Econ 712 Macroeconomic Theory- Midterm Exam. University of Wisconsin

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1 Instructions

- Please answer all questions. If you get stuck in one section move to the next one. Do not waste time on questions that you find hard to solve.
- Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- This is a closed book exam. Students may bring one page (both sides) or two pages (single sides) of notes.
- Please hand in the exam promptly at 12 Noon
- Each question is worth 50 points. The point total for each section is indicated at the beginning of the section. Look at these "prices" when deciding how to allocate your time!!
- If you believe that a question is wrong or poorly worded, please make the "minimal" necessary changes to make it "beautiful" and well posed. Of course, unnecessary changes will result in a lower grade.
- *Suggestion*: Do not try the extra credit sections until you finished the regular exam!
- Please use one blue book for each question and write only on the "right" page. (The odd numbered page in a newspaper.)
- Please remember to put your name in each blue book.
- Good luck !

2 Questions

Problem 1 (Adjustment Costs and Productivity) *(50 points) Consider an economy populated by a large number of identical individuals. Each household has preferences given by*

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1,$$

where c_t denotes consumption of market goods. The function u is assumed to be strictly concave, strictly increasing and as many times differentiable as needed. The output of consumption and investment goods satisfies,

$$c_t + x_t \leq f(k_t),$$

where f is assumed to be strictly concave, strictly increasing and as many times differentiable as needed. Capital, k_t, is used in the production of consumption (and investment) goods in the "morning" of period t, and it is used again, as an input in the production of new capital goods, in the "afternoon." The stock of capital available at the beginning of period $t+1$ is given by

$$k_{t+1} \leq \phi k_t h(\frac{x_t}{k_t}),$$

where h is strictly concave, as differentiable as you need it to be, and satisfies $h'(z) > 0$. This technology captures the idea that the "yield" of an additional unit of investment is decreasing in the size of the existing capital stock. This is a form of adjustment cost. Note that after this "afternoon" activity the stock of capital is fully depreciated.

1. *(10 points) Describe the planner's problem.*
2. *(13 points) Go as far as you can analyzing whether a steady state exists and is unique. If you need to make additional assumptions, please justify your choices.*
3. *(13) Go as far as you can describing the impact of an increase in the productivity parameter ϕ on the steady state levels of capital and consumption.*
4. *(14) Define a competitive equilibrium in which:*
   - *(a) Households buy and sell bonds and collect profits from firms.*
   - *(b) Firms in the consumption/investment producing sector maximize profits.*
   - *(c) Firms in the capital producing sector maximize profits.*

     Go as far as you can describing the equilibrium prices in the steady state.

   ***Hint**: The price of capital in the two activities, producing consumption/investment and producing capital is not the same.*

5. ***Extra Credit***. *Consider the economy in part (4) but assume that consumers can trade shares in these firms. Go as far as you can characterizing share prices.*

Solution 2 1. *Define the function* g *by*

$$g(\frac{k_{t+1}}{\phi k_t})k_t = x_t.$$

Since h *is monotone increasing,* g *is well defined. Moreover, the following identities hold*

$$\begin{aligned} z &\equiv h(g(z)), \\ g'(z) &= [h'(g(z)]^{-1} > 0, \\ g''(z) &= -\frac{g'(z)^2 h''(z)}{h'(z)} > 0. \end{aligned}$$

With this notation, the planner's problem is

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to

$$f(k_t) - c_t - g(\frac{k_{t+1}}{\phi k_t})k_t.$$

2. *The first order conditions are*

$$\begin{aligned} u'(c_t) &= \lambda_t, \\ \lambda_t g'(\frac{k_{t+1}}{\phi k_t})\phi^{-1} &= \beta\lambda_{t+1}[f'(k_{t+1}) - g(\frac{k_{t+2}}{\phi k_{t+1}}) - g'(\frac{k_{t+2}}{\phi k_{t+1}})\frac{k_{t+2}}{\phi k_{t+1}}]. \end{aligned}$$

Let $\gamma = \phi^{-1}$. *It follows that, at the steady state,*

$$g'(\gamma)\gamma(1-\beta) + \beta g(\gamma) = \beta f'(k^*).$$

Since we showed that $g' > 0$, *the left hand side is strictly positive. The right hand side is a decreasing function of* k *and the expression has a solution if*

$$\begin{aligned} \lim_{k\to 0} \beta f'(k) &> g'(\gamma)\gamma(1-\beta) + \beta g(\gamma), \\ \lim_{k\to\infty} \beta f'(k) &> g'(\gamma)\gamma(1-\beta) + \beta g(\gamma), \end{aligned}$$

which is clearly satisfied if the production function has the "usual" Inada conditions.

