

Econ 712 - Homework # 3

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Problem 1 (Open Economy with Adjustment Costs) Consider the following simple economy. Preferences are given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t).$$

The economy is open to international borrowing and lending. The representative consumer has a budget constraint given by

$$c_t + x_{kt} + \frac{\phi}{2} \frac{x_{kt}^2}{k_t} + b_{t+1} \leq w_t + r_t k_t + R_t^* b_t,$$
$$k_{t+1} \leq (1 - \delta_k) k_t + x_{kt}.$$

The term

$$\frac{\phi}{2} \frac{x_{kt}^2}{k_t}$$

captures adjustment costs in capital. Thus, large increases in investment (x_{kt}) can be very costly. The productive technology in this economy is given (in per capita terms) by

$$y_t \leq z f(k_t).$$

1. Assume that $\beta R_t^* = 1$. Go as far as you can describing the dynamics of output in this economy starting from an arbitrary initial condition. Describe the behavior of interest rates and real wages. Discuss the following claim: Interest rates always equal the marginal product of capital.
2. Go as far as you can describing the impact of a once and for all permanent increase in productivity (z) on consumption, output, interest rates and wages.
3. Go as far as you can describing the impact of a temporary (that will last for T periods) increase in productivity (z) on consumption, output, interest rates and wages.

4. Define the current account as the change in a country's external asset position. In terms of the notation in this exercise, it is given by

$$M_t = b_{t+1} - b_t.$$

Go as far as you can describing how the two types of productivity shocks you studied in the previous two points affect the current account.

5. Do the same for the trade balance, B_t , defined as

$$B_t = zf(k_t) - [c_t + x_{kt} + \frac{\phi}{2} \frac{x_{kt}^2}{k_t}]$$

Problem 2 (Durable Goods and Productivity Shocks) Consider an economy populated by a large number of identical dynasties with utility functions given by

$$U \equiv \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(z_t)], \quad 0 < \beta < 1,$$

where $u(c)$ and $v(z)$ are strictly increasing and strictly concave functions. In this context, c_t denotes consumption of non-durable (e.g. food) goods, while z_t is the stock of durables (e.g. houses) at time t . The economy's aggregate feasibility constraint is given by

$$\begin{aligned} c_t + qx_{zt} + x_{kt} &\leq Af(k_t), \\ k_{t+1} &\leq (1 - \delta_k)k_t + x_{kt}, \\ z_{t+1} &\leq z_t h\left(\frac{x_{zt}}{z_t}\right), \quad h(0) = 0, \quad h(\delta_z) = 1, \quad 0 < \delta_z < 1 \\ 0 &< (k_0, z_0), \quad \text{given} \end{aligned}$$

where f is a standard (i.e. twice differentiable, increasing, strictly concave, $\lim_{k \rightarrow 0} f'(k) = \infty$ and $\lim_{k \rightarrow \infty} f'(k) = 0$) production function, and $0 < \delta_k < 1$ is the depreciation rate corresponding to capital. The quantity x_{zt} is interpreted as investment (purchases) of new durables. The function h is assumed to be strictly increasing and strictly concave. In this specification, δ_z corresponds to the depreciation rate of the durable good given that, when $x_{zt} = \delta_z z_t$, the stock of the durable good is unchanged (i.e. $z_{t+1} = z_t$). Let the non-durable good be the numeraire. In this interpretation, q is the price of durables in terms of nondurables.

1. Define a competitive equilibrium in which households own the stocks of capital and durables and trade (at least) one period bonds
2. Show that a steady state exists and is unique. What does the model imply about the effect of changing TFP (the constant A) upon the steady state quantities consumed of durables and non-durables? Explain your argument.

3. Does theory pin down the impact of an increase in TFP (A) on the **ratio** of durables to nondurables (i.e. c^*/z^* , where a $*$ denotes a steady state value). Explain your argument. To answer this section you may assume that the relevant utility functions are

$$u(c) = \frac{c^{1-\theta}}{1-\theta}, \quad v(z) = \frac{z^{1-\eta}}{1-\eta}, \quad (\theta, \eta) > 0.$$

4. Assume now that the economy is open and that individuals can borrow and lend (subject to the standard transversality condition) in the international bond market. Assume that the world interest rate, R^* , satisfies $R^*\beta = 1$. It is claimed that, since the interest rate is constant, the country's level of output and stock of durables will converge in **one period** to their steady state values (not necessarily the same steady state as the closed economy). Discuss this claim.
5. For the isoelastic utility functions described above, go as far as you can describing the dynamic behavior of the stock of durables for an economy that starts with the steady state level of capital (but not durables) and is open to international borrowing and lending with $R^*\beta = 1$.
6. Consider next the effect of a permanent increase in productivity on the dynamic path of output and the stock of durables in an open economy similar to the one described in the previous section.