Econ 712 Macroeconomic Theory- First Exam. University of Wisconsin

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1 Instructions

- Please answer all questions. If you get stuck in one section move to the next one. Do not waste time on questions that you find hard to solve.
- Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- This is a closed book exam. Students may bring one page (both sides) or two pages (single sides) of notes.
- Please hand in the exam promptly at 12 Noon
- Each question is worth 50 points. The point total for each section is indicated at the beginning of the section. Look at these "prices" when deciding how to allocate your time!!
- If you believe that a question is wrong or poorly worded, please make the "minimal" necessary changes to make it "beautiful" and well posed. Of course, unnecessary changes will result in a lower grade.
- *Suggestion*: Do not try the extra credit sections until you finished the regular exam! They are more "challenging" than the regular questions, and they are there so that you can show off.
- Please use one blue book for each question and write only on the "right" page. (The odd numbered page in a newspaper.)
- Please remember to put your name in each blue book.
- Good luck !

2 Questions

Problem 1 (Productivity in the Capital Goods Industry and Growth) *Consider an economy populated by a large number of households with utility functions given by*

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1,$$

where u is twice differentiable, increasing and strictly concave (if necessary you may assume that it satisfies the Inada condition). There are two goods in this economy: consumption and investment. Feasibility is completely described by

$$\begin{aligned} c_t &\leq z^1 F^1(k_{1t}, n_{1t}), \\ x_t &\leq z^2 F^2(k_{2t}, n_{2t}), \\ k_{t+1} &\leq (1-\delta)k_t + x_t, \\ k_{1t} + k_{2t} &\leq k_t, \\ n_{1t} + n_{2t} &\leq 1. \end{aligned}$$

The notation is standard: c_t is consumption at time t, x_t is the output of (new) capital goods at t, and k_{it} (n_{it}) are the quantities of capital (labor) allocated to sector i at time t. It is assumed that the functions F^i, $i = 1, 2$, are twice differentiable, concave, and homogeneous of degree one. Note that capital and labor are fully malleable and can be (costlessly) reallocated across sectors.

1. *(10 points) Define a competitive equilibrium for a closed economy in which households buy new capital goods from the capital producing firms, and rent 'old' capital to the firms in both sectors.*

2. *(10 points) Go as far as you can showing that a steady state exists.*

3. *(5 points) Go as far as you can analyzing the effect of an increase in the productivity of the capital producing sector (i.e. an increase in z^2) on the price of new capital goods (relative to consumption goods) in the steady state.*

4. *(5 points) Go as far as you can analyzing the effect of an increase in the productivity of the capital producing sector (i.e. an increase in z^2) on the capital-labor ratio in each of the two sectors in the steady state.*

5. *(10 points) Go as far as you can analyzing the effect of an increase in the productivity of the capital producing sector (i.e. an increase in z^2) on employment in each of the two sectors in the steady state.*

6. *(10 points) Consider now an economy that can trade in goods with the rest of the world but that it cannot borrow or lend. Thus, this economy can import and*

export both consumption and capital goods, but trade must be balanced. Let the international price of the consumption good be 1, and the price of the investment good be $\hat{p}_k$. Describe the steady state.

7. ***Extra Credit:*** *Consider the economy described in section 6. Assume that, at time 0, it is learned that there will be an decrease in the price of capital goods at time T.*

   (a) *Go as far as you can exploring the impact of this announcement on consumption and output in the short run. If you cannot get definitive results explain the source of the ambiguity.*

   (b) *Go as far as you can exploring the impact of this announcement on short and long interest rates. If you cannot get definitive results explain the source of the ambiguity.*

Note: *In your answer to all the sections, you may assume that the equilibrium is interior.*

