

Econ 712 Problem Set #2

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Problem 1 - Capital Utilization

The planner's problem is given by

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1$$

$$\begin{aligned} \text{s. t.} \quad & c_t + p_k x_t \leq f(k_t v_t) \\ & k_{t+1} = (1 - \delta(v_t))k_t + G(x_t, a) \\ & (c_t, k_{t+1}, v_t, x_t) \geq (0, 0, 0, 0) \end{aligned}$$

Assume an interior solution (the reason why we can assume this will be seen later on) and write down the Lagrangean as following:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma_t [f(k_t v_t) - c_t - p_k x_t] + \mu_t [(1 - \delta(v_t))k_t + G(x_t, a) - k_{t+1}]).$$

This gives the following FOCs.

$$[c_t] \quad u'(c_t) = \gamma_t \quad (1)$$

$$[k_{t+1}] \quad \beta \gamma_{t+1} v_{t+1} f'(v_{t+1} k_{t+1}) - \mu_t + \beta \mu_{t+1} (1 - \delta(v_{t+1})) = 0 \quad (2)$$

$$[x_t] \quad -\gamma_t p_k + \mu_t G_1(x_t, a) = 0 \quad (3)$$

$$[v_t] \quad \gamma_t f'(v_t k_t) - \mu_t \delta'(v_t) = 0 \quad (4)$$

and

$$[Feasibility] \quad c_t + p_k x_t = f(k_t v_t) \quad (5)$$

$$k_{t+1} = (1 - \delta(v_t))k_t + G(x_t, a) \quad (6)$$

$$[TVC] \quad \lim_{T \rightarrow \infty} \beta^T \mu_T k_{T+1} = 0. \quad (7)$$

The steady state solution should satisfy the following conditions.

$$u'(c^*) = \gamma^* \quad (8)$$

$$\beta \gamma^* v^* f'(v^* k^*) - \mu^* + \beta \mu^* (1 - \delta(v^*)) = 0 \quad (9)$$

$$-\gamma^* p_k + \mu^* G_1(x^*, a) = 0 \quad (10)$$

$$(\implies -\gamma^* p_k + \mu^* = 0 \quad \text{if } G(x, a) = x)$$

$$(\implies -\gamma^* p_k + \mu^* (1 - \theta) x^{1-\theta} a^\theta = 0 \quad \text{if } (x, a) = x^{1-\theta} a^\theta)$$

$$\gamma^* f'(v^* k^*) - \mu^* \delta'(v^*) = 0 \quad (11)$$

$$c^* + p_k x^* = f(v^* k^*) \quad (12)$$

$$k^* \delta(v^*) = G(x^*, a) \quad (13)$$

$$(\implies k^* \delta(v^*) = x^* \quad \text{if } G(x, a) = x)$$

$$(\implies k^* \delta(v^*) = x^{1-\theta} a^\theta \quad \text{if } G(x, a) = x^{1-\theta} a^\theta)$$

1-1. $G(x, a) = x$

Putting (11) into (9) and rearranging it, we have

$$\beta(v^* \delta'(v^*) + 1 - \delta(v^*)) = 1$$

$$\text{or } \lambda v^{*\lambda+1} = 1/\beta - 1. \quad (14)$$

Thus, there exists a unique solution $v^* \in (0, 1)$ in (14), which is obviously independent of p_k as well as the functional form of $G(x, a)$. Once we get v^* , then everything is straightforward. (10) and (11) yields

$$f'(v^* k^*) = p_k \delta'(v^*) \implies k^* = \left(\frac{\alpha z}{p_k (\lambda + 1)} \right)^{\frac{1}{1-\alpha}} v^{*\frac{1-\lambda-\alpha}{1-\alpha}}.$$

Then, (13) determines x^* and finally (12) determines c^* .

1-2. $G(x, a) = x^{1-\theta} a^\theta$

First we note that utility function is strictly increasing and strictly concave with infinite marginal utility at $c_t = 0$ (see (1)), and the law of motion satisfied at equality (see (6)). Also, we see that the marginal installation cost at $x_t = 0$ is infinite. (see (3)) Thus, these conditions guarantee an interior solution. First we look at the steady state solutions and then we will see the dynamics of the interior solution.

Steady State Analysis

Regarding the effect of the price of investment good at the steady state, as shown in #1-1 the capital utilization v^* is independent of p_k since it has nothing to do with the functional form of $G(x, a)$.

