JOB MATCHING AND THE WAGE DISTRIBUTION[*]

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Abstract

The objective of this paper is to bring together the microeconomic-labor and the macroeconomic-equilibrium views of matching in labor markets. We nest a job matching model à la Jovanovic (1984) into a Mortensen and Pissarides (1994)-type equilibrium search environment. The resulting framework preserves the implications of job matching theory for worker turnover and wage dynamics, while allowing for aggregation and general equilibrium analysis, in particular of the wage distribution. We obtain two testable implications of job matching and search frictions for wage inequality. First, learning about match quality and turnover map Gaussian output noise into an ergodic wage distribution of empirically accurate shape: unimodal, skewed, with a Paretian right tail. In this sense, *ex post* worker self-selection is a plausible alternative to the *ex ante* sorting emphasized by the dominant Roy model tradition. Second, high idiosyncratic productivity risk hinders learning and sorting and therefore, in contrast to the predictions of standard equilibrium search models with observable match quality, tends to reduce wage inequality. We also obtain analytic solutions for the wage distribution and for aggregate worker flows – quits to unemployment and to other jobs, displacements and hires – which provide the likelihood function of the model, and allow for its parametric structural estimation.

Keywords: wage distribution, wage inequality, job matching, specific human capital, worker flows, unemployment, job search, Bayesian learning, ergodic analysis.

JEL Classifications: C73, D31, D83, E24, J63, J64.

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1. Introduction

The search-and-matching model is the canonical framework for the analysis of labor markets, both in micro-labor economics and in macroeconomics. In spite of their common roots and similarities, the micro and macroeconomic approaches to matching in labor markets have evolved in parallel, and have addressed different issues. In this paper, we bring together these two views of the labor market, and we extend their scope. In particular, we show that the matching approach can reconcile the observed patterns of worker flows and wage dynamics with classic stylized facts concerning wage inequality. To this purpose, we introduce the first general equilibrium analysis of job matching in the sense of Jovanovic (1979, 1984) [J79, J84], and we bring it into the tradition of equilibrium unemployment theory of Mortensen and Pissarides (1994) [MP94].

The job matching theory originating with J79 provides the benchmark model of worker turnover in labor economics, which motivated a vast body of applied microeconomic research. The worker-firm match is modelled as an experience good, whose characteristics are initially unknown, and are revealed over time by realized output. Optimal inference and turnover lead to a dynamic selection of matches that accounts for a wide range of robust empirical correlations: positive between worker tenure and wage (Topel 1991), initially positive and soon negative between tenure and the hazard rate of separation (Farber 1994), negative between tenure or wage and the worker propensity to search on the job (Pissarides and Wadsworth 1994).

The macroeconomic approach to matching in labor markets has focused instead on unemployment and on wage dispersion. The "equilibrium" search-and-matching literature, originating from the work of Diamond (1971, 1982), Mortensen (1982), and Pissarides (1985), has in fact branched out into two influential research programs. The equilibrium theory of unemployment, as best represented by MP94, has become the standard framework of analysis for aggregate labor markets. The equilibrium theory of "pure" wage dispersion builds on the lack of coordination inherent in search frictions to explain the failure of the law of one price in labor markets, through either asymmetric equilibrium strategies adopted by identical workers and firms (Burdett and Mortensen 1998) or imperfectly assortative matching between *ex-ante* heterogeneous workers and firms (Postel-Vinay and Robin 2002a, Shi 2002).

The exchanges between the micro-labor and the macro-equilibrium approaches to matching in labor markets have been infrequent and unsatisfactory. The implications of job matching for wage inequality and for the magnitude of worker flows are unknown, and may well be totally counterfactual. On the other side, unemployment theory has acknowledged the role of evolving idiosyncratic uncertainty in match productivities as an engine of job flows. In view of its empirical success, the J79 job matching framework would be the natural structural model for this type of uncertainty; unfortunately, it lacks the tractable aggregation properties that are required for general equilibrium analysis. Therefore, macro-equilibrium search models feature idiosyncratic uncertainty (if any) only in reduced forms that contradict the observed evolution of wages and separation rates *within* a job. In MP94, for instance, the productivity of a match drifts down. Wage-posting models of equilibrium wage dispersion rule out, by construction, the on-the-job wage dynamics and voluntary separations that inspired Jovanovic's work.

The bottom line is that micro and macroeconomists have re-tooled the matching model to answer their own, different questions. However, a synthesis of these two views appears natural, for several reasons. On *theoretical* grounds, job matching is a natural competing model of equilibrium wage dispersion. Also, as forcefully argued by MP94, the motives for job separations are key to understand unemployment and job flows, and the applied labor literature has identified job matching as a major source of worker turnover.[1] On *empirical* grounds, the counterfactual predictions of equilibrium search for worker turnover and wage dynamics beg for some reconciliation. Finally, on purely *methodological* grounds, the common "matching" assumption of a fixed proportion technology and constant returns to scale naturally suggests a feasible synthesis.

Our starting point is J84, where the J79 job matching model is enriched with two job-contact rates, both finite and exogenous in the style of partial equilibrium search, in order to account for unemployment and job-to-job quits. Bridging the remaining gap from this hybrid framework to equilibrium search entails three challenges.

The first, key challenge is of technical nature, and concerns aggregation. This is the likely reason why job matching theory failed to make progress beyond partial equilibrium analysis. In Jovanovic's well-known Gaussian setup, posterior beliefs about match productivity are two-dimensional, mean and variance, and evolve according to a classical linear filter. In equilibrium, the mean equals the wage and the variance is inversely proportional to job tenure. These features make the model so elegant and empirically appealing, but also not amenable to aggregation. The cross-sectional belief distribution is bivariate and impossible to characterize analytically; thus, we cannot solve for the univariate wage distribution and for the aggregate worker flows implied by the model.

We introduce a key modeling simplification of the J84 setup, which allows to aggregate

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[1] See for example Flinn (1986), Lane and Parkin (1998). This empirical literature typically focuses on younger workers, as suggested by the theory, because older workers are much more likely to have found their good match. This does not imply that job matching is not relevant for older workers: while their separations are less frequent, the consequences are often more dire.

the job matching model and to finally uncover its general equilibrium implications. We assume that *the unknown match quality may take one of only two values.* In effect, each worker-firm match runs a sequential probability ratio test, based on the data provided by cumulative output, of two simple hypotheses: the match is either successful or not. From an economic viewpoint, this simplification is painless: the economic trade-offs, implications and insights uncovered by job matching theory are preserved in our model. As it happens sometimes, two states of the world are sufficient to capture the intuition on the general effects of risk. The main benefit of our simplification is that the posterior belief about match quality is now ordered − the chance that the match is successful − and follows a simple non-linear filter. In turn, this affords an analytic solution of the ergodic and stationary distribution of posterior beliefs and (expected) match productivities.

When we map posterior beliefs about match success into wages, we discover a second, conceptual inconsistency between J84 and general equilibrium search. In J84, the equilibrium wage is "competitive" and equals expected productivity conditional on output history; hence, active firms make zero expected profits, and idle firms lack the rents required to cover the entry cost of creating a vacancy. Costless vacancies, far from solving the problem, would generate frictionless matching, defeating J84's purpose of extending job matching theory to encompass frictional unemployment. Recruiting costs and free entry require that firms have some bargaining power. At the other extreme, job matching requires that workers have some bargaining power, else the wage would equal the value of leisure and would fail to reflect the evolution of expected match productivity. To reconcile the two views, we need rent-sharing. Following the tradition in equilibrium unemployment theory, we assume that wages are set by generalized Nash bargaining, which subsumes J84's competitive wages as a special (but problematic) case.

In turn, rent-sharing raises a third and final conceptual issue. A worker earns and generates rents for his employer, and potentially for alternative employers. Therefore, an outside offer to an employed worker triggers a tri-lateral renegotiation problem. Competitive wage-setting in J84 resolved the issue totally in favor of the worker. We illustrate what we consider a robust solution to this sub-game, and compare it to the other solutions proposed so far in the literature. We stress *very strongly* that both rent-sharing on the job and the explicit analysis of renegotiation after an outside offer−two clear departures from J84−are *indispensable* to a comprehensive and consistent explanation for involuntary unemployment, worker and job turnover, the dynamics and the distribution of wages.

In the model just described and motivated, the equilibrium wage is an affine function of the expected productivity of a match, just like in J84. But, in our simpler framework, this *does* allow for a simple analytic characterization of the wage distribution, which in turn reveals our two substantive results.

First and foremost, the equilibrium wage density generically exhibits the three main features of empirical wage distributions: it is unimodal and right-skewed like a log-normal, and has a long and "fat" right tail, of Pareto functional form. The selection of good matches, through optimal quits to unemployment and to other jobs, redistributes mass of workers from the lower to the upper part of the distribution of beliefs about match quality, which determine wages. This explains the skewness and the fat Pareto tail, in spite of symmetric and Gaussian (thin-tailed) noise in output. Remarkably, our job matching model explains the typical shape of an empirical wage distribution more accurately and intuitively than many equilibrium search models specifically designed to this purpose.[2]

The second result is a general equilibrium implication of imperfect information about match productivity: the larger the idiosyncratic productivity risk that clouds the underlying match quality, the higher the (Pareto) rate of decay of the upper tail of the wage distribution. Intuitively, the harder the inference problem faced by firms and workers, the less effective the market selection process, the fewer workers have time to identify a good match, before being exogenously separated from their jobs. The upper tail of the wage distribution contains precisely those matches that have been almost ascertained to be successful. This implication is reversed in search models that assume evolving but *observable* idiosyncratic productivity or demand shocks, such as MP94. This is important, because all the predictions of job matching that have found empirical support in a vast labor literature – the stylized facts mentioned earlier, and new ones addressed by a recent revival of interest in learning of workers's productivities (Altonji and Pierret 2001, Nagypal 2000) – are also consistent with observable idiosyncratic uncertainty. Our implication may be confronted with the data, for example by cross-industry comparisons, to conclusively accept or reject incomplete information and learning about match quality as a relevant source of job-specific productivity dynamics.

In addition to these two results, we contribute to the ongoing effort to render equilibrium models of wage dispersion empirically operational. Seminal early attempts in this direction rely on wage-posting models that allow for no on-the-job wage dynamics (Christensen *et alii* 2002) and treat the distribution of worker and firm productivities as unobservables, to be estimated necessarily by non-parametric methods (Postel-Vinay and Robin 2002a). In contrast, the analytic equilibrium solution of our job matching

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[2] Mortensen (1998) shows that a typical wage-posting search equilibrium implies an increasing or U-shaped wage distribution. Upfront firm-specific investments may restore unimodality and long, fat upper tail. Postel-Vinay and Robin (2002b) introduce employed search and *ex-post* competition for employed workers, but no *ex-post* productivity risk. The implied equilibrium wage distribution is hump-shaped, but lacks the strong skewness and especially the Pareto tail that we observe in the data.

model allows for structural estimation *by maximum likelihood* of the model parameters, while explaining returns to tenure. An econometrician estimating a Mincerian wage equation without knowing match output histories would dump into the "error" term the wage dispersion created by evolving beliefs about match quality. Our equilibrium wage distribution, instead, provides the likelihood function to be maximized. Its skewness reflects the selection process via job shopping, and illustrates formally the bias that would arise from the estimation of a wage equation by linear regression, under the standard assumption of Gaussian (symmetric) residuals. The empirically accurate shape of our equilibrium wage distribution suggests an excellent fit of the model to the data.