Solution to Problem 1 (Sketch) The representative household solves the following problem

$$\max_{\{c_t\},\{x_t\},\{k_{t+1}\}\{b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

subject to

$$\begin{aligned}
c_t + p_{kt}x_t + b_{t+1} &\leq w_t + r_t k_t + R_t b_t \qquad t = 0, 1, ... \\
k_{t+1} &\leq (1-\delta)k_t + x_t \qquad t = 0, 1, ... \\
\lim_{T\to\infty} \beta^T u'(c_T) b_{T+1} &= 0, \\
k_0 &> 0 \text{ and } b_0 \qquad \text{given}, \\
(c_t, x_t, k_{t+1}) &\geq (0, 0, 0) \qquad t = 0, 1, ...
\end{aligned}$$

It is easier (for me) to consider the case of two firms, one in the consumption sector and the other in the capital goods sector. (Of course, given constant returns to scale this is without loss of generality.) The firm in the consumption sector solves

$$\max c_t - w_t n_{1t} - r_t k_{1t}, \tag{2}$$

subject to

$$c_t \leq z^1 F^1(k_{1t}, n_{1t}),$$

The representative firm in the capital goods sector solves

$$\max p_{kt} x_t - w_t n_{2t} - r_t k_{2t}, \tag{3}$$

subject to

$$x_t \leq z^2 F^2(k_{2t}, n_{2t}).$$

Definition 2 *A recursive competitive equilibrium is a collection of price sequences* $[\{w_t^*\}, \{r_t^*\}, \{p_{kt}^*\}, \{R_t^*\}]$, $t = 0, 1, ...,$ *an allocation* $[\{c_t^*\}, \{x_t^*\}, \{k_{1t}^*\}, \{k_{2t}^*\}, \{n_{1t}^*\}, \{n_{2t}^*\}]$, $t = 0, 1, ...,$ *and a sequence of bond holdings* $\{b_{t+1}^*\}$ *such that,*

1. *Given prices, the allocation and the sequence* $\{b_{t+1}^*\}$ *solve (1) with* $k_{1t}^* + k_{2t}^* = k_t^*$ *[utility maximization].*

2. *Given prices, the allocation solves (2) and (3) [profit maximization].*

3. *The allocation is feasible [market clearing].*

4. $b_0^* = b_0 = 0, k_0^* = k_0 > 0$ *is given.*

Given that it is assumed that the solution is interior, it is immediate to derive from the firms' first order conditions

$$z^1 F_k^1(k_{1t}, n_{1t}) = r_t, \tag{4}$$

$$z^1 F_n^1(k_{1t}, n_{1t}) = w_t, \tag{5}$$

$$p_{kt} z^2 F_k^2(k_{2t}, n_{2t}) = r_t, \tag{6}$$

$$p_{kt} z^2 F_n^2(k_{2t}, n_{2t}) = w_t. \tag{7}$$

The (relevant) first order conditions for the household is (this is corresponds to the optimal choice of capital)

$$p_{kt} u'(c_t) = \beta[(1 - \delta) p_{kt+1} u'(c_{t+1}) + u'(c_{t+1}) r_{t+1}]. \tag{8}$$

If a steady state exists, then these equations —and the feasibility constraints— must be satisfied for constant values of all the variables. Thus, (8) and (6) imply

$$1 = \beta[1 - \delta + z^2 F_k^2(\kappa_2, 1)], \tag{9}$$

where $\kappa_i = k_i/n_i$ is the capital labor ratio. This equation uniquely pins down κ_2^*. Using (4)-(7), it follows that

$$m_1(\kappa_1) = m_2(\kappa_2^*), \tag{10}$$

where

$$m_i(\kappa) \equiv \frac{F_k^i(\kappa, 1)}{F_n^i(\kappa, 1)}$$

is a strictly decreasing function. Thus, (10) pins down κ_1^*. Feasibility implies that

$$k = \kappa_1 n_1 + \kappa_2 (1 - n_1), \tag{11}$$

and

$$\delta k = z^2 F^2(\kappa_2, 1)(1 - n_1).$$

These two equations imply that

$$n_1^* = \frac{z^2F^2(\kappa_2^*,1) - \delta\kappa_2^*}{z^2F^2(\kappa_2^*,1) - \delta\kappa_2^* + \delta\kappa_1^*} \in (0,1), \tag{12}$$

and k^* is given by (11). The aggregate level of consumption is

$$c^* = z^1F^1(\kappa_1^*,1)n_1^*, \tag{13}$$

while the price of capital goods is

$$p_k^* = \frac{z^1F_k^1(\kappa_1^*,1)}{z^2F_k^2(\kappa_2^*,1)} \tag{14}$$

It follows that a steady state exists and it is unique.

