Econ 712 Macroeconomic Theory- First Exam. University of Wisconsin

Instructor: Rody Manuelli

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1 Instructions

- Please answer all questions. If you get stuck in one section move to the next one. Do not waste time on questions that you find hard to solve.
- Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- This is a closed book exam. Students may bring one page (both sides) or two pages (single sides) of notes.
- Please hand in the exam promptly at 12 Noon
- Each question is worth 50 points. The point total for each section is indicated at the beginning of the section. Look at these "prices" when deciding how to allocate your time!!
- If you believe that a question is wrong or poorly worded, please make the "minimal" necessary changes to make it "beautiful" and well posed. Of course, unnecessary changes will result in a lower grade.
- Please use one blue book for each question and write only on the "right" page. (The odd numbered page in a newspaper.)
- Please remember to **put your name** in each blue book.
- Good luck !

2 Questions

Problem 1 (Habit Persistence) *Consider an economy populated by a large number of households with utility functions given by*

$$\sum_{t=0}^{\infty} \beta^t u(c_t - \phi z_t), \qquad 0 < \beta < 1,\ \phi \geq 0$$

where $u : \mathbb{R}_+ \to \mathbb{R}$, *is twice differentiable, increasing and strictly concave (if necessary you may assume that it satisfies the Inada condition). The variable* c_t *is individual consumption, and* z_t *is a measure of lagged consumption. To be precise,*

$$z_t \geq \sum_{j=0}^{\infty} (1 - \delta_c)^j c_{t-1-j}, \qquad 0 \leq \delta_c \leq 1.$$

It follows that, alternatively, it is possible to describe the law of motion for z_t *as*

$$z_{t+1} \geq (1 - \delta_c) z_t + c_t.$$

In this setting, z_t *is a measure of 'habit persistence,' as it implies that the marginal utility of any given level of consumption decreases the higher the level of past consumption. The technology in this economy is standard and given by*

$$\begin{aligned} c_t + x_t &\leq f(k_t), \\ k_{t+1} &\leq (1 - \delta_k) k_t + x_t, \end{aligned}$$

where the functions f *is strictly concave, increasing and satisfies Inada conditions. Assume* $\delta_k \in (0, 1)$

1. *(15 points) Let the planner maximize the utility of the representative agent subject to all the feasibility constraints. Argue that, under some condition on* (ϕ, δ_c) *an* ***interior*** *steady state exists and is unique. Describe the condition that* (ϕ, δ_c) *has to satisfy.*

2. *(10 points) What does the model say about the impact of cross-country differences in how much people care about past consumption —as measured by* ϕ*— on the steady state output per worker.*

3. *(10 points) Define a competitive equilibrium in which, in each period, households trade (at least) one period bonds, capital, consumption and investment goods. Assume that consumption is taxed at the rate* τ*, that is, the cost of purchasing* c *units of consumption is* $(1 + \tau)c$*. The revenue produced by this tax is rebated in a lump-sum fashion to the households.*

4. *(15 points) Economist A argues that current consumption produces an 'externality,' in the sense that it lowers the marginal utility of future consumption. Given this, he/she suggests that a tax on consumption (with proceeds rebated to the consumer in lump-sum fashion) will guarantee that the **steady state** of the competitive equilibrium of this economy will coincide with the **steady state** of the planner's problem, and that the tax rate that attains this equality of the two steady states minimizes the value of z_t (at the steady state). Go as far as you can analyzing this claim.*

***Note**: Section 3 requires you to describe the steady state version of the competitive equilibrium with constant consumption taxes in this economy. You need not need to show that every competitive equilibrium converges to a steady state. It suffices to assume that a steady state exists, and derive its properties.*

Solution 2 (Sketch) *The Lagrangian corresponding to the planner's problem (ignoring the non-negativity constraints) is*

$$\begin{aligned}\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \{u(c_t - \phi z_t) + \lambda_t[f(k_t) + (1-\delta_k)k_t - c_t - k_{t+1}] + \\ & \theta_t[z_{t+1} - (1-\delta_c)z_t - c_t]\}.\end{aligned}$$

The first order conditions are:

$$\begin{aligned} c_t &: \quad u'(c_t - \phi z_t) = \lambda_t + \theta_t, \\ k_{t+1} &: \quad \lambda_t = \beta\lambda_{t+1}[(1-\delta_k) + f'(k_{t+1})], \\ z_{t+1} &: \quad \theta_t = \beta\theta_{t+1}(1-\delta_c) + \phi\beta u'(c_{t+1} - \phi z_{t+1}), \end{aligned}$$

and the feasibility constraints at equality. Note that the consumer would like to choose z_t as small as possible, since this lowers utility; it follows that the law of motion of this variable must hold as an equality. In a steady state all variables and Lagrange multipliers are constant. Thus, at a steady state (if one exists), it follows that

$$\begin{aligned} \rho + \delta_k &= f'(k^*), \\ c^* &= f(k^*) - \delta_k k^*, \\ \delta_c z^* &= c^*. \end{aligned}$$

Existence of a unique strictly positive vector that satisfies the first two equations follows from standard conditions on the production function. It remains to check that $c^* - \phi z^* > 0$. However, this is equivalent to $c^*(1 - \phi/\delta_c) > 0$. A necessary and sufficient condition is that $\phi/\delta_c < 1$.