The paper is organized as follows. Section 2 illustrates the model, Section 3 equilibrium wages and turnover, Section 4 the stationary and ergodic wage distribution, Section 5 closes the general equilibrium of the model with a matching function, Section 6 concludes, an Appendix collects proofs of the propositions.

2. The Economy

A consumption good is produced in continuous time by pairwise firm-worker matches (*jobs*). The average productivity or "quality" of each match, μ, is specific and *ex-ante* uncertain: upon matching, the firm and the worker share a common prior belief on μ, independent of their past histories and concentrated on two points, $p_0 = \Pr(\mu = \mu_H) = 1 - \Pr(\mu = \mu_L) \in (0,1)$, where μ_L denotes a "bad" match and $\mu_H (> \mu_L)$ a "good" match.

The performance of the match is also subject to two additional and orthogonal sources of idiosyncratic noise. First, cumulative output in the time interval $[0, t]$ is a normal random variable, a Brownian Motion with drift μ and known variance σ^2:

$$X_t = \mu t + \sigma Z_t \sim N\left(\mu t, \sigma^2 t\right).$$

Here Z_t is a Wiener process, a continuous additive noise that keeps μ hidden and creates an inference problem. Over time, parties observe output realizations $\langle X_t \rangle$, generating a filtration $\{\mathcal{F}_t^X\}$, and update in a Bayesian fashion their belief from the prior p_0 to the posterior $p_t \equiv \Pr(\mu = \mu_H \mid \mathcal{F}_t^X)$. The second, more drastic source of idiosyncratic shocks is a Poisson jump process, forcing jobs out of business at rate $\delta > 0$. This shock captures many important idiosyncratic sources of match dissolution; a few examples are, on the labor demand side, technological obsolence, natural disasters, changes in specific tax code provisions, idiosyncratic product demand shocks; on the labor supply side, human capital shocks such as worker disability, retirement, death, or other events like spousal relocation.

The economy is populated by a large mass of *ex-ante* homogeneous firms, ensuring free entry, and by a unit measure of *ex-ante* homogeneous workers. If δ contains a worker attrition component, the population is replenished by new workers. A jobless worker enjoys a flow value of leisure b, while idle firms get zero flow returns. Workers and firms are risk-neutral optimizers and discount future payoffs at rate $r > 0$. We assume $b \in [\mu_L, (1-p_0)\mu_L + p_0\mu_H]$, so the matching choice is non trivial: a new match should always be accepted, because it produces more than the joint value of inactivity b, and should dissolved if $\mu = \mu_L \leq b$. In practice, parties perform a sequential probability ratio test of simple hypotheses on the viability of the match.

A worker is hired at finite Poisson rate λ when unemployed, and at rate $\psi\lambda$ when searching on the job. In both cases job search is costless, except for its time-consuming aspect and for discounting. Here ψ is the chance at every point in time that an employed worker who wants a new job has the opportunity to actively search for one. We defer to Section 5 the description of the matching process generating λ. There is no recall of past offers. The firm must pay a flow sunk cost κ to keep a vacancy open and attract applications from workers, unemployed and employed alike. Every new match, whether the worker joins from unemployment or from another job, restarts from a common prior chance p_0 of success. For the sake of simplicity, there is no initial "screening" phase as in J84, nor choice of search intensity. Search frictions create rents that the parties split according to a generalized Nash bargaining rule.

The natural state variable of the bargaining game is the posterior belief p_t of match success. By Theorem 9.1 in Liptser and Shyryaev (1977), conditional on the output process X, the posterior probability that a match was successful evolves from any prior $p_0 \in (0,1)$ as a martingale diffusion solving:

$$dp_t = p_t(1-p_t)s d\bar{Z}_t \tag{2.1}$$

where

$$s \equiv \frac{\mu_H - \mu_L}{\sigma}$$

is the *signal/noise ratio* of output, and

$$d\overline{Z}_t \equiv \frac{1}{\sigma}\left[dX_t - p_t\mu_H dt - (1-p_t)\mu_L dt\right]$$

is the *innovation* process, the normalized difference between realized and unconditionally expected flow output. This is a standard Wiener process w.r. to the filtration $\{\mathcal{F}_t^X\}$. Intuitively, beliefs move faster the more uncertain match quality (the term $p(1-p)$ peaks at $p = 1/2$), and the more informative production, as measured by s.[3]

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[3] In this binary structure, unlike in the Gaussian model of J79, J84, posterior belief precision $[p_t(1-p_t)]^{-1}$ does not necessarily increase over time as evidence accumulates. However, the qualitative implications of Jovanovic's model depend on the martingale property of beliefs and on optimal selection, *not* on the specific match distribution assumed. In fact, these properties survive intact in this binary framework (see Proposition 3). In contrast, aggregation is tractable only in this binary structure.

3. Wages and Job Separation

3.1. On-the-Job Search and the "Poaching Subgame"

We analyze the steady state general equilibrium of this economy, where aggregate variables (including the wage distribution) do not change over time, while worker turnover and job churning are continuously driven by purely idiosyncratic uncertainty.

Let $W(p)$ denote the discounted total payoffs that a worker receives in the equilibrium of the bargaining-and-search game, when employed in a match that is successful with current posterior chance p. Similarly, let U denote the worker's value of unemployment, independent of p because of the match-specific nature of μ, $J(p)$ the rents of the firms, V the value to the firm of holding an open vacancy, and $S(p) = W(p) + J(p) - U - V$ the total surplus of this match. By free entry, $V = 0$. We may then write Bellman equations for worker and firm given an arbitrary wage function $w(p)$ of the belief p; the equilibrium wage is pinned down by a generalized Nash bargaining solution, giving the worker a fraction $\beta \in [0, 1]$ of total match surplus: $W(p) - U = \beta S(p)$, $J(p) = (1 - \beta)S(p)$, implying

$$\beta J(p) = (1 - \beta)[W(p) - U]. \tag{3.1}$$

Before solving for the wage from (3.1), by backward induction we first address the subgame following an outside offer to a worker, who is searching on the job, to match at a renewed prior p_0. The current employer's maximum valuation for its match with the worker equals $S(p) + U$, and the new firm's maximum valuation equals $S(p_0) + U$. As long as both of these values strictly exceed the joint outside option U, one of the two firms suffers a discrete loss whether the worker decides to quit or not. If the worker quits, his current employer loses the rents $J(p)$ it was earning; if the worker stays with his current employer and rejects the offer by the other firm, the employer may lose a wage raise necessary to retain the worker, and the other firm loses the rents that the new match could have generated. This congestion suggests that a cooperative bargaining solution to this tri-lateral renegotiation, like the standard solution (3.1) assumed for the bi-lateral bargaining between firm and worker, is inappropriate.

This situation describes a symmetric information game between two buyers (the firms) competing for a worker, under common knowledge of the total gains from either trade, $S(p) + U$ and $S(p_0) + U$. If the worker earns no rents, $\beta = 0$, there is no issue of search on-the-job. So assume $\beta > 0$. When $W(p) \geq S(p_0) + U$, breaking ties in favor of the current employer, the latter has nothing to fear, because the worker would lose from quitting even if he could capture all the rents from the new match. Then the worker will not search on the job. Similarly, when $S(p) + U < W(p_0)$, the worker would quit anyway, because the value of *ex-post* renegotiation at the new job $W(p_0)$ exceeds the total value of the existing match $S(p)+U$. In this case the worker will search from employment for sure.

Therefore, the interesting question is whether the worker would search while employed in the other cases, namely (using 3.1) when

$$\beta\left[W(p_0)-U\right] \leq W(p)-U < \frac{W(p_0)-U}{\beta}. \tag{3.2}$$

Clearly, the worker accepts the outside offer if and only if $S(p) < S(p_0)$, or by (3.1) $W(p) < W(p_0)$, because the match with higher valuation can always generate sufficient payoffs to attract the worker. Hence, *quits are always socially efficient.* However, the worker may search on the job even if $W(p) \geq W(p_0)$, in order to generate outside offers and extract more rents from his current employer. We face a circularity: the Bellman values $S(p)$ and $W(p)$ determine the worker's propensity to quit, hence the match separation rate, and in turn the current match surplus $S(p)$ itself.

The literature has analyzed two main strategic forms of this poaching game under symmetric information and no recall of past offers. In the J84 job matching model with search frictions, drawing directly from the frictionless J79 model, firms compete for the worker to the point that the latter's wage captures the entire expected output: $w_t = \mathbb{E}[\mu \mid \mathcal{F}_t^X]$. In our setting, as we will show later, this is formally equivalent to the worker making a take-it-or-leave-it offer, $\beta = 1$, and the firm earning no rents. In this case, the double inequality (3.2) is empty and the issue is moot. But, as we argued in the Introduction and will demonstrate shortly, $\beta = 1$ is inconsistent with free entry by firms.

