Econ 712 Macroeconomic Theory- Midterm Exam. (Solution)

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1 Instructions

- Please answer all questions. If you get stuck in one section move to the next one. Do not waste a lot of time on questions that you find hard to solve.
- Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- This is a closed book exam. Students may bring one page (both sides) or two pages (single sides) of notes.
- Please hand in the exam promptly at 10:45 AM.
- Each question is worth 50 points. The point total for each section is indicated at the beginning of the section. Look at these "prices" when deciding how to allocate your time!!
- If you believe that a question is wrong or poorly worded, please make the "minimal" necessary changes to make it "beautiful" and well posed. Of course, unnecessary changes will result in a lower grade.
- *Suggestion*: Do not try the extra credit sections until you finished the regular exam!
- Please use one blue book for each question and write only on the "right" page. (The odd numbered page in a newspaper.)
- Please remember to put your name in each blue book.
- Good luck !

2 Questions

Problem 1 (Home Production and Aggregate Output) *(50 points) Consider an economy populated by a large number of identical individuals. Each household has preferences given by*

$$\sum_{t=0}^{\infty} \beta^t u(c_t^{\mu} m_t^{1-\mu}), \quad 0 < \mu < 1, \quad 0 < \beta < 1,$$

where c_t denotes consumption of market goods, and m_t denotes consumption of home produced goods. The function u is assumed to be strictly concave, strictly increasing and as many times differentiable as needed. Assume that each household produces home goods according to the following technology,

$$m_t \leq A_m z_t^{\phi}(1-n_t)^{1-\phi}, \quad 0 < \phi < 1,$$

where z_t is market goods used in the production of home goods, and $1 - n_t$ is the fraction of the household's time allocated to producing home goods. Each household is assumed to maximize utility subject to the following sequence of budget constraints

$$\begin{aligned} c_t + z_t + x_t + b_{t+1} &\leq r_t k_t + (1-\tau) w_t n_t + R_t b_t + T_t, \\ k_{t+1} &\leq (1-\delta) k_t + x_t, \end{aligned}$$

where k_t is the stock of capital, b_t is the stock of bonds, r_t and w_t are the rental prices of capital and labor, τ ($0 < \tau < 1$) is a tax rate on labor income, and T_t is a transfer from the government. We assume that the government tax revenue is used to transfer resources to each household. Following the competitive assumption, each household takes the sequence $\{T_t\}$ as given.

Assume that there is a large number of firms in this economy. Each firm maximizes profits given by

$$\pi_t = F(k_t, n_t) - r_t k_t - w_t n_t,$$

where F is assumed concave (as many times differentiable as you need) and homogeneous of degree one. Given these assumptions, the ***equilibrium*** *level of profits is zero.*

i) (10 points)Define a competitive equilibrium.

ii) (15 points) Assume that the steady state exists and is unique. Analyze the following proposition: An increase in the productivity of home production — an increase in A_m— frees up household time that can be used to produce more market goods. Thus, the steady state level of output (of market goods) increases. ***Note****: This is a question that asks you to compare different steady states. You may assume that all variables are interior (and this includes n). Do not waste time checking interiority.*

iii) (15 points) Analyze this claim: A change in the tax rate of labor income will change real wages in the steady state.

iv) (10 points) Argue (this could be tricky) that this economy will converge to a steady state. ***Hint:*** *To prove this you need to set up a "two planner" problem.*

Extra Credit: *Suppose that the utility function was given by*

$$\hat{u}(c,m) = u([\mu c^{-\rho} + (1-\mu)m^{-\rho}]^{-1/\rho}), \qquad \rho > -1.$$

What is your answer to ii)?

Solution (sketch): i) Let the household solve the following problem

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t^{\mu} m_t^{1-\mu}),$$

subject to,

$$m_t \leq A_m z_t^{\phi}(1-n_t)^{1-\phi},$$

$$\begin{aligned} c_t + z_t + x_t + b_{t+1} &\leq r_t k_t + (1-\tau)w_t n_t + R_t b_t + T_t, \\ k_{t+1} &\leq (1-\delta)k_t + x_t. \end{aligned}$$