The model implies that the steady state level of output per worker is independent of ϕ.

In a competitive equilibrium, households solve

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t - \phi z_t),$$

subject to

$$\begin{aligned} (1+\tau)c_t + x_t + b_{t+1} &\leq w_t n_t + r_t k_t + R_t b_t + v_t, \\ z_{t+1} &\geq (1-\delta_c) z_t + c_t, \\ k_{t+1} &\leq (1-\delta_k) k_t + x_t \\ \lim_{T\to\infty} \Pi_{j=t}^{T} R_j^{-1} b_{T+1} &\geq 0, \qquad n_t \leq 1 \end{aligned}$$

where v_t is a transfer received from the government.

Firms solve

$$\max c_t + p_{kt} x_t - w_t n_t - r_t k_t,$$

subject to

$$c_t + x_t \leq F(k_t, n_t)$$

where $F(k,1) = f(k)$, and F is homogeneous of degree one.

An equilibrium is an allocation $[\{c_t^*\}, \{x_t^*\}, \{k_{t+1}^*\}, \{n_t^*\}]_{t=0}^{\infty}$, a price system $[\{R_{t+1}^*\}, \{p_{kt}^*\}, \{w_t^*\}, \{r_t^*\}]$ and a sequence of bond holdings $\{b_{t+1}^*\}$ such that

1. *Given prices, households maximize utility.*
2. *Given prices, firms maximize profits.*
3. *Markets clear.*

The first order conditions for utility maximization are (evaluated at the steady state)

$$\begin{aligned} u'(c^* - \phi z^*) &= (1+\tau)\lambda_H^* + \theta_H^*, \\ \lambda_H^* &= \beta \lambda_H^* [(1-\delta_k) + r^*], \\ \lambda_H^* &= \beta \lambda_H^* R^*, \\ \delta_c z^* &= c^*. \end{aligned}$$

Given that $r^* = f'(k^*)$ from the firm's problem, and that feasibility holds, it follows that

$$\begin{aligned} \rho + \delta_k &= f'(k^*), \\ c^* &= f(k^*) - \delta_k k^*. \end{aligned}$$

Thus, the steady state of a competitive equilibrium coincides with the solution of the planner's problem for all tax rates. The reason for this is simple: In the steady *state, habit persistence does not play any role in determining consumption since the marginal rate of substitution is given by the discount factor. Moreover, with inelastic labor supply, consumption taxes are not distortionary. For this to be the case, they would have to create a wedge between current and future consumption. However, a constant tax rate does not distort the choice between present and future consumption, as both are taxed at the same rate.*

Problem 3 (Housing Prices) *Consider an economy populated by a large number of identical households with utility function given by*

$$E_0\{\sum_{t=0}^{\infty}\beta^t u(c_t, h_t)\}, \qquad 0 < \beta < 1,$$

where c_t is consumption of a nondurable good, and h_t is consumption of housing. The function u is twice differentiable, increasing and strictly concave (if necessary you may assume that it satisfies the Inada condition). In this economy, there are I trees per household, with each of them 'producing' d_{it} units of nondurable consumption at t. Assume that $c_t = \sum_{i=1}^{I} d_{it}$. Each household trades in shares to all the trees, bonds of all maturities, a full set of Arrow-Debreu state contingent securities and housing. One unit of housing costs q_t units of non-durable good. Thus, the total cost of a house of 'size' h_t is $q_t h_t$. In equilibrium, the supply of housing per household is given by a stochastic process $\{h_t\}$.

1. *(10 points) Go as far as you can describing how to price trees and houses as a function of the exogenous stochastic processes $[\{d_{it}\}, \{h_t\}]$.*

2. *(15 points) Assume that*

$$u(c, h) = \frac{[c^{\alpha}h^{1-\alpha}]^{1-\theta}}{1-\theta}, \qquad 0 < \alpha < 1,\ \theta > 0,$$

*and that $h_t = \bar{h} > 0$. Go as far as you can deriving the implications of the model for house prices. How is q_t related to the value of the 'stock market.' **Note**: define the value of the stock market as $p_t = \sum_{i=1}^{I} p_{it}$, where p_{it} is the price of tree i.*

3. *(15 points) Assume that*

$$u(c, h) = \frac{[c^{\alpha}h^{1-\alpha}]^{1-\theta}}{1-\theta}, \qquad 0 < \alpha < 1,\ \theta > 0,$$

and that $h_t = \mu c_t,\ \mu > 0$. Go as far as you can deriving the implications of the model for house prices. How is q_t related to $\{R_{jt}^{-1}\}_{j=1}^{\infty}$ —the collection of prices of j-period bonds?