The second possibility, consistent with firms receiving rents and entering the market, is *ex-post* Bertrand competition between the two firms (Postel-Vinay and Robin 2002b). In this case, the worker always searches on the job when (3.2) holds. If $S(p) \geq S(p_0)$, the employer retains his employee by offering him an amount $S(p_0) - W(p)$, positive by (3.2), to match the best possible offer that the other firm could ever make. If $S(p) < S(p_0)$, the worker quits his current job, accepts the new offer and receives a human wealth gain $S(p) - W(p_0)$, positive by (3.2). In either case, both firms fight so fiercely as to effectively maximize the worker's returns − hence his propensity − to search on the job.[4]

Albeit the model can be as easily solved under the assumption of Bertrand competition,

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[4] Dey and Flinn (2002) assume that the new offer is the result of Nash bargaining, with the total surplus from the old match $S(p)$ as the outside option of the worker. This solution further raises the returns to the worker from searching on the job, but seems to require some recall, that we rule out, and does not survive the backward induction refinement that we impose. In Pissarides (1994) employed search takes place as long as it creates a positive surplus for the current match, to be abandoned, while the costs and returns of the new employer do not play any role. Felli and Harris (1996) characterize the unique, perfect sequential equilibrium of the repeated poaching game without any commitment but with perfect recall, or equivalently without search frictions as in J79.

we depart from this solution because of two unappealing features. First, suppose that firms can make offers and counteroffers, as in an ascending auction, first or second price. Then, backward induction implies that to no bids are made in equilibrium. The key is symmetric information: all players know in advance which firm will win the auction. For the losing firm, bidding is weakly dominated, and strictly dominated for any arbitrarily small cost of bidding. It is common knowledge that the winning firm can always respond successfully to any hostile bid. Firms' bids only redistribute rents to the worker.[5]

The second problem of *ex-post* Bertrand competition for an employed worker is more specific to dynamic matching markets. If every firm in the economy could credibly commit *ex-ante*, for example through reputation, not to match outside offers to its employee, then there may exist an equilibrium in which all firms do in fact commit and are better off than in the Bertrand outcome. The reason is that the firm's *ex-post* temptation to respond to outside offers creates *ex-ante* incentives for the worker to generate offers via on-the-job search. In turn, the worker rent-seeking behavior reduces all firms' payoffs.[6]

In light of these considerations, we assume that, when an employed worker successfully contacts another open vacancy, the two firms play the unique backward-induction equilibrium of the ascending auction, equivalent to two separate bi-lateral renegotiation games between the worker and each of the two firms. We state this first result as a:

Proposition 1. (Equilibrium of the Poaching Subgame) *The unique subgame perfect equilibrium of the ascending auction poaching subgame is socially efficient, and has the following outcome. When a worker matched with a firm at posterior belief* p *receives an outside offer by another firm to re-match at* p_0*: if* $W(p) < W(p_0)$*, the current employer does not respond, the worker quits, restarts bi-lateral renegotiation with the new firm, and earns rents* $W(p_0)$*; otherwise the worker and his employer disregard the outside offer. Therefore, the employed worker keeps searching for another job if and only if* $W(p) < W(p_0)$*, no outside offers are matched by employers, and no lump-sum transfers between firms and workers take place.*

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[5] This reasoning points to asymmetric information as the engine of competition for employed workers. In Burdett and Mortensen (1998) a poaching firm makes a unilateral offer to an employed applicant, independently of the wage he is earning; as emphasized by Postel-Vinay and Robin (2002b), this implies that the firm in practice ignores that wage, but the worker's current employer does not respond to the outside offer by assumption. Here, backward induction yields no response to outside offers as an equilibrium strategy under symmetric information.

[6] In a two-sided matching model with search frictions and non-transferable utility, Burdett *et alii* (1999) show that the externality caused by the propensity to search for alternative partners while matched can be so powerful to generate a continuum of steady state equilibria.

3.2. The Equilibrium Wage

We may now work backward, using Proposition 1 to compute continuation payoffs, and address the bi-lateral bargaining game between a worker and his employer when no outside offer is on the table. We solve for the equilibrium wage that guarantees (3.1). The worker's values of being (respectively) unemployed and matched well with probability p solve the Hamilton-Jacobi-Bellman (HJB) equations:[7]

$$\begin{aligned} rU &= b + \lambda[W(p_0) - U] \\ rW(p) &= w(p) + \Sigma(p)W''(p) - \delta[W(p) - U] + \psi\lambda \max\langle W(p_0) - W(p), 0\rangle \end{aligned} \tag{3.3}$$

where

$$\Sigma(p) \equiv \frac{1}{2}s^2p^2(1-p)^2$$

is half the *ex-ante* variance of the change in posterior beliefs, roughly speaking "the speed of learning" about match quality. The opportunity cost of unemployment, rU, equals its flow benefit b plus the capital gain $W(p_0) - U$ from a new match, which has prior belief p_0 of being successful, accruing at rate λ. Similarly, the opportunity cost $rW(p)$ of working in a job that is successful with posterior chance p equals the flow wage $w(p)$, plus a diffusion-learning term $\Sigma(p)W''(p)$, minus the capital loss following exogenous separation at rate δ, plus the capital gain following a profitable quit to another job, which resets the prior to p_0 (see Proposition 1.) The learning speed $\Sigma(p)$ is converted into payoff units by the convexity of the Bellman value $W''(p)$, because information (here in the form of output) spreads posterior beliefs and empowers more informed decisions by the worker.

The worker optimally quits to unemployment at every belief $\underline{p}_W \in [0,1]$ such that $W(\underline{p}_W) = U$ (*value matching*) and $W'(\underline{p}_W) = 0$ (*smooth pasting*), and keeps searching on the job whenever $W(p)$ falls short of the value $W(p_0)$ that he can obtain from a fresh start at a new firm. In this case he gains exactly $W(p_0) - W(p)$ (see again Proposition 1.)

The problem of the firm is similar. The free entry condition $V = 0$ will be used later to close the general equilibrium. The value to the employer $J(p)$ of an active match that is successful with posterior chance p solves the HJB equation

$$rJ(p) = \bar{\mu}(p) - w(p) + \Sigma(p)J''(p) - \delta J(p) - \psi\lambda J(p)\mathbb{I}\{W(p) < W(p_0)\} \tag{3.4}$$

with $\mathbb{I}\{\cdot\}$ an indicator function, so $\mathbb{I}\{W(p) < W(p_0)\} = 1$ if and only if the worker seeks outside offers. The opportunity cost of production $rJ(p)$ equals expected flow output

$$\bar{\mu}(p) \equiv p\mu_H + (1-p)\mu_L$$

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[7] Unless otherwise noted, Karlin and Taylor (1981) is the main reference for the standard technical results in diffusion theory exploited in this paper.

minus the wage $w(p)$, plus the return from learning the quality of the match $\Sigma(p)J''(p)$, minus expected capital losses due to exogenous separation ($\delta J(p)$) and to a quit by the worker to another job ($\psi\lambda J(p)$ when $W(p) < W(p_0)$ and the worker keeps searching). The firm optimally fires the worker at every $\underline{p}_J \in [0,1]$ such that $J(\underline{p}_J) = 0$ and $J'(\underline{p}_J) = 0$.

By (3.1), worker and firm agree to separate and to become idle when the posterior belief hits the same threshold(s) $\underline{p}= \underline{p}_W = \underline{p}_J$. When the worker quits to another job, he forfeits positive rents $W(p) - U > 0$ for even larger ones $W(p_0) - U$ in the new match, while his employer suffers an unrecoverable loss $J(p) \propto W(p) - U > 0$. Observe that (3.1) implies $\mathbb{I}\{W(p) < W(p_0)\} = \mathbb{I}\{J(p) < J(p_0)\}$ and $\beta J''(p) = (1-\beta)W''(p)$. Using these facts and (3.1) into the HJB equations (3.3) and (3.4), plus some (omitted) algebra, yield a simple and intuitive expression for the equilibrium wage:

$$w(p) = b + \beta\left[\bar{\mu}(p) - b + \lambda J(p_0)(1 - \psi\mathbb{I}\{J(p) < J(p_0)\})\right]. \tag{3.5}$$

The wage received by the worker exceeds his opportunity cost of time b by his bargaining share β of flow expected output $\bar{\mu}(p)$ in excess of b, plus the endogenous outside option from unemployed job search $\lambda J(p_0)$, reduced by a fraction ψ when the match looks unpromising and the worker searches on the job, $W(p) < W(p_0)$ or $J(p) < J(p_0)$, in order to compensate the firm for the potential loss of a valuable employee. The wage is affine and increasing in the posterior belief, and jumps up at p_0 as the worker ceases on-the-job search and the firm no longer faces the potential quit of its employee. Employed search improves the worker's outside option, at the expense of joint match surplus. Notice that the discontinuity in the wage w at p_0 guarantees continuity in the value functions W and J, so there is no incentive to renegotiate further on lump-sum transfers when $p_t = p_0$.

If $\beta = 1$, from (3.1) $J(p) \equiv 0$, and (3.5) reduces to $w(p_t) = \bar{\mu}(p_t) = \mathbb{E}[\mu \mid \mathcal{F}_t^X]$, namely J84's "competitive" wage equal to expected match productivity. In this case the firm lacks the positive rents $J(p_0) > 0$ that are necessary to cover the cost κ of posting a vacancy and entering the market. Conversely, if $\beta = 0$, then again from (3.1) we see that the worker gets paid his opportunity cost of time $w(p) = b$, independently of past performance, tenure, etc. In both extreme cases, the wage is disconnected from labor market tightness λ.

Replacing the wage function (3.5) into the worker's and the firm's HJB equations transforms their bargaining-separation game into two separate optimal stopping problems. Using (3.1), (3.5) and boundaries turns the firm's HJB equation (3.4) into:

$$J(p) = \frac{(1-\beta)[\bar{\mu}(p) - b] + \Sigma(p)J''(p) - \beta\lambda J(p_0)(1 - \psi\mathbb{I}\{J(p) < J(p_0)\})}{r + \delta + \psi\lambda\mathbb{I}\{J(p) < J(p_0)\}} \tag{3.6}$$

subject to value matching and smooth pasting at $\underline{p}$. An additional boundary condition is

$$J(1) = (1-\beta)\frac{\mu_H - b}{r+\delta} - \frac{\beta\lambda}{r+\delta}J(p_0),$$

because the worker would never quit a "perfect" match ($W(1) > W(p_0)$) due to the absorbing property of the extreme belief $p = 1$. This allows to solve for the value function, which equals the sum of the present discounted value of flow returns and of the option value of separating should things go wrong, including a direct quit for $p < p_0$. So let:

$$\alpha_0 \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(r+\delta+\psi\lambda)}{s^2}}; \quad \alpha_1 \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(r+\delta)}{s^2}}.$$

Proposition 2. (Bargaining and Separation Equilibrium) *A firm and a worker match sharing a common prior belief p_0 of a good outcome, continuously observe output in $[0,t]$, update the posterior belief p_t according to (2.1), renegotiate the wage $w(p)$ according to (3.5), and separate when the posterior declines to a low cutoff $\underline{p} \in (0, p_0)$ (when the wage falls to a reservation value $w(\underline{p})$). The worker searches on the job for a new match at prior p_0 (again) if and only if $p_t < p_0$, and always accepts outside offers, to which his employer never responds. The value function of the firm is the increasing and convex function of beliefs:*

$$J(p) = \left[c_{0J} p^{1-\alpha_0}(1-p)^{\alpha_0} + k_{0J} p^{\alpha_0}(1-p)^{1-\alpha_0}\right] \mathbb{I}\{\underline{p} \le p < p_0\} + c_{1J} p^{1-\alpha_1}(1-p)^{\alpha_1} \mathbb{I}\{p_0 \le p \le 1\}, \\ + \frac{(1-\beta)[\bar{\mu}(p) - b] - \beta\lambda J(p_0)(1 - \psi\mathbb{I}\{\underline{p} \le p < p_0\})}{r + \delta + \psi\lambda\mathbb{I}\{\underline{p} \le p < p_0\}} \quad (3.7)$$

where the coefficients c_{0J}, k_{0J}, c_{1J} and the optimal stopping point $\underline{p} \in (0, p_0)$ uniquely solve the system of four algebraic equations:

$$J(\underline{p}) = 0, \qquad J'(\underline{p}+) = 0, \qquad J(p_0-) = J(p_0+), \qquad J'(p_0-) = J'(p_0+). \qquad (3.8)$$

3.3. Tenure, Wages and Turnover

The bargaining equilibrium described in Proposition 2 implies a stochastic process for the worker's employment status and, conditional on employment, for the posterior belief of a good match p_t. The belief starts from p_0, evolves as the diffusion (2.1) following output realizations, is "killed" at rate δ by exogenous destruction and is absorbed into unemployment for a random duration of mean $1/\lambda$. The same happens if dismal output drives the belief down to $\underline{p}$ and leads parties to separate and to restart search. If $p_t < p_0$ the worker also seeks outside job offers, and finds one at rate $\psi\lambda$, resetting beliefs to p_0.

Before illustrating our new results, we verify that our model preserves the qualitative correlations between tenure, wages, separation rate and employed search that are observed in the data and that are central to extant theories of worker turnover, as summarized in the Introduction. We also find an analytic expression for expected tenure $\mathcal{T}(p)$ as a function of the current belief p and, by (3.5), of the wage.

In the absence of endogenous separation at $\underline{p}$, $\mathcal{T}(p)$ should equal $1/\delta$ for $p > p_0$ when outside offers are rejected, and $1/(\delta + \psi\lambda)$ for $p < p_0$ when they are accepted. But the match also terminates endogenously, when the belief falls to $\underline{p}$. Overall, $\mathcal{T}(p)$ solves:

$$\Sigma(p)\mathcal{T}''(p) - (\delta + \psi\lambda\mathbb{I}\{\underline{p} \leq p < p_0\})\mathcal{T}(p) = -1$$

Lemma 1. (Expected Tenure) *The expected future duration of a match is the increasing and convex function of the current belief that the match is productive:*

$$\mathcal{T}(p) = \mathbb{I}\{p_0 \leq p \leq 1\}\frac{1}{\delta}\left\{1 + c_{1\mathcal{T}}p^{1-\alpha_1}(1-p)^{\alpha_1}\right\}$$
$$+\mathbb{I}\{\underline{p} \leq p < p_0\}\frac{1}{\delta+\psi\lambda}\left\{1 + c_{0\mathcal{T}}p^{1-\alpha_0}(1-p)^{\alpha_0} + k_{0\mathcal{T}}p^{\alpha_0}(1-p)^{1-\alpha_0}\right\}$$

where $\{c_{0\mathcal{T}}, k_{0\mathcal{T}}, c_{1\mathcal{T}}\}$ *solve* $\mathcal{T}(\underline{p}) = 0$, $\mathcal{T}(p_0-) = \mathcal{T}(p_0+)$, $\mathcal{T}'(p_0-) = \mathcal{T}'(p_0+)$.

Standard in Bayesian learning, in expectation with respect to current beliefs p_t, posterior beliefs $p_{t+\Delta t}$ are a martingale: $\mathbb{E}[p_{t+\Delta t}|0 \leq p_{t+\Delta t} \leq 1, p_t] = p_t$ for all $\Delta t \geq 0$. But, if we condition on match continuation from t to $t + \Delta t > t$, the belief is a strict *sub*martingale, because it is bounded below by $\underline{p} > 0$ and reflects only good output outcomes. In fact:

$$\mathbb{E}[p_{t+\Delta t}|\text{producing in } [t, t+\Delta t], p_t] > \mathbb{E}[p_{t+\Delta t}|\underline{p} \leq p_{t+\Delta t} \leq 1, p_t] > \mathbb{E}[p_{t+\Delta t}|0 \leq p_{t+\Delta t} \leq 1, p_t] = p_t$$

where the first inequality holds because quits occur only for low beliefs, namely $p_s < p_0$ for $s \in [t, t + \Delta t]$, and the chance of exogenous match dissolution is independent of p_s, so match continuation is more likely for high beliefs in $[t, t + \Delta t]$; the second inequality follows from $\underline{p} > 0$; and the equality is the martingale property. Hence the confidence in a good match rises on average with tenure, although there is always a positive chance of a decline. Also standard in Bayesian learning, the value function is convex in beliefs p, hence a submartingale too. The flow wage (3.5) is affine in beliefs, due to the combined assumptions of expected utility and linear sharing rule, hence it is also a submartingale for continuing matches.

Unconditionally on match quality, starting from a current belief p_t, the probability of separating endogenously at some future date $T > t$ ($p_T = \underline{p}$) before finding out that the match is good for sure ($p_T = 1$) equals $(p_t - \underline{p})/(1 - \underline{p})$; therefore, the probability of endogenous separation to unemployment is decreasing in p_t. The hazard rate of a quit $\psi\lambda\mathbb{I}\{\underline{p} \leq p_t < p_0\}$ is also decreasing in p_t. The hazard rate of exogenous separation, δ, is independent of p_t. Overall, separation is less likely the larger the expected productivity of the match, and thus (on average, by Proposition 1) the longer the worker's tenure. The only exception is at the beginning of a match, when instantaneous endogenous separation is impossible by continuity of the belief process' sample paths. Thus, on average, the hazard rate of separation initially increase and then decrease with tenure, as in the J79 job matching model and as observed in the data (Farber 1994).

We summarize these findings in the following:

Proposition 3. (Tenure, Wages, and Search Behavior) *Unconditionally on true match quality, but conditional on match continuation, the human wealth of the employed worker* $W(\cdot)$*, his flow wage* $w(\cdot)$*, and the rents of his employer* $J(\cdot)$ *rise on average with tenure. On-the-job search is more common among low-tenured workers. The hazard rate of match separation rate initially increases and then decreases monotonically with tenure. Expected future tenure is increasing in the current wage.*

4. The Ergodic Wage Distribution

The stochastic process describing the equilibrium evolution of the posterior belief of a good match is clearly Markovian and strongly recurrent. Therefore, the stationary density is also ergodic: from any non-degenerate prior $p_0 \in (0,1)$, the posterior belief converges a.s. to a random variable p_∞ with support $[\underline{p}, 1]$ and total probability mass equal to total employment, plus an atom of unemployment. If p_∞ has a density, say f, then in a large population of workers f can be interpreted also as the ergodic and stationary cross-sectional distribution of employed workers (matches, posterior beliefs). For the following results we refer to Feller (1954)'s classic treatment of diffusions on an interval and to his physical interpretation of the dynamic equations. Imposing stationarity in the Fokker-Planck (Kolmogorov forward) equation of the process, which describes the dynamics of the transition density, we obtain the following equation for the stationary and ergodic density f of the belief process:

$$\frac{d^2}{dp^2}[\Sigma(p)f(p)] - (\delta + \psi\lambda\mathbb{I}\{\underline{p} \leq p < p_0\})f(p) = 0, \tag{4.1}$$

subject to the following three boundary conditions:

1. no time spent at the separation boundary $\underline{p} > 0$: $\Sigma(\underline{p})f(\underline{p}+) = 0$, thus by $\Sigma(\underline{p}) > 0$,

   $$f(\underline{p}+) = 0;$$

   this is a standard condition for "attainable" boundaries, which can be hit in finite time with positive probability and are either absorbing or reflecting;

2. balance of total flows (respectively) in and out of employment:

$$\Sigma(p_0)[f'(p_0-)-f'(p_0+)]=\psi\lambda\int_{\underline{p}}^{p_0}f(p)dp+\delta\int_{\underline{p}}^{1}f(p)dp+\Sigma(\underline{p})f'(\underline{p}+),$$

   equating the total inflow into employment on the LHS to the total outflow on the RHS, due to (resp.) quits to other jobs, exogenous job destructions, and quits to unemployment at $\underline{p}$.

3. balance of total flows (respectively) in and out of unemployment:

$$\delta\int_{\underline{p}}^{1}f(p)dp+\Sigma(\underline{p})f'(\underline{p}+)=\lambda(1-\int_{\underline{p}}^{1}f(p)dp),$$

   equating the inflow into unemployment, both involuntary due to job dissolution at rate δ and voluntary through the separation boundary $\Sigma(\underline{p})f'(\underline{p}+)$, to the outflow, exit rate λ times unemployment. This is a standard restriction in search models, which gives rise to a Beveridge curve.

The total flow in or out of employment exceeds that in or out of unemployment by an amount equal to job-to-job quits, because these are the only separations that do not entail an unemployment spell.[8]

To solve (4.1) for f, let:

$$\gamma_0\equiv\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{2(\delta+\psi\lambda)}{s^2}};\qquad\gamma_1\equiv\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{2\delta}{s^2}}.$$

Proposition 4. (The Ergodic Distribution of Posterior Beliefs about Match Quality) *For* $p\in[\underline{p},1]$*:*

$$f(p)=c_{0f}p^{-1-\gamma_0}(1-p)^{\gamma_0-2}\left[\left(\frac{1-\underline{p}}{\underline{p}}\frac{p}{1-p}\right)^{2\gamma_0-1}-1\right]\mathbb{I}\{\underline{p}\le p<p_0\}+c_{1f}p^{-1-\gamma_1}(1-p)^{\gamma_1-2}\mathbb{I}\{p_0\le p\le 1\}\tag{4.2}$$

where the coefficients c_{0f} *and* c_{1f} *are the unique and positive solution of the linear algebraic system* $\Xi(c_{0f},c_{1f})'=(\lambda,0)'$ *(the matrix* Ξ *is appendicized in (A.3)).* f *is globally continuous, with a kink at* p_0*. In* $[\underline{p},p_0]$*,* f *is always increasing; in* $[p_0,1]$*,* f *is decreasing if* $\gamma_1\ge 2$*, namely if the rate of attrition exceeds the squared signal/noise ratio of output* $\delta\ge s^2$*, U-shaped if* $\min\{3p_0-1,1\}<\gamma_1<2$*, and increasing if* $1<\gamma_1\le\min\{3p_0-1,1\}$*.*

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[8] Although the second condition may appear equivalent to the third, if the second boundary condition is violated then the distribution f will generate different flows of quits $\psi\lambda\int_{\underline{p}}^{p_0}f(p)dp$ and new hires from employment, namely total hires $\Sigma(p_0)[f'(p_0-)-f'(p_0+)]$ minus exits from unemployment $\lambda(1-\int_{\underline{p}}^{1}f(p)dp)$.