3. *An increase in* γ *increases* $g'(\gamma)\gamma(1-\beta) + \beta g(\gamma)$. *Thus, an increase in* ϕ *—a decrease in* γ*— decreases* $f'(k)$ *and, hence, it increases* k^*.

4. Households solve the following problem

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + b_{t+1} \leq R_t b_t + \Pi_{1t} + \Pi_{2t}.$$

Firms producing consumption initially own the capital stock. Let the price of one unit of capital at time t that will be productive at $t+1$ be p_{kt}. Let the morning price of capital be r_{Mt}, and the afternoon price, r_{At}. Finally, let q_t be the inverse of the t period interest rate,

$$q_t = \Pi_{j=1}^{t} R_j^{-1}.$$

Then the firm producing consumption goods solves the problem has profits at time t given by

$$\Pi_{1t} = \max_k [f(k) - r_{Mt}k + r_{At}k], \qquad t \geq 1$$

while profits in the first period are

$$\Pi_{10} = f(k_0).$$

Investment goods produced at t are not sold until $t+1$. Thus, the current price is $p_{kt} = R_{t+1}^{-1} r_{Mt+1}$. The representative firm in the investment sector solves the following problem

$$\Pi_{2t} = \max_{k,x} R_{t+1}^{-1} r_{Mt+1} \phi k h(\frac{x}{k}) - x - r_{At}k, \qquad t \geq 0.$$

The first order conditions for these two maximization problems imply that, at the steady state

$$\begin{aligned} f'(k) &= r_M - r_A, \\ \beta r_M \phi h'(\frac{x}{k}) &= 1, \\ \beta r_M \left(\phi h(\frac{x}{k}) - \phi h'(\frac{x}{k}) \frac{x}{k} \right) &= r_A. \end{aligned}$$

A simple manipulation shows that

$$\begin{aligned} x^* &= g(\gamma) k^*, \\ r_A^* &= \gamma g'(\gamma) - g(\gamma), \\ r_M^* &= \beta^{-1} \gamma g'(\gamma). \end{aligned}$$

5. *For any sequence of profits, the value of each firm is given by,*

$$P_{it} = \sum_{j=0}^{\infty} \frac{q_{t+j}}{q_t} \Pi_{it}.$$

It follows that $\Pi_{2t} = 0$ and hence this firm has zero value. In the case of the representative firm in the consumption sector, the steady state price is just,

$$P_1^* = \frac{\beta}{1-\beta}[f(k^*) - k^* f'(k^*)]$$

Problem 3 (Terms of Trade and Asset Prices) *(50 points) Consider a representative household economy. Preferences are given by*

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\alpha} z_t^{1-\alpha})^{1-\theta}}{1-\theta}, \qquad 0 < (\beta, \alpha) < 1, \quad \theta > 0,$$

where c_t denotes consumption of the domestic good, and z_t is consumption of an imported good. Let output of the domestic good —fruit that drops from a tree— be given by the stochastic process $\{x_t\}$ where

$$x_{t+1} = \bar{x}^{\phi} x_t^{1-\phi} \epsilon_{t+1}, \qquad 0 < \phi < 1,$$

where the ϵ_t are i.i.d. and such that $E[\epsilon_t] = 1$.

Assume that this country is open to trade flows but completely closed to capital flows. This means that the per-capita feasibility constraint is given by,

$$c_t + p_t z_t \leq x_t,$$

where p_t is the price of imports in terms of domestic goods. In this setting, an increase in p_t corresponds to a worsening of this country's terms of trade. The price of the imported good, p_t, is an i.i.d. random variable, which is also independent of the realization of x_t.