Consider now the effect of an increase in z^2. From (9) it follows that $d\kappa_2/dz^2 > 0$. Next, equation (10) implies that $d\kappa_1/dz^2 > 0$. Given that (9) implies that $z^2F_k^2(\kappa_2^*,1)$ is constant, the increase in κ_1 implies that the price of capital goods decreases. Finally, the impact on n_1 is ambiguous as it depends on the elasticity of substitution.

If this is an open economy and the international price is $\hat{p}_k$ the equilibrium is the same as in the closed economy case if the solution is interior, i.e. $\hat{p}_k = p_k^*$.

Problem 3 (New Technologies and Asset Prices) *Consider an economy populated by a large number of households with utility functions given by*

$$E_0\{\sum_{t=0}^{\infty}\beta^t u(c_t)\}, \qquad 0 < \beta < 1,$$

where u is twice differentiable, increasing and strictly concave (if necessary you may assume that it satisfies the Inada condition). Initially, in this economy there is only one technology —which we label technology 1. Output of the consumption good using this technology satisfies,

$$y_{1t} \leq z^1F(a_{1t}, n_{1t}),$$

where a_{1t} is the amount of land used at time t and n_{1t} is the number of workers (hours) allocated to this technology. It is assumed that F is twice differentiable, homogeneous of degree one, concave, and its has strictly decreasing marginal productivities of the two factors. In addition, assume that

$$\lim_{n\to 0} z^1F_n(a,n) = \infty.$$

Land and labor are in fixed supply. We normalize the aggregate quantities to one. In this economy, individuals trade —among other assets— bonds of all maturities, shares in farms (i.e. land), and labor. However, agents are free to trade any assets (e.g. options) they want.

At time 0 it is learned that at time $T > 0$ a new technology will become available. This technology is completely described by

$$y_{2t} \leq z_t^2 n_{2t},$$

where z_t^2 is a stochastic process with the property that

$$z_t^2 > z^1 F_n(1, 1).$$

*The **announcement** specifies the* distribution *of $\{z_t^2\}$ for $t \geq T$, but not the realizations. (You may assume that the actual values of z_t^2 will not be known until time t.)*

1. *(10 points) Go as far as you can describing the equilibrium levels of output, land prices, wages and interest rates at time 0, **before** the announcement.*
2. *(20 points) Describe the impact of the announcement on*
   (a) *R_{j0}^{-1} (the price of j period bonds), for $j = 1$, and $j = T + k$, $k > 0$. Explain your findings.*
   (b) *The expected wage rate $E_0\{w_t\}$ for $t = 0$ and $t = T + k$, $k > 0$. Explain your findings.*
   (c) *The value of the stock market (aggregate value of land) at $t = 0$.*
3. *(10 points) Explain the economic forces that drive four findings in 2. In particular, in what sense —if any— can the stock market be used as an indicator of the 'future health' of this economy.*
4. *(10 points) Suppose that $\{z_t^2\}$ is a sequence of i.i.d. random variables. Assume that at time t every agent in the economy observes a signal, which we denote x_t, which is positively correlated with z_T^2 (the 'first' value of the technology). More specifically, at time t every individual knows the distribution of z_T^2 conditional on x_t. Discuss and explain whether these signals will affect*
   (a) *Prices of bond of short maturity.*
   (b) *Prices of bonds of long maturity.*
   (c) *Stock (land) prices.*
5. ***Extra Credit**: Suppose that technology 2 is described by*

$$\begin{aligned} y_{2t} &\leq z_t^2 F(a_{2t}, n_{2t}), \\ z_t^2 &> z^1. \end{aligned}$$