The Equilibrium Wage Density and its Economic Implications. Rational (Bayesian) learning and optimal match selection map Gaussian output X_t into a piece-wise Lévy-stable distribution f of posterior beliefs, which belongs to the Lévy-Pareto *type*. The interpretation of f is empirically more meaningful in wage space. Without loss in generality, we can normalize the scale of output so that $\beta\sigma s = \beta(\mu_H - \mu_L) = 1$. Then, the equilibrium wage function (3.5) becomes a pure location transformation $w(p) = \mathfrak{w}_{\mathbb{I}\{\underline{p} \leq p < p_0\}} + p$ where:

$$\mathfrak{w}_{\mathbb{I}\{\underline{p} \leq p < p_0\}} \equiv b + \beta \left[\mu_L - b + \lambda J(p_0)(1 - \psi \mathbb{I}\{\underline{p} \leq p < p_0\}\right].$$

For $w \geq \underline{w} \equiv \mathfrak{w}_1 + \underline{p}$, and given $w_0 \equiv \mathfrak{w}_0 + p_0$, the wage density is:

$$\phi(w) = f\left(w - \mathfrak{w}_{\mathbb{I}\{w < w_0\}}\right). \tag{4.3}$$

Therefore, ϕ also belongs to the Pareto type. In fact, both f and ϕ have a *fat right tail, which is decaying generically (for $\delta \geq s^2$) but always at slower rate than a Gaussian.*

Proposition 4 has three important implications, the three new results of our analysis. First, the theoretical equilibrium wage distribution $\phi(w)$ replicates the typical shape of an empirical wage distribution, including its well-known Paretian right tail. Quits to other jobs and to unemployment weed out disproportionately bad matches, censor the left tail, and skew the distribution. To the best of our knowledge, this is the first explanation of this stylized fact based on self-selection in terms of *ex-post* productivity heterogeneity.[9] The upper panel of Figure A.1 depicts the ergodic density f for a parameterization of the model implying $\delta > s^2$ (see Moscarini 2002 for a precise quantitative evaluation of this model). The lower panel illustrates, for the same parameterization, the frequency distribution of a long simulation of a discrete time belief process, designed to converge in distribution to the solution of our continuous-time filter (2.1) as the time interval vanishes.

The second result follows from the definition of γ_1: the *right tail of the wage distribution $\phi(w)$ decays faster the larger the ratio δ/s^2* between the exogenous match dissolution rate δ and the (squared) informativeness of output s^2. The values of δ and s^2 also affect the scale of $\phi(w)$ through the constants of integration c_{0f}, c_{1f}, but the rate of decay of $\phi(w)$ in w equals $\gamma_1 - 2$ and depends on δ and s^2 only directly, through their ratio. Intuitively, when jobs are at high risk of exogenous destruction (δ is large), or when the output process is very noisy and uninformative, so beliefs move slowly (σ is large and, given the earlier normalization $\mu_H - \mu_L = \beta^{-1}$, the signal/noise ratio s is low), the learning-selection process has no time to produce its effects.

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[9] Roy (1951) explains the skewness of the empirical wage distribution as the result of the self-selection of workers in terms of their *ex-ante* known comparative advantages to work in different sectors. Under the assumption of log-normal bi-variate skills, Roy produced a wage distribution that lacks the Pareto-like tail and fails to fit the US wage distribution. Heckman and Sedlacek (1985) amend this shortcoming of the Roy model by introducing unobserved worker heterogeneity.

This unambiguous prediction is unique to an incomplete information environment in general equilibrium. A "noisy" economy is "sclerotic": high idiosyncratic output uncertainty unrelated to firm and worker characteristics (high δ and σ) clouds the intrinsic inequality in productivities (μ) and prevents it from being reflected by equilibrium prices. Wages remain concentrated around their starting value w_0; income inequality tends to be dampened, rather than enhanced, by high idiosyncratic risk, with an efficiency cost in terms of poor sorting. In contrast, this prediction is reversed in search models with perfectly observable match-specific productivity, such as MP94. In that kind of environment, a larger variance of idiosyncratic output shocks raises the incentives to maintain the job active, in order to save on new search costs; in turn, this standard option value effect leads to a reduction in the optimal destruction cutoff, a widening of the range of wages, thus of their inequality. Indeed, the implications of job matching exploited by the empirical turnover literature, e.g. Flinn (1986), all stem from the existence of a stochastic state variable describing the viability of the match and affecting the wage. But this is consistent also with observable *ex-post* idiosyncratic randomness in either productivity or opportunity cost of working.[10] We could always re-define the belief process in J79 to be an observable, exogenous productivity process, and the empirical methodologies applied to test the job matching model would not detect this difference. Our equilibrium implication, instead, crucially depends on imperfect information. It may be tested empirically, for example comparing different industries or occupations, to conclusively accept or reject learning about match quality as a relevant source of wage dynamics and turnover.

The third important feature of our equilibrium solution (4.3) is its simple econometric implementation. The matching rate λ, endogenous to the model as illustrated in the next section, for the sake of estimation can be pinned down by measured unemployment duration. Then, after choosing μ_L and μ_H to normalize scale and location of output, the remaining parameters of the model $(b, \sigma, \beta, p_0, \delta, \psi)$ can be estimated from wage data, by maximizing the likelihood function $\phi(w)$. The lower bound of its support, $\underline{w}$, is also a function of parameters, through the nested bargaining-separation game solved in Proposition 2. The analytic solution of the equilibrium allocation makes the structural estimation of the model parametric, thus simple and transparent, particularly in the accuracy of the estimates, a nontrivial and often unresolved issue in the non-parametric case.

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[10] In support of job-matching, Nagypal (2000) finds that the correlation between tenure and probability of being fired is far from perfect, as it should be (e.g.) in a deterministic training or learning-by-doing model. Similarly, Altonji and Pierret (2001) find that employers practice statistical discrimination and base their hiring decisions on initially observable characteristics of the candidates; however, over time, the wage becomes increasingly dependent on what the firm learns about the worker.

5. The Matching Function and General Equilibrium

The description of the economy is completed by a frictional matching process, and the equilibrium is closed by a free entry condition that determines the job-finding rate λ, so far taken as given. Another major advantage of this model is that proving existence and uniqueness of the stationary general equilibrium, as well as computing it, requires solving a simple *scalar* fixed-point problem for λ. The three functions of posterior beliefs that are part of the equilibrium, the wage $w(\cdot)$, the value $J(\cdot)$ and the ergodic distribution $f(\cdot)$, are computed in a second step by simple algebra, once λ is known. This is due to the fact that each worker-firm relationship needs to track only one scalar aggregate variable, the job-finding rate λ, and not the wage distribution, to determine wages and optimal search and separation policies.

Following the equilibrium search tradition, an increasing, concave and CRS matching function $m(a, v)$, satisfying Inada conditions, yields the flow of new matches as a function of the stocks of open vacancies v and of *job applicants* a, both unemployed and employed:

$$a = 1 - \int_{\underline{p}}^{1} f(p)dp + \psi \int_{\underline{p}}^{p_0} f(p)dp. \tag{5.1}$$

For concreteness, albeit this is inessential to the main results, let $m(a, v) = a^{\eta}v^{1-\eta}$ for $\eta \in (0, 1)$. As all workers are ex-ante identical, we assume random matching. Due to CRS in matching, only *labor market tightness* $\theta \equiv v/a$ matters for equilibrium,

$$\lambda = \frac{m\left(a, v\right)}{a} = m\left(1, \frac{v}{a}\right) = \theta^{1-\eta}. \tag{5.2}$$

The value of a vacancy V solves the standard arbitrage equation

$$rV = -\kappa + \frac{m\left(a, v\right)}{v}\left[J(p_0) - V\right]$$

and therefore the free entry condition $V = 0$ equates the cost of the vacancy to the expected rents from filling the job:

$$\kappa = \frac{m\left(a, v\right)}{v} J(p_0) = \theta^{-\eta} J(p_0). \tag{5.3}$$

Definition 1. *A Stationary General Equilibrium (SGE) is a vector of scalars* $\left\{\lambda^*, \theta^*, \underline{p}^*, a^*, v^*\right\}$, *and a triple of functions* $\left\{J^*, w^*, f^*\right\}$ *defined on the unit interval, which satisfy (3.5), (3.7), (3.8), (4.2), (5.1), (5.2), (5.3), and* $v^* = a^*\theta^*$.

By CRS in matching, the firm's vacancy-filling rate depends only on θ, thus (by 5.2) on λ. The free-entry condition (5.3) can then be rewritten as:

$$J(p_0) = \kappa\lambda^{\frac{\eta}{1-\eta}}$$

which describes a continuous increasing relationship between λ and the starting rents $J(p_0)$, going from 0 to ∞. This relationship is termed the "job creation curve."

Proposition 2 allows to uniquely solve, *given a value of* λ, for the value function $J(\cdot|\lambda)$, thus for the starting value $J(p_0|\lambda)$, another continuous relationship between λ and $J(p_0|\lambda)$ that we dub the "profit curve".

A SGE requires an intersection between the job creation and the wage curves: formally, we seek λ^* such that $J(p_0|\lambda^*)(\lambda^*)^{-\frac{\eta}{1-\eta}} = \kappa$. The other variables forming a SGE are found recursively from the unique λ^*. The proof of the final Proposition shows that an increasing job-finding rate λ reduces the initial rents of a firm $J(p_0|\lambda)$ from a positive value $J(p_0|0) > 0$ to $J(p_0|\infty) = 0$. Intuitively, a higher λ strengthens the worker's bargaining power at the expenses of the firm's profits. Therefore $J(p_0|\lambda)\lambda^{-\frac{\eta}{1-\eta}}$ decreases from ∞ to 0 as λ rises from 0 to ∞, and we conclude:

Proposition 5. *There exists a unique Stationary General Equilibrium, which features positive employment.*

6. Conclusions

This paper introduces a tractable analytical framework to reconcile the microeconomic-labor and macroeconomic-equilibrium views of matching in labor markets. The model inherits, from the former, desirable properties relating to the implied correlations between wages, tenure, and turnover, and from the latter an equilibrium structure that is able to account for involuntary unemployment, worker flows between jobs and in/out of jobs, and job creation. The new contribution is the ability to account also for some empirically robust features of wage inequality, and to link the wage distribution to aggregate worker flows.

The model provides a natural candidate explanation for additional stylized facts, such as the sizable and persistent wage loss caused by exogenous displacement, which has been detected even after controlling for observed and unobserved worker heterogeneity (Jacobson *et alii* 1993). The tractability of the model makes it also an open-ended, flexible tool, which can be extended in several directions; for example, it easily accommodates *ex-ante* heterogeneity in worker skills. In this sense, the empirical wage distribution that we are able to replicate here has to be interpreted as conditional on observable worker and firm characteristics. The emerging empirical literature exploiting matched employer-employee data (e.g. Abowd *et alii* 1999, Postel-Vinay and Robin 2002a) has showed that such residual wage dispersion is pervasive and sizable. Moscarini (2002) exploits the flexibility of the model to explain, both qualitatively and quantitatively, a host of additional facts concerning cross-skill inequality in labor markets, such as the strong inequality across worker groups in entry rates into unemployment and in within-group unexplained wage dispersion.

Turning to the model's robustness, both a continuous search effort choice and a screening phase − the newly met firm and worker instantaneously draw an informative signal of their match quality, before starting production − would add some smoothness to the model, and potentially eliminate the gap in the support of the wage distribution corresponding to the prior belief p_0. For example, a discrete screening signal would just expand the number of starting "prior beliefs", and the corresponding number of continuity conditions of the value function and its slope when solving for the Bellman value J and the distribution of beliefs f. However, these extensions do not promise to add any important insights. More importantly, the search effort-cost function and the distribution of the initial screening signal would add "free parameters", namely unobserved features of the environment affecting the distribution of wages. We consider more instructive to isolate the effects of learning and selection in the presence of a single source of uncertainty.

The model easily accommodates various alternative solutions of the poaching game, including those already proposed in the literature, such as Bertrand competition with or without ensuing renegotiation. Each different solution delivers a different magnitude of the separation cutoff $\underline{p}$, given parameters. The results illustrated in Section 4 relating to the wage distribution survive qualitatively intact.

An important direction of future research concerns the empirical implementation of the model. The model parameters can be estimated from labor market price information, by maximizing the equilibrium wage likelihood function, and then input into a calibration to predict labor market quantities. The derivation of the wage density also produces explicit formulae for the flows of workers between employment states, which enter the boundary conditions of the belief distribution: voluntary quits to unemployment, quits to other jobs, exogenous displacements, and hires from unemployment. These flows have been carefully measured in the equilibrium unemployment literature. The cross-restrictions provided by price and quantity data allow to substantially raise the standards in testing the empirical accuracy of the model, and therefore potentially enhance its reliability for policy analysis.

References

ABOWD, JOHN, FRANCIS KRAMARZ AND DAVID MARGOLIS, 1999, "High Wage Workers and High Wage Firms", *Econometrica*, 67(2), 251-333.

ALTONJI, JOSEPH AND CHARLES PIERRET, 2001, "Employer Learning and Statistical Discrimination", *Quarterly Journal of Economics*, 116(1), 313-350.

BURDETT, KENNETH AND DALE MORTENSEN, 1998, "Wage Differentials, Employer Size, and Unemployment", *International Economic Review*, 39(2), 257-273.

BURDETT, KENNETH, RYIOCHI IMAI AND RANDALL WRIGHT, 1999, "Unstable Relationships", mimeo U. of Pennsylvania, Dept. of Economics.

CHRISTENSEN, BENT JESPER, RASMUS LENTZ, DALE MORTENSEN, GEORGE NEUMANN AND AXEL WERVATZ, 2002, "On the Job Search and the Wage Distribution", mimeo NWU, Department of Economics.

DEY, MATTHEW AND CHRISTOPHER FLINN, 2002, "An Equilibrium Model of Health Insurance Provision and Health Determination". Mimeo NYU, Department of Economics.

DIAMOND, PETER, 1971, "A Model of Price Adjustment", *Journal of Economic Theory* 3, 156-168.

DIAMOND, PETER, 1982, "Aggregate Demand Management in Search Equilibrium", *Journal of Political Economy* 90(5), 881-894.

FARBER, HENRY, 1994, "The Analysis of Interfirm Worker Mobility", *Journal of Labor Economics*, 12(4), 554−593.

FELLER, WILLIAM, 1954, "Diffusion Processes in One Dimension", *Transactions of the American Mathematical Society*, 1-31.

FELLI, LEONARDO AND CHRISTOPHER HARRIS, 1996, "Learning, Wage Dynamics, and Firm-Specific Human Capital", *Journal of Political Economy*, 104(4), 838-868.

FLINN, CHRISTOPHER, 1986, "Wages and Job Mobility of Young Workers", *Journal of Political Economy*, 94(3) Part I, S88-S110.

HECKMAN, JAMES AND GUILHERME SEDLACEK, 1985, "Heterogeneity, Aggregation and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market", *Journal of Political Economy*, 93(6), 1077-1125.

JACOBSON, LOUIS, ROBERT LALONDE AND DANIEL SULLIVAN, 1993, "Earnings Losses of Displaced Workers", *American Economic Review*, 83(4), 685-709.

JOVANOVIC, BOYAN, 1979, "Job Matching and the Theory of Turnover", *Journal of Political Economy*, 87(5) Part I, 972-990. [J79]

JOVANOVIC, BOYAN, 1984, "Matching, Turnover and Unemployment", *Journal of Political Economy*, 92(1), 108-122. [J84]

KARLIN, SAMUEL AND HOWARD TAYLOR, 1981, *A Second Course in Stochastic Processes*, Academic Press.

LANE, JULIA AND MICHAEL PARKIN, 1998, "Turnover in an Accounting Firm", *Journal of Labor Economics*, 16(4), 702-717.

LIPTSER, ROBERT AND ALBERT SHYRYAEV, 1977, *Statistics of Random Processes*, 2nd volume, Springer-Verlag.

MORTENSEN, DALE, 1982, "Efficiency of Mating, Racing, and Related Games", *American Economic Review*, 72, 968-979.

MORTENSEN, DALE, 1998, "Equilibrium Unemployment with Wage Posting: Burdett-Mortensen Meet Pissarides", mimeo Northwestern University.

MORTENSEN, DALE AND CHRISTOPHER PISSARIDES, 1994, "Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies* 61(3), 397-415. [MP94]

MOSCARINI, GIUSEPPE, 2002, "Skill and Luck in the Theory of Turnover", mimeo Yale University, Department of Economics.

NAGYPAL, EVA, 2000, "Learning-By-Doing Versus Learning About Match Quality: Can We Tell Them Apart?". Mimeo Stanford University.

PISSARIDES, CHRISTOPHER, 1985, ""Short Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages"", *American Economic Review*, 75(4), 676-690.

PISSARIDES, CHRISTOPHER, 1994, "Search Unemployment with On-the-Job Search", *Review of Economic Studies,* 61(3), 457-475.

PISSARIDES, CHRISTOPHER AND JONATHAN WADSWORTH, 1994, "On-the-job Search: Some Empirical Evidence from Britain", *European Economic Review* 38(2), 385-401.

POSTEL-VINAY, FABIEN AND JEAN-MARC ROBIN, 2002a, "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity". *Econometrica*, 70(6), 2295-2350.

POSTEL-VINAY, FABIEN AND JEAN-MARC ROBIN, 2002b, "The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counteroffers". *International Economic Review*, 43(4), 989-1016.

ROY, ANDREW, 1951, "Some Thoughts on the Distribution of Earnings", *Oxford Economic Papers* (New Series), **3**, 135-146.

SHI, SHOUYONG, 2002, "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Based Technology". *Review of Economic Studies*, 69(2): 467-91.

SHYRYAEV, ALBERT, 1978, *Optimal Stopping Rules*, Springer-Verlag.

TOPEL, ROBERT, 1991, "Specific Capital, Mobility and Wages: Wages Rise with Job Seniority", *Journal of Political Economy*, 99(1), 145-176.

A. Appendix: Proofs of the Propositions

Proof of Proposition 2. J is the value function of an optimal Bayesian experimentation problem with flow returns $\bar{\mu}(p)$ that are linear in beliefs p by the expected utility hypothesis: ergo J is convex in beliefs by a standard improvement argument. It follows that $\underline{p}$ is unique and J is everywhere continuous and almost everywhere *twice* differentiable. So standard HJB Verification Theorems for optimal stopping problems apply, including value matching and smooth pasting at $\underline{p}$ (Shyryaev (1978), 3.8). Since J is convex, non-negative (a firm could always separate to obtain zero), and flat at the lower bound $\underline{p}$ where it is also zero, it must be globally increasing where strictly positive, so $\mathbb{I}\{J(p) < J(p_0)\} = \mathbb{I}\{p < p_0\}$ and it is optimal to stop on-the-job search at and only at p_0. Continuity of J and J' at p_0 are value matching and smooth pasting conditions for this stopping choice.

By direct verification and using $\mathbb{I}\{\underline{p} \le p < p_0\} = \mathbb{I}\{J(p) < J(p_0)\}$, the general solution to the HJB Equation (3.6) is:

$$\begin{aligned} J(p) \;=\; & \frac{(1-\beta)[\bar{\mu}(p) - b] - \beta\lambda J(p_0)(1 - \psi\mathbb{I}\{\underline{p} \le p < p_0\})}{r + \delta + \psi\lambda\mathbb{I}\{\underline{p} \le p < p_0\}} \\ & + c_{iJ} p^{1-\alpha_i}(1-p)^{\alpha_i} + k_{iJ} p^{\alpha_i}(1-p)^{1-\alpha_i}. \end{aligned}$$

for $i = \mathbb{I}\{p_0 \le p \le 1\}$ and constants of integration c_{iJ}, k_{iJ}. Imposing the boundary condition

$$J(1) = \frac{(1-\beta)(\mu_H - b) - \beta\lambda J(p_0)}{r + \delta} < \infty$$

and continuity $J(1) = J(1-)$ yields $k_{1J} = 0$, else the term $k_{1J} p^{\alpha_1}(1-p)^{1-\alpha_1}$ would explode to $\pm\infty$ as $p \uparrow 1$ by $\alpha_1 > 1$, violating continuity and monotonicity of J, which imply $J(p) \in [0, J(1)]$.

We have left five unknowns, the three remaining constants $\{c_{0J}, k_{0J}, c_{1J}\}$, $J(p_0)$ and the separation point $\underline{p}$. To find them we have five equations. The simplest algorithm is as follows. Fix an arbitrary positive value $J(p_0) = \bar{J}_0$ and consider the linear system of three equations in $\{c_{0J}, k_{0J}, c_{1J}\}$:

1. Continuity from the left $J(p_0-) = \bar{J}_0$:

$$\begin{aligned} & [r + \delta + \beta\lambda + \psi\lambda(1-\beta)]\bar{J}_0 = (1-\beta)[\bar{\mu}(p_0) - b] + \\ & + (r + \delta + \psi\lambda)[p_0^{1-\alpha_0}(1-p_0)^{\alpha_0} c_{0J} + p_0^{\alpha_0}(1-p_0)^{1-\alpha_0} k_{0J}] \end{aligned}$$

2. Continuity from the right, $\bar{J}_0 = J(p_0+)$, which implies value matching for stopping on-the-job search at p_0:

$$(r + \delta + \beta\lambda)\bar{J}_0 = (1-\beta)[\bar{\mu}(p_0) - b] + (r + \delta)\, p_0^{1-\alpha_1}(1-p_0)^{\alpha_1} c_{1J}.$$

3. Smooth pasting for stopping on-the-job search at p_0, $J'(p_0+) = J'(p_0-)$:

$$
\begin{aligned}
& p_0^{\alpha_0-1}(1-p_0)^{-\alpha_0}(\alpha_0-p_0)k_{0J} + p_0^{-\alpha_0}(1-p_0)^{\alpha_0-1}(1-\alpha_0-p_0)c_{0J} + \frac{(1-\beta)(\mu_H-\mu_L)}{r+\delta+\psi\lambda} \\
= \; & p_0^{-\alpha_1}(1-p_0)^{\alpha_1-1}(1-\alpha_1-p_0)c_{1J} + \frac{1-\beta}{r+\delta}(\mu_H-\mu_L)
\end{aligned}
$$

Solve this system for $\{c_{0J}, k_{0J}, c_{1J}\}$ given the guess $\bar{J}_0$, and plug both into value matching at separation, $J(\underline{p}) = 0$:

$$
-\beta\lambda\,(1-\psi)\,\bar{J}_0 + c_{0J}\underline{p}^{1-\alpha_0}(1-\underline{p})^{\alpha_0}(r+\delta+\psi\lambda) + k_{0J}\underline{p}^{\alpha_0}(1-\underline{p})^{1-\alpha_0}(r+\delta+\psi\lambda) = -(1-\beta)[\bar{\mu}(\underline{p})-b]
$$

and smooth pasting $J'(\underline{p}) = 0$:

$$
\frac{(1-\beta)(\mu_H-\mu_L)}{r+\delta+\psi\lambda} + c_{0J}\underline{p}^{-\alpha_0}(1-\underline{p})^{\alpha_0-1}(1-\alpha_0-\underline{p}) + k_{0J}\underline{p}^{\alpha_0-1}(1-\underline{p})^{-\alpha_0}(\alpha_0-\underline{p}) = 0.
$$

Finally iterate over values of $\bar{J}_0$ and corresponding vector $\{c_{0J}, k_{0J}, c_{1J}\}$ until the last two equations yield the same value of $\underline{p}$.

Proof of Lemma 1. For the ODE solved by $\mathcal{T}(p)$ see Karlin and Taylor (1986, Chapt. 15). The general solution in the claim can be verified directly. The boundary conditions are those stated in the claim plus $\mathcal{T}(1) = 1/\delta$, because a job that is good for sure can be destroyed only exogenously ($p = 1$ is an absorbing belief as $\Sigma(1) = 0$). By the same reasoning as in the previous proof this implies $k_{1\mathcal{T}} = 0$. To see why $\mathcal{T}$ is increasing, notice that $\mathcal{T}(\cdot) \leq 1/\delta$ because job destruction is always a risk, with strict inequality somewhere, so from the general solution we get $c_{1\mathcal{T}} < 0$. Next:

$$
\begin{aligned}
\mathcal{T}'(p) \;=\; & \frac{1}{\delta+\psi\lambda}\left[c_{0\mathcal{T}}p^{-\alpha_0}(1-p)^{\alpha_0-1}(1-\alpha_0-p) + k_{0\mathcal{T}}p^{\alpha_0-1}(1-p)^{-\alpha_0}(\alpha_0-p)\right]\mathbb{I}\{\underline{p} \leq p < p_0\} \\
& + \frac{1}{\delta}c_{1\mathcal{T}}p^{-\alpha_1}(1-p)^{\alpha_1-1}(1-\alpha_1-p)\mathbb{I}\{p_0 \leq p \leq 1\}
\end{aligned}
$$

so $\mathcal{T}'(p) > 0$ for $p > p_0$ by $1-\alpha_1-p < 1-\alpha_1 < 0$. By contradiction, suppose $0 \geq \mathcal{T}'(p)$ for some $p \in (\underline{p}, p_0)$. Since $\mathcal{T}'(\underline{p}) \geq 0$ and $\mathcal{T}'(p_0-) = \mathcal{T}'(p_0+) > 0$, by continuity of $\mathcal{T}'$ in $[\underline{p}, p_0]$ and the Mean Value Theorem either there is only one such p, with $\mathcal{T}'(p) = 0$, equivalent to:

$$
c_{0\mathcal{T}} = -k_{0\mathcal{T}}\left(\frac{p}{1-p}\right)^{2\alpha_0-1}\frac{\alpha_0-p}{\alpha_0+p-1}, \tag{A.1}
$$

in which case the claim obtains, or there are two roots of (A.1). But tedious algebra shows that the RHS of (A.1) is either globally increasing or decreasing in p, according to the sign of $k_{0\mathcal{T}}$, hence (A.1) may have at most one root.

Proof of Proposition 4. Let $\nu(p) = p^2(1-p)^2 f(p)$; we obtain from (4.1) a familiar-looking ODE:

$$p^2(1-p)^2\nu''(p) = \frac{2(\delta + \psi\lambda\mathbb{I}\{p < p_0\})}{s^2}\nu(p).$$

The general solution is:

$$\nu(p) = \nu_i(p) = \tilde{c}_{if}p^{1-\gamma_i}(1-p)^{\gamma_i} + \tilde{k}_{if}p^{\gamma_i}(1-p)^{1-\gamma_i}$$

for $i = \mathbb{I}\{p \geq p_0\}$. Therefore the ergodic density is:

$$f_i(p) = \nu(p)p^{-2}(1-p)^{-2} = \tilde{c}_{if}p^{-1-\gamma_i}(1-p)^{\gamma_i-2} + \tilde{k}_{if}p^{\gamma_i-2}(1-p)^{-1-\gamma_i}$$

the sum of Inverted-Beta-1 functions. Integrating $f_1(p)$ between p_0 and 1 yields an exploding second term in f, because $\int_{p_0}^{p}(1-x)^{-1-\gamma_1}\,dx = (\gamma_1)^{-1}(1-x)^{-\gamma_1}\,|_{p_0}^{p} \to \infty$ as $p \uparrow 1$ by $\gamma_1 > 0$. Hence we must have $\tilde{k}_{1f} = 0$ to satisfy the requirement that the density has finite mass $0 < \int_{\underline{p}}^{1} f(z)\,dz < 1 < \infty$. In contrast, $\int_{p_0}^{1}(1-x)^{\gamma_1-2}\,dx < \infty$ by $2-\gamma_1 < 1$, or $\gamma_1 > 1$, so $\tilde{c}_{1f}$ can be non-zero. Next let:

$$\xi_1 = \left(\frac{1-\underline{p}}{\underline{p}}\right)^{2\gamma_0-1}$$

By the change of variable $p' = p/(1-p)$:

$$\begin{aligned}
\xi_2 &\equiv \int_{\underline{p}}^{p_0} p^{-1-\gamma_0}(1-p)^{\gamma_0-2}dp = \int_{\frac{\underline{p}}{1-\underline{p}}}^{\frac{p_0}{1-p_0}} \left(\frac{p'}{1+p'}\right)^{-1-\gamma_0}\left(\frac{1}{1+p'}\right)^{\gamma_0-2}\frac{dp'}{(1+p')^2} \\
&= \int_{\frac{\underline{p}}{1-\underline{p}}}^{\frac{p_0}{1-p_0}} (p')^{-1-\gamma_0}(1+p')dp' = \frac{\left(\frac{\underline{p}}{1-\underline{p}}\right)^{-\gamma_0} - \left(\frac{p_0}{1-p_0}\right)^{-\gamma_0}}{\gamma_0} + \frac{\left(\frac{\underline{p}}{1-\underline{p}}\right)^{1-\gamma_0} - \left(\frac{p_0}{1-p_0}\right)^{1-\gamma_0}}{\gamma_0 - 1}.
\end{aligned}$$

Notice that the mean of f can be obtained analogously. Similarly:

$$\xi_3 \equiv \int_{\underline{p}}^{p_0} p^{\gamma_0-2}(1-p)^{-1-\gamma_0}dp = \frac{\left(\frac{p_0}{1-p_0}\right)^{\gamma_0-1} - \left(\frac{\underline{p}}{1-\underline{p}}\right)^{\gamma_0-1}}{\gamma_0-1} + \frac{\left(\frac{p_0}{1-p_0}\right)^{\gamma_0} - \left(\frac{\underline{p}}{1-\underline{p}}\right)^{\gamma_0}}{\gamma_0}.$$

$$\begin{aligned}
\xi_4 &\equiv \int_{p_0}^{1} p^{-1-\gamma_1}(1-p)^{\gamma_1-2}dp = \frac{1}{\gamma_1}\left(\frac{p_0}{1-p_0}\right)^{-\gamma_1} + \frac{1}{\gamma_1-1}\left(\frac{p_0}{1-p_0}\right)^{1-\gamma_1} \\
\xi_5 &\equiv \frac{s^2}{2}\underline{p}^{-\gamma_0}(1-\underline{p})^{\gamma_0-1}(1-2\gamma_0) \\
\xi_6 &\equiv \frac{s^2}{2}\left[p_0^{-\gamma_0}(1-p)^{\gamma_0-1}(3p_0-1-\gamma_0) - \xi_1 p_0^{\gamma_0-1}(1-p)^{-\gamma_0}(3p_0-2+\gamma_0)\right].
\end{aligned}$$

The boundary conditions then read:

1. no time spent at the separation boundary $f(\underline{p}+) = 0$:

$$\tilde{k}_{0f} = -\xi_1 \tilde{c}_{0f}. \tag{A.2}$$

   which implies $\tilde{c}_{0f}\tilde{k}_{0f} < 0$. Using (A.2) replace $\tilde{k}_{0f}$ out. Thus:

$$\int_{\underline{p}}^{1} f(p)dp = \tilde{c}_{0f}\left(\xi_2 - \xi_1\xi_3\right) + \tilde{c}_{1f}\xi_4$$

$$\begin{aligned}
\Sigma(\underline{p})f'(\underline{p}+) &= \tilde{c}_{0f}\frac{s^2\underline{p}^2(1-\underline{p})^2}{2}\Big[\underline{p}^{-2-\gamma_0}(1-\underline{p})^{\gamma_0-3}(3\underline{p}-1-\gamma_0) \\
&\quad + \tilde{k}_{0f}\underline{p}^{\gamma_0-3}(1-\underline{p})^{-2-\gamma_0}(3\underline{p}+\gamma_0-2)\Big] \\
&= \tilde{c}_{0f}\frac{s^2}{2}\underline{p}^{-\gamma_0}(1-\underline{p})^{\gamma_0-1}\left[3\underline{p}-1-\gamma_0-(3\underline{p}+\gamma_0-2)\right] = \tilde{c}_{0f}\xi_5.
\end{aligned}$$

2. Balance of flows in and out of employment:

$$\tilde{c}_{0f}\xi_6 = \psi\lambda\tilde{c}_{0f}\left(\xi_2 - \xi_1\xi_3\right) + \delta[\tilde{c}_{0f}\left(\xi_2 - \xi_1\xi_3\right) + \tilde{c}_{1f}\xi_4] + \tilde{c}_{0f}\xi_5.$$