Let the representative firm solve the following problem

$$\max c_t + z_t + x_t - w_t n_t - r_t k_t,$$

subject to,

$$c_t + z_t + x_t \leq F(k_t, n_t).$$

Define a competitive equilibrium as a collection of price sequences $[\{w_t^*\}, \{r_t^*\}, \{R_t^*\}]$, $t = 0, 1, ...$, an allocation $[\{c_t^*\}, \{x_t^*\}, \{n_t^*\}, \{z_t^*\}, \{m_t^*\}, \{k_{t+1}^*\}]$, $t = 0, 1, ...$, and a sequence of bond holdings $\{b_{t+1}^*\}$ such that,

a) Given prices, the allocation and the sequence $\{b_{t+1}^*\}$ solve the household's maximization problem. [Utility maximization].

b) Given prices, the allocation solves the firm's maximization problem [profit maximization].

c) The allocation is feasible [market clearing]

$$\begin{aligned} m_t &\leq A_m z_t^{\phi}(1-n_t)^{1-\phi}, \\ k_{t+1} &\leq (1-\delta)k_t + x_t, \\ c_t + z_t + x_t &\leq F(k_t, n_t) \end{aligned}$$

d) $b_0^* = b_0 = 0, k_0^* = k_0 > 0$ is given.

ii) Taking the first order conditions from the consumer's problem and substituting in the equilibrium prices (i.e. $w_t = F_n(k_t, n_t)$ and $r_t = F_k(k_t, n_t)$) one gets the following first order conditions (here λ_t is the marginal utility of wealth)

$$\begin{aligned} u'(c_t^{\mu}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu})\mu c_t^{\mu-1}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu} &= \lambda_t \\ u'(c_t^{\mu}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu})\phi(1-\mu)c_t^{\mu}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu}/z_t &= \lambda_t \\ u'(c_t^{\mu}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu})(1-\phi)(1-\mu)c_t^{\mu}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu}/(1-n_t) &= \lambda_t(1-\tau)F_n(k_t, n_t) \\ \beta\lambda_{t+1}[1-\delta+F_k(k_{t+1}, n_{t+1})] &= \lambda_t. \end{aligned}$$

Simplification of the first three conditions show that they are equivalent to

$$\begin{aligned}\mu z_t &= \phi(1-\mu)c_t, \\ (1-\phi)(1-\mu)c_t &= \mu(1-\tau)F_n(k_t,n_t)(1-n_t).\end{aligned}$$

Thus, at the steady state, it must be the case that

$$\begin{aligned}\mu z^* &= \phi(1-\mu)c^* \\ (1-\phi)(1-\mu)c^* &= \mu(1-\tau)F_n(k^*,n^*)(1-n^*) \\ 1 &= \beta[1-\delta+F_k(k^*,n^*)] \\ c^*+z^*+\delta k^* &= F(k^*,n^*).\end{aligned}$$

Since A_m is **not** an argument of this system, changes in A_m cannot possibly have any impact on the steady state allocation. The reason is simple: Changes in A_m corresponds to changes in the utility function that do not affect the marginal rate of substitution between goods; as such, it cannot have any impact on the steady state allocations.

iii) The condition

$$1=\beta[1-\delta+F_k(k^*,n^*)]$$

pins down the capital labor ratio given that F is homogeneous of degree one. Since $w=F_n(k^*,n^*)$ is a function of the capital labor ratio, then changes in τ will have no impact on before tax wages.

iv) Consider the following planner's problem

$$\max\sum_{t=0}^{\infty}\beta^t u(c_t^{\mu}(A_m z_t^{\phi}(1-n_t)^{1-\phi})^{1-\mu}),$$

subject to,

$$c_t+z_t+k_{t+1}\leq(1-\tau)F(k_t,n_t)+\theta_t k_t+(1-\delta)k_t+T_t,$$

for given sequences $\{\theta_t\}$ and $\{T_t\}$. This problem is clearly well defined. Moreover if $\{\theta_t\}=\{\tau F_k(k_t^*,n_t^*)\}$where $\{(k_t^*,n_t^*)\}$ is the sequence of chosen (i.e. optimal) values of the capital stock and employment in the goods producing sector. This is a standard —albeit complicated— planner's problem. The solution converges.