1. *(13 points) Go as far as you can computing the one period rate of return on safe bonds. What does the theory say about the relationship between terms of trade shocks and domestic interest rates?*

2. *(13 points) An economist argues that since terms of trade shocks, p_t, are i.i.d. they cannot affect "long" interest rates (e.g. the price of a two period risk free bond). Evaluate that claim.*

3. *(13 points) What is the impact of an improvement in the terms of trade (a low p_t) on the stock market (i.e. on the value of a share of a tree that drops dividends according to $\{x_t\}$).*

4. *(11 points) What do you think will happen to domestic interest rates if the economy was completely open to both trade and capital flows under the assumption that there are markets for all the Arrow-Debreu securities and that —in equilibrium— aggregate world output was constant?*

   ***Note**: For this last part assume that it is possible for the world output of all goods to be constant and, at the same time, for the relative price of two goods to fluctuate.*

Solution 4 *1. The first order conditions for the maximization problem faced by the representative household are (assuming it trades, one and j period bonds and shares in the domestic tree)*

$$\begin{aligned}
c_t &: u_c(t) = \lambda_t, \\
z_t &: u_z(t) = \lambda_t p_t, \\
b_{1t} &: \lambda_t = \beta R_{1t} E_t[\lambda_{t+1}], \\
b_{jt} &: \lambda_t = \beta^j R_{jt} E_t[\lambda_{t+j}], \\
s_t &: q_t \lambda_t = \beta E_t[\lambda_{t+1}(q_{t+1} + x_{t+1})].
\end{aligned}$$

Imposing the equilibrium condition

$$c_t + p_t z_t = x_t,$$

it follows that the equilibrium quantities consumed of each good are given by

$$\begin{aligned}
c_t &= \alpha x_t, \\
z_t &= (1-\alpha) x_t p_t^{-1}.
\end{aligned}$$

The equilibrium marginal utility of domestic consumption is then

$$u_c(t) = \hat{u} \frac{x_t^{-\theta}}{p_t^{(1-\theta)(1-\alpha)}},$$

where

$$\hat{u} = [\alpha^\alpha (1-\alpha)^{1-\alpha}]^{1-\theta}.$$

Using the expressions from the first order conditions, it follows that the per period rate of return of a j-period bond, p_{jt}^b satisfies

$$\hat{R}_{jt}^b = p_t^{-\frac{(1-\theta)(1-\alpha)}{j}} \frac{\left(\hat{u} x_t^{-\theta}\right)^{1/j}}{\beta \left(E_t[u_c(t+j)]\right)^{1/j}}.$$

Note that the term $E_t[u_c(t+j)]$ is independent of p_t given our i.i.d. assumption. Thus, simple algebra shows that

$$\frac{\partial \hat{R}_{jt}^b}{p_t} \frac{p_t}{\hat{R}_{jt}^b} = -\frac{(1-\theta)(1-\alpha)}{j}. \tag{1}$$

This expression completely summarizes the implications of the theory for the rate of return on bonds.

2. *From (1) it follows that increases in p_t can either increase or decrease domestic interest rates depending on whether $\theta \gtreqless 1$. It is clear that terms of trade shocks affect the rate of return on bonds of all maturities.*

3. *Iterating on the Euler equation that prices shares, it is immediate to obtain that*

$$q_t \hat{u} x_t^{-\theta} = p_t^{(1-\theta)(1-\alpha)} \sum_{j=1}^{\infty} \beta^j E_t \left[u_c(t+j) x_{t+j} \right].$$

   However, as argued in the case of bonds, the terms $E_t \left[u_c(t+j) x_{t+j} \right]$ are all independent of p_t. It follows that

$$\frac{\partial q_t}{p_t} \frac{p_t}{q_t} = (1-\theta)(1-\alpha),$$

   which can have either sign. In all cases, shares and interest rates move in opposite directions.

4. *If there are complete markets and no aggregate uncertainty, consumption in each country is constant. It follows that the marginal utility is constant as well, and one period interest rates are β^{-1}.*