3. Balance of flows in and out of unemployment.

$$\lambda[\mu - \tilde{c}_{0f}\left(\xi_2 - \xi_1\xi_3\right) - \tilde{c}_{1f}\xi_4] = \delta[\tilde{c}_{0f}\left(\xi_2 - \xi_1\xi_3\right) + \tilde{c}_{1f}\xi_4] + \tilde{c}_{0f}\xi_5.$$

To obtain the expression in claim, let $c_{0f} = -\tilde{c}_{0f}$ and $c_{1f} = \tilde{c}_{1f}$. So $\Xi\binom{c_{0f}}{c_{1f}} = \binom{\lambda}{0}$, where:

$$\Xi \equiv \begin{pmatrix} -\left(\xi_2 - \xi_1\xi_3\right)\left(\lambda+\delta\right) - \xi_5, & \left(\lambda+\delta\right)\xi_4 \\ \xi_6 - (\psi\lambda+\delta)(\xi_2-\xi_1\xi_3) - \xi_5, & \delta\xi_4 \end{pmatrix}. \tag{A.3}$$

The solution is:

$$\begin{aligned}
c_{0f} &= \frac{\lambda\delta}{\left(\xi_2 - \xi_1\xi_3\right)\left(\lambda+\delta\right)\psi\lambda + \lambda\xi_5 - \left(\lambda+\delta\right)\xi_6} \\
c_{1f} &= \frac{\lambda}{\xi_4}\frac{\xi_5 + (\psi\lambda+\delta)(\xi_2-\xi_1\xi_3) - \xi_6}{\left(\xi_2 - \xi_1\xi_3\right)\left(\lambda+\delta\right)\psi\lambda + \lambda\xi_5 - \left(\lambda+\delta\right)\xi_6}
\end{aligned}$$

Finally, a substantial amount of algebra (omitted) shows that the boundary conditions also imply that the density is continuous at p_0: $f(p_0-) = f(p_0+)$, therefore the inflow at p_0 creates a kink but not a jump in the density. This also implies $c_{0f}, c_{1f} > 0$ by a simple contradiction argument using $c_{0f}k_{0f} > 0$ found earlier.

Proof of Proposition 5. Since the worker-firm pair cannot produce more than μ_H flow expected output, $J(p_0|\lambda)$ is bounded uniformly in λ above by $\mu_H(r+\delta)^{-1}$. Then $\lim_{\lambda\to\infty} J(p_0|\lambda)\lambda^{-\frac{\eta}{1-\eta}} = 0$.

When $\lambda = 0$, the following facts are true: $\alpha_0 = \alpha_1$, after manipulation of value matching and smooth pasting at p_0 we have $k_{0J} = c_{1J}(\alpha_0 - \alpha_1) = 0$, $c_{0J} = c_{1J}$, and finally for all $p \geq \underline{p}$

$$J(p|0) = \frac{(1-\beta)[\bar{\mu}(p) - b]}{r+\delta} + c_{1J}p^{1-\alpha_1}(1-p)^{\alpha_1}.$$

We show that this implies $J(p_0|0) > 0$ and therefore $\lim_{\lambda\to 0} J(p_0|\lambda)\lambda^{-\frac{\eta}{1-\eta}} = \infty$. By contradiction suppose

$$J(p_0|0) = 0 = \frac{(1-\beta)[\bar{\mu}(p_0) - b]}{r+\delta} + c_{1J}p_0^{1-\alpha_1}(1-p_0)^{\alpha_1}.$$

The assumption $\bar{\mu}(p_0) \geq b$ then implies $c_{1J} \leq 0$. By value matching this also implies $\underline{p} = p_0$, and then by smooth pasting

$$J'(p_0|0) = 0 = \frac{(1-\beta)(\mu_H - \mu_L)}{r+\delta} + c_{1J}p_0^{-\alpha_1}(1-p_0)^{\alpha_1 - 1}(1-\alpha_0 - p_0)$$

which implies $c_{1J} > 0$, the desired contradiction, because $\mu_H - \mu_L > 0$ and $\alpha_0 > 1$.

From these two limits and from continuity, it follows that the required fixed point $\lambda^* > 0$ always exists.

To establish that it is unique, it suffices to show that the profit curve $J(p_0|\lambda)$ is decreasing in λ, because then it cuts exactly once the job creation curve. Consider two values λ_0 and λ_1 with $\lambda_1 > \lambda_0 > 0$. By contradiction, suppose that $J(p_0|\lambda_1) \geq J(p_0|\lambda_0)$. Then $\lambda_1 J(p_0|\lambda_1) > \lambda_0 J(p_0|\lambda_0)$. This implies

$$J\left(1|\lambda_1\right) = \frac{(1-\beta)(\mu_H - b) - \beta\lambda J(p_0|\lambda_1)}{r+\delta} < \frac{(1-\beta)(\mu_H - b) - \beta\lambda J(p_0|\lambda_0)}{r+\delta} = J\left(1|\lambda_0\right),$$

so by continuity there exists $p' \in [p_0, 1)$ such that $J(p'|\lambda_1) = J(p'|\lambda_0)$ and $J'(p'|\lambda_1) \leq J'(p'|\lambda_0)$. Now there are two possibilities.

First, for all $p < p'$, $J'(p|\lambda_1) \leq J'(p|\lambda_0)$, which implies $J(p|\lambda_1) > J(p|\lambda_0)$ and $\underline{p}(\lambda_1) < \underline{p}(\lambda_0)$. From the ODE (3.6) solved by $J(.|\lambda)$ for $\lambda = \lambda_0, \lambda_1$:

$$\Sigma(p)J''(p|\lambda) = \left[r + \delta + \psi\lambda\mathbb{I}\left\{p < p_0\right\}\right] J(p|\lambda) - (1-\beta)[\bar{\mu}(p) - b] + \beta\lambda J(p_0|\lambda)(1 - \psi\mathbb{I}\left\{p < p_0\right\}).$$

we deduce $J''(p|\lambda_1) > J''(p|\lambda_0)$ for all $p > \underline{p}(\lambda_0)$ and then from $J'(\underline{p}(\lambda_0)|\lambda_1) > 0 = J'(\underline{p}(\lambda_0)|\lambda_0)$ we obtain $J'(p|\lambda_1) > J'(p|\lambda_0)$, a contradiction.

Second, there exists $p'' < p'$ such that $J'(p''|\lambda_1) > J'(p''|\lambda_0)$ and $J(p''|\lambda_1) = J(p''|\lambda_0)$. But then again from the ODE above we infer $J''(p|\lambda_1) > J''(p|\lambda_0)$ at $p = p''$ and, by the instability of the ODE, for all larger values of p, implying the same contradiction: $J'(p|\lambda_1) > J'(p|\lambda_0)$ for all values of $p > p''$. ■

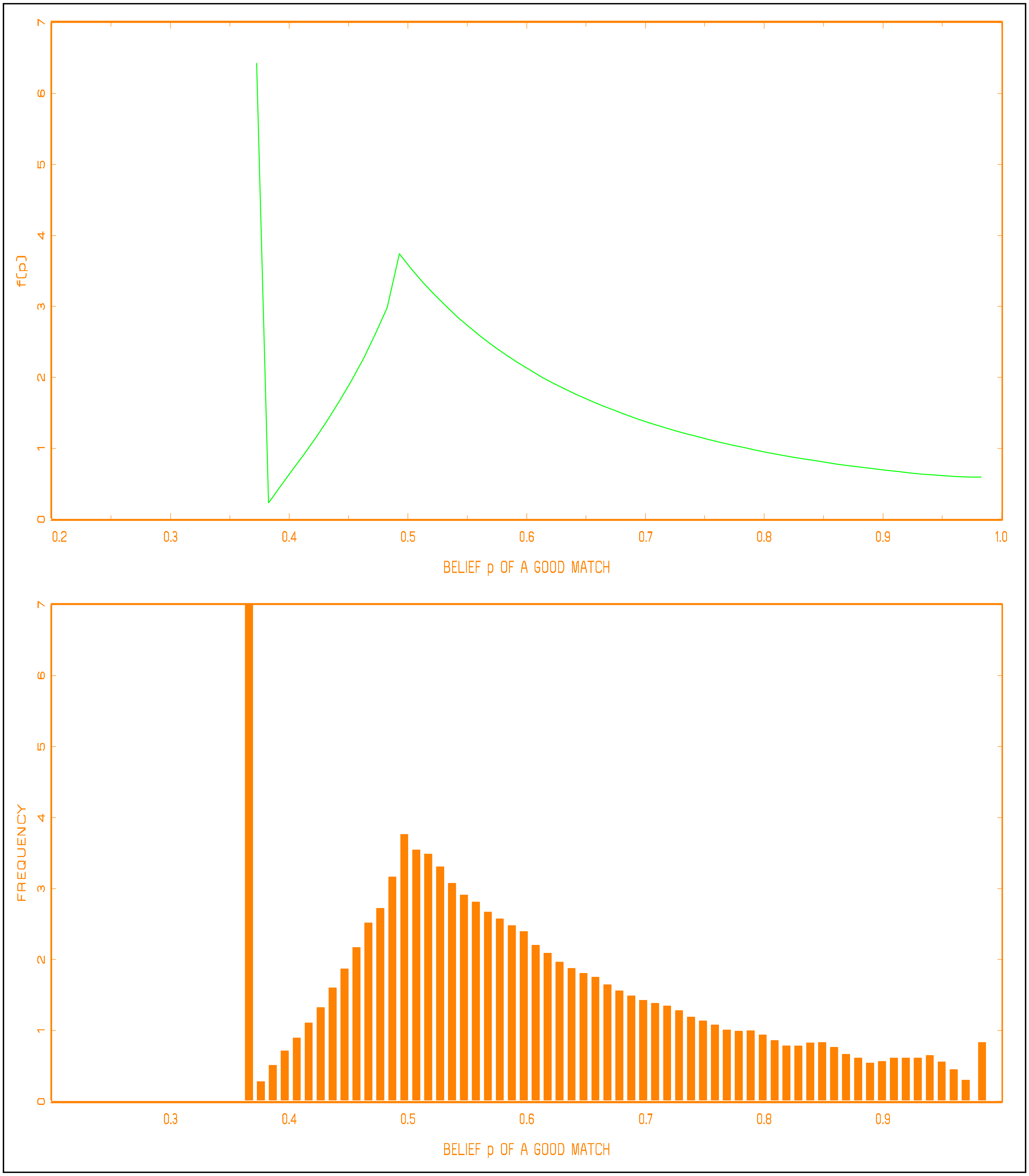

Figure A.1: Ergodic distribution of posterior beliefs about match quality (upper panel) and frequency distribution of simulated belief process (lower panel). In both panels, the atom at the lower bound $\underline{p}$ is the mass of unemployed workers.