Problem 2 (Health and Dynamics) *(50 points) Consider a representative household economy. Preferences are given by*

$$\sum_{t=0}^{\infty}\beta^t u(c_t),\quad 0<\beta=\frac{1}{1+\rho}<1,$$

where c_t denotes consumption. The function u is assumed to be strictly concave, strictly increasing and as many times differentiable as needed. The aggregate feasibility constraints in this economy are given by,

$$\begin{aligned}c_t+x_t &\leq f(k_t), \\ k_{t+1} &\leq (1-\delta)k_t+x_t,\end{aligned}$$

where f is increasing, strictly concave and as many times differentiable as necessary.

i) (10 points)Define a competitive equilibrium in which all households inelastically supply one unit of labor and have the same initial wealth.

ii) (15 points) Assume that the initial capital stock, k_0, is small. Go as far as you can describing the time path of interest rates and wage rates in this economy.

ii)(25 points) Let the discount factor, β, capture both "pure" time discount, and the probability of dying. More specifically, interpret an "improvement in health care" —a lower probability of dying— as a decrease in ρ (an increase in β). Assume that at $t = 0$ the economy learns that at $t = T$ there will be an "improvement in health care" (i.e. a decrease in ρ). Go as far as you can describing the impact of this announcement on the time path of interest rates and wages

Solution (sketch): i) and ii) These are standard. The appropriate answer can be found in the notes. I will discuss the solution to iii). To describe the dynamic behavior define the following two loci

$$\begin{aligned} c(\lambda_1(k,\rho)) &\equiv f(k) - \delta k, \\ c(\lambda_2(k,\rho)) &\equiv f(k) - \delta k + k - k^*(\rho), \end{aligned}$$

where $k^*(\rho)$ is the steady state level of capital per worker corresponding to a discount factor equal to ρ. It is given by the solution to

$$\rho + \delta \equiv f'(k^*(\rho)).$$

It follows that $k^*(\rho)$ is a decreasing function of ρ. Thus, from the previous equations it is clear that a decrease in ρ from ρ to ρ' will have no effect on the $\lambda_1(k,\rho)$ locus, and it will shift up the $\lambda_2(k,\rho)$ locus.

In Figure 1 both sets of manifolds are displayed. In addition, the figure shows two possible paths. Path A corresponds to the case in which T is relatively short. In this case, consumption decreases in all periods until time T, while the capital stock increases. Since consumption is decreasing until T the interest rate —given by $R_{t+1} = \beta^{-1}u'(c_t)/u'(c_{t+1})$ is less than β^{-1}, the long run interest rate. Since k_t is increasing, the interest rate is decreasing. The wage rate is an increasing function of the stock of capital per worker and it increases over the whole period.

Path B corresponds to the "large" T case. Since the capital stock is increasing —although at a slower pace— the interest rate is decreasing. However, in this case, initially interest rates exceed β^{-1} as consumption increases; in the last few periods before T the interest rate is below β^{-1}.

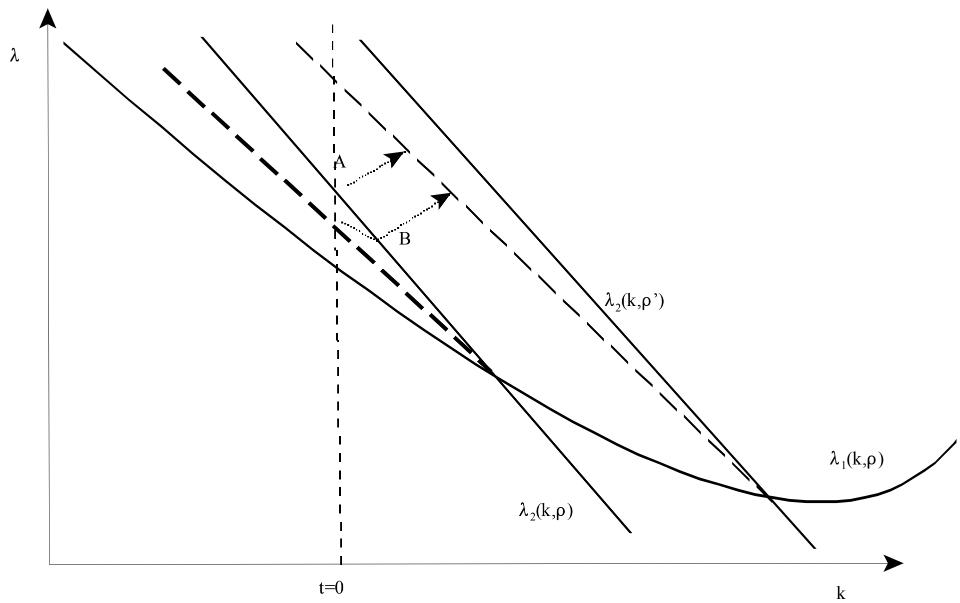

Figure 1: