R., Prescott, E.C., 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics*, 15 (1985), 145-161.
- [21] Poterba, J., Summers, L.H., 1988. Mean Reversion in Stock Returns: Evidence and Implications. *Journal of Financial Economics* 22, 27-60.
- [22] Schwert, G. W., 1989. Why Does Stock Market Volatility Change Over Time?. *Journal of Finance* 44, 115-1153.
- [23] Sundaresan, S.M., 1989. Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth. *The Review of Financial Studies* 41, 75-110.

- [24] Weil, P., 1989. The Equity Premium Puzzle and the Risk-free Rate Puzzle. *Journal of Monetary Economics* 24, 401-421.

Table 1. Parameter

Parameter	Variables	Value
Mean consumption growth (%)	g	1.89
Standard deviation of consumption growth (%)	σ	1.22
Time impatience parameter (%)	ρ	2.72
Mean-reverting coefficient (%)	κ	5.00
Steady-state surplus consumption ratio (%)	\bar{S}	0.10
Volatility of the log surplus consumption ratio ratio (%)	π	8.00
Curvature of the power utility function	γ	5.16

Table 2: Means and standard deviations of artificial and historical data

Statistic	Artificial Data	Postwar Sample	Long Sample
$E(\Delta c)$	1.89*	1.89	1.72
$\sigma(\Delta c)$	1.22*	1.22	3.32
$E(r_f)$	0.94*	0.94	2.92
$\sigma(r_f)$	6.65	3.01	5.01
$E(r_s - r_f)$	6.69*	6.69	3.90
$\sigma(r_s - r_f)$	16.5	15.7	18.0
$E(r_s - r_f)/\sigma(r_s - r_f)$	0.41	0.43	0.22
$\exp(E(p - d))$	18.7	24.7	21.1
$\sigma(p - d)$	0.55	0.26	0.27

* means that we choose the parameter values to match these moments. The historical moments for the two data sets are from CC. All the returns are based on annual frequency. Annual consumption is the average of monthly consumption. The discrete-time version of the CC model is used to generate annual data. Δc is log consumption growth, r_s is the log stock return, r_f is the log risk-free rate, and $p - d$ is the log price-dividend ratio.

Table 3: Auto and cross correlations of artificial and historical data

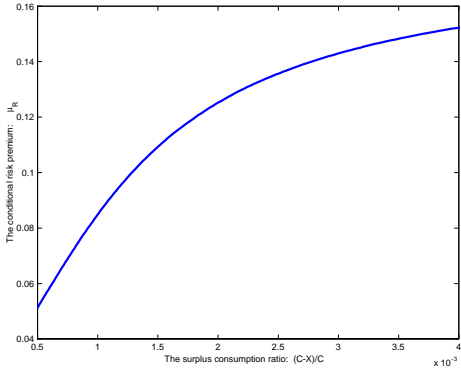
Statistic	Lag j (years)				
	1	2	3	5	7
$(r - r_f)_t, (r - r_f)_{t+j}$					
Artificial data	0.026	0.015	0.022	0.023	0.018
Postwar sample	-0.11	-0.28	0.15	0.02	-0.10
Long sample	0.05	-0.21	0.08	-0.14	-0.11
$\sum_{i=1}^j \rho[(r_s - r_f)_t, (r_s - r_f)_{t+j}]$					
Artificial data	0.026	0.041	0.063	0.086	0.104
Postwar sample	-0.11	-0.39	-0.24	-0.22	-0.32
Long sample	0.05	-0.17	-0.09	-0.23	-0.44
$(p - d)_t, r_s - r_f _{t+j}$					
Artificial data	0.21	0.20	0.20	0.18	0.16
Postwar sample	-0.16	0.09	0.11	-0.05	-0.02
Long sample	-0.12	0.02	-0.06	-0.10	-0.05
$(p - d)_t, (r_s - r_f)_{t+j}$					
Artificial data	0.068	0.067	0.065	0.061	0.059
Postwar sample	-0.42	-0.25	-0.13	-0.35	-0.17
Long sample	-0.20	-0.21	-0.10	-0.19	-0.08

The autocorrelations and cross-correlations for the two data sets are from CC.

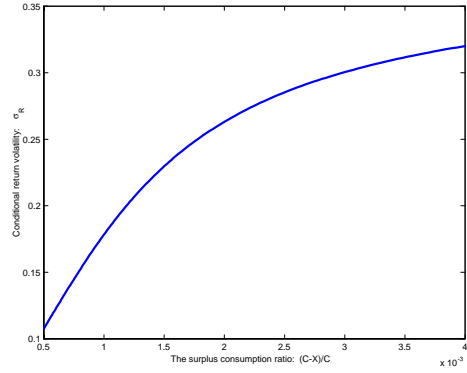
Table 4: Long-horizon return regression

Horizon (years)	Artificial data		Postwar sample		Long sample	
	Coefficient	R^2	Coefficient	R^2	Coefficient	R^2
1	0.022	0.005	-0.26	0.18	-0.13	0.04
2	0.043	0.009	-0.43	0.27	-0.28	0.08
3	0.064	0.013	-0.54	0.37	-0.35	0.09
5	0.104	0.020	-0.90	0.55	-0.60	0.18
7	0.163	0.025	-1.21	0.65	-0.75	0.23

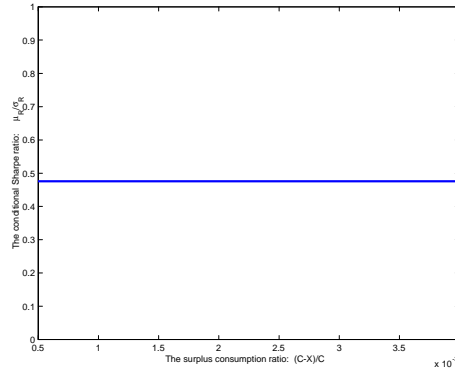
The regression coefficients and R^2 for the two data sets are from CC.



(a) The conditional risk premium



(b) Conditional return volatility



(c) The conditional Sharpe ratio

Figure 1. In part (a), the conditional risk premium is plotted as a function of the surplus consumption ratio, which ranges from 0.0005 to 0.004. As shown in this picture, the conditional risk premium is increasing with the surplus consumption ratio, a counterfactual result. In part (b), conditional return volatility is plotted as a function of the surplus consumption ratio, which has a value from 0.0005 to 0.004. As shown there, conditional return volatility is increasing with the surplus consumption ratio, a result that is inconsistent with the empirical fact. Finally, in part (c), the conditional Sharpe ratio (the market price of risk) is plotted against the surplus consumption ratio, which ranges from 0.0005 to 0.004. There the conditional Sharpe ratio is a constant.