4. *(10 points) Let the function u be arbitrary, and let the stochastic processes $[\{d_{it}\}, \{h_t\}]$ be given. Assume that households do not own the houses they live in; they rent houses from developers. That is, in every period, they pay a certain amount of rent, r_t. Go as far as you can to derive the equilibrium per period rent and the price of a house. Does the stochastic process for q_t in this environment differ from the one you computed in 1? [Assume that the utility function and the stochastic processes are the same]*

Solution 4 (Sketch) 1. *The household solves*

$$\max E_0\{\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)\}$$

subject to

$$\begin{aligned} & c_t + q_t h_{t+1} + \sum_{j=1}^{\infty} R_{jt}^{-1} b_{jt} + \sum_{i=1}^{I} p_{it} s_{it+1} + \int_X q(x_t, x') z_t(x_t, x') dx' \\ \leq & \sum_{j=1}^{\infty} R_{j-1t}^{-1} b_{jt-1} + \sum_{i=1}^{I} s_{it+1}(p_{it} + d_{it}) + z_{t-1}(x_{t-1}, x_t) + q_t h_t. \end{aligned}$$

This budget constraint is standard, and we follow the convention that, in period t, the household sells its house (and receives $q_t h_t$), and then it purchases the house that it will live in next period. The first order conditions include (among others)

$$\begin{aligned} u_c(c_t, h_t) q_t &= \beta E_t\{u_c(c_{t+1}, h_{t+1}) q_{t+1} + u_h(c_{t+1}, h_{t+1})\}, \\ u_c(c_t, h_t) p_{it} &= \beta E_t\{u_c(c_{t+1}, h_{t+1})[p_{it+1} + d_{it+1}]\}, \\ R_{jt}^{-1} &= \beta^j E_t\left\{\frac{u_c(c_{t+j}, h_{t+j})}{u_c(c_t, h_t)}\right\}. \end{aligned}$$

Repeated substitution in the first equation implies that the price of the house is given by

$$q_t = \sum_{j=1}^{\infty} \beta^j E_t\left\{\frac{u_h(c_{t+j}, h_{t+j})}{u_c(c_t, h_t)}\right\}.$$

This expression can also be written as

$$q_t = \sum_{j=1}^{\infty} \beta^j E_t\left\{\frac{u_c(c_{t+j}, h_{t+j})}{u_c(c_t, h_t)} \frac{u_h(c_{t+j}, h_{t+j})}{u_c(c_{t+j}, h_{t+j})}\right\}.$$

Since the price of a tree satisfies

$$p_{it} = \sum_{j=1}^{\infty} \beta^j E_t\left\{\frac{u_c(c_{t+j}, h_{t+j})}{u_c(c_t, h_t)} d_{it+j}\right\},$$

it follows that houses are priced like stocks (trees) with a dividend equal to the marginal rate of substitution between consumption of nondurables and housing.

2. *Using the specific utility function and the assumption that the stock of housing is constant, it follows that*

$$q_t = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u_c(c_{t+j}, \bar{h})}{u_c(c_t, \bar{h})} \frac{(1-\alpha)c_{t+j}}{\alpha \bar{h}} \right\}.$$

Since the value of the stock market is

$$p_t = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u_c(c_{t+j}, \bar{h})}{u_c(c_t, \bar{h})} c_{t+j} \right\},$$

it follows that the value of housing is a constant proportion of the value of the stock market. More precisely, the model implies that

$$q_t \bar{h} = \frac{1-\alpha}{\alpha} p_t.$$

3. *In this case, simple calculations show that*

$$q_t = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{c_{t+j}^{-\theta}}{c_t^{-\theta}} \frac{(1-\alpha)}{\alpha \mu} \right\},$$

or

$$q_t = \frac{(1-\alpha)}{\alpha \mu} \sum_{j=1}^{\infty} R_{jt}^{-1}.$$

4. *If the household rents the house, then its budget constraint is given by*

$$\begin{aligned} & c_t + r_t h_t + \sum_{j=1}^{\infty} R_{jt}^{-1} b_{jt} + \sum_{i=1}^{I} p_{it} s_{it+1} + \int_X q(x_t, x') z_t(x_t, x') dx' \\ \leq \quad & \sum_{j=1}^{\infty} R_{j-1t}^{-1} b_{jt-1} + \sum_{i=1}^{I} s_{it+1}(p_{it} + d_{it}) + z_{t-1}(x_{t-1}, x_t), \end{aligned}$$

where r_t is the per period rent. It is immediate to verify that, in an interior solution,

$$u_h(c_t, h_t) = r_t u_c(c_t, h_t)$$

Now, since a house is identical to a tree that drops dividends given by $\{r_t\}$, the standard pricing formula for trees (as applied to houses) is

$$q_t = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u_c(c_{t+j}, h_{t+j})}{u_c(c_t, h_t)} r_{t+j} \right\},$$

or,

$$q_t = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u_c(c_{t+j}, h_{t+j})}{u_c(c_t, h_t)} \frac{u_h(c_{t+j}, h_{t+j})}{u_c(c_{t+j}, h_{t+j})} \right\},$$

which is the formula derived in 1. Thus, in this economy, housing prices are independent of the structure of ownership.