   Go as far as you can exploring the effects of this new technology (relative to the linear technology described above) on
   (a) *The value of land.*
   (b) *The expected wage rate $E_0\{w_t\}$ for $t = 0$ and $t = T + k$, $k > 0$.*

Solution to Problem 1 (Sketch) The standard first order conditions for pricing bonds and stocks are

$$R_{jt}^{-1} = \frac{\beta^j E_t[u'(c_{t+j})]}{u'(c_t)} \tag{15}$$

and

$$p_t = \sum_{j=1}^{\infty} \beta^j \frac{E_t[u'(c_{t+j})d_{t+j}]}{u'(c_t)}. \tag{16}$$

It follows that to determine asset prices all we need is to determine the equilibrium consumption allocation. Consider the pre-announcement equilibrium. It is clear that $n_{1t} = 1$, as there is only one technology. It follows that

$$\begin{aligned} c_t &= z^1 F(1,1), \\ d_t &= z^1 F_a(1,1), \\ w_t &= z^1 F_n(1,1). \end{aligned}$$

Given these equilibrium quantities, asset prices at $t = 0$ are,

$$\begin{aligned} R_{j0}^{-1} &= \beta^j, \\ p &= \frac{\beta}{1-\beta} z^1 F_a(1,1). \end{aligned}$$

Let's consider next the equilibrium allocation after period T. Since the marginal product of labor must equal the wage in both technologies (workers are mobile), it follows that

$$\tilde{w}_t = \begin{cases} z^1 F_n(1,1) & t < T \\ z_t^2 = z^1 F_n(1, n_{1t}) & t \geq T, \end{cases}$$

where the condition $z_t^2 = z^1 F_n(1, n_{1t})$ determines (uniquely) n_{1t}. It follows from our assumptions that $0 < n_{1t} < 1$. (That n_1 cannot be zero follows from the assumption of unbounded marginal product of labor, while $n_1 < 1$ is a direct consequence of $z_t^2 > z^1 F_n(1,1)$.) Thus, for $t \geq T$, $\tilde{w}_t \geq w_t$ (while they are equal for $t < T$).

Dividends (land rents) are

$$\tilde{d}_t = \begin{cases} z^1 F_a(1,1) & t < T \\ z^1 F_a(1, n_{1t}) & t \geq T. \end{cases}$$

It follows that $\tilde{d}_t \leq d_t$, with strict inequality for $t \geq T$ (this is due to $n_{1t} < 1$), while consumption is

$$\tilde{c}_t = \begin{cases} z^1 F(1,1) & t < T \\ \max_n z^1 F(1,n) + z_t^2 (1-n) & t \geq T. \end{cases}$$

In this case we have that $\tilde{c}_t \geq c_t$, with a strict inequality for $t \geq T$.

What is the effect on asset prices? For $j < T$, $R_{j0}^{-1} = \beta^j = \tilde{R}_{j0}^{-1}$, as consumption is unchanged. However, for $j \geq T$, $\tilde{c}_j \geq c_j$, and $\tilde{R}_{j0}^{-1} < R_{j0}^{-1}$ (see (15)). Finally, the post-announcement price of land is

$$\tilde{p}_0 = \sum_{j=1}^{T-1} \beta^j z^1 F_a(1,1) \sum_{j=T}^{\infty} \beta^j \frac{E_0[u'(\tilde{c}_j)\tilde{d}_j]}{u'(c_0)},$$

which is clearly lower since $\tilde{c}_j \geq c_j$ and $\tilde{d}_j \leq d_j$ for $j \geq T$.

In this economy a drop in the stock market is "good news" as it signals the advent of a new, more productive, technology.

Finally, if a signal x_t is available, it is clear that:

- It will not affect the price of short term bonds (they are constant),
- It will affect the price of long bonds in so far as it affects the conditional expectation of the marginal utility of consumption. Formally, it will affect long bond prices since is affects $E_t[u'(\max_n z^1 F(1,n) + z^2_{t+j}(1-n)) \mid x_t]$.
- It will affect stock prices for the same reason as above.