Now, from (13) we have

$$x^* = a^{\frac{-\theta}{1-\theta}} (\delta(v^*)k^*)^{\frac{1}{1-\theta}}$$

(10) and (11) together with the above equation gives

$$p_k = \frac{(1-\theta)a^\theta x^{*\theta} f'(v^*k^*)}{\delta'(v^*)} = \dots = A(v^*, \theta, \alpha, z) k^{*\alpha-1-\frac{\theta}{1-\theta}},$$

where $A(v^*, \theta, \alpha, z)$ is some positive constant depending on v^*, θ, α and z . Since $\alpha-1-\frac{\theta}{1-\theta} < 0$, the above equation implies that p_k and k^* is negatively correlated. Thus, the capital-output ratio and the investment-output ratio have the following relationship with p_k :

$$\begin{aligned} \frac{k^*}{f(v^*k^*)} &= Bk^{*1-\alpha} \downarrow \quad \text{as } p_k \uparrow \\ \frac{x^*}{f(v^*k^*)} &= Ck^{*\frac{1}{1-\theta}-\alpha} \downarrow \quad \text{as } p_k \uparrow \quad \text{if } \frac{1}{1-\theta} > \alpha \\ &= Ck^{*\frac{1}{1-\theta}-\alpha} \uparrow \quad \text{as } p_k \uparrow \quad \text{if } \frac{1}{1-\theta} < \alpha \end{aligned}$$

Note that this comparative statics give us the intuition about the cross country variation with different p_k . This never tells us the evolution of the solution.

Dynamics of the Solution

Define function $c(\gamma_t)$ using (1) such that $u'(c(\gamma_t)) = \gamma_t$. From (4) we have

$$v_t = \left(\frac{\alpha z \gamma_t}{(\lambda + 1) \mu_t} \right)^{\frac{1}{\lambda+1-\alpha}} k_t^{\frac{-(1-\alpha)}{\lambda+1-\alpha}} \quad (15)$$

and

$$v_{t+1} = \left(\frac{\alpha z \gamma_{t+1}}{(\lambda + 1) \mu_{t+1}} \right)^{\frac{1}{\lambda+1-\alpha}} k_{t+1}^{\frac{-(1-\alpha)}{\lambda+1-\alpha}}$$

Note that

$$\frac{\gamma_t}{\mu_t} = \frac{1}{p_k} G_1 \left(\frac{f(v_t k_t) - c(\gamma_t)}{p_k}, a \right) \quad (16)$$

and

$$\frac{\gamma_{t+1}}{\mu_{t+1}} = \frac{1}{p_k} G_1 \left(\frac{f(v_{t+1} k_{t+1}) - c(\gamma_{t+1})}{p_k}, a \right)$$

Also, plugging (4) into (2) we get

$$\mu_{t+1} = \mu_t / \beta (\lambda v_{t+1}^{\lambda+1} - 1). \quad (17)$$

Finally, the feasibility constraints give us

$$k_{t+1} = G \left(\frac{f(v_t k_t) - c(\gamma_t)}{p_k} + (1 - \delta(v_t)) k_t, a \right). \quad (18)$$

Putting above equations all together, the following numerical algorithm can be proposed:

- (i) Given (γ_0, k_0)
- (ii) Get v_0 from

$$v_0 = \left(\frac{\alpha z}{(\lambda + 1) p_k} \frac{1}{p_k} G_1 \left(\frac{f(v_0 k_0) - c(\gamma_0)}{p_k}, a \right) \right)^{\frac{1}{\lambda+1-\alpha}} k_0^{\frac{-(1-\alpha)}{\lambda+1-\alpha}} \quad (19)$$

- (iii) Get k_1 from (18), putting (γ_0, k_0, v_0) .
- (iv) Get (γ_1, μ_1, v_1) by solving (17) together with time $t + 1$ version of (15) and (16), putting k_1 into them.

(v) Iterate (iii) and (iv) to get k_{t+1} and $(\gamma_{t+1}, \mu_{t+1}, v_{t+1})$ consecutively.

Note that we have to determine γ_0 at the equilibrium path which is the most crucial part of the procedure as in the model in the lecture. But, this is not quite easy. Another problem arises when we solve the system of equation for Step (iv). It is not quite obvious about the existence and the uniqueness. However, we can easily verify that there is an unique solution if we confine the case where $\theta = 0$ or $G(x, a) = x$.

Equation (19) is obtained by combining (15) and (16) for $t = 0$. It is easy to see that it has a unique solution.

1-3. $G(x, a) = x^{1-\theta}a^\theta$

As you have seen in #1-2, the problem is 4-dimensional, i.e., we have to search the equilibrium path on (γ, μ, v, k) -coordinate. This is not easy stuff unless we introduce extra assumptions on the functional forms, etc.

Problem 2. Open Economy with Durable Goods

The problem is

$$\max \sum_{t=0}^{\infty} \beta^t (u(c_t) + v(z_{t+1}))$$

subject to

$$\begin{aligned} c_t + x_{kt} + qx_{zt} + b_{t+1} + \frac{d}{2}(z_{t+1} - z_t)^2 &\leq zf(k_t) + R^*b_t \\ k_{t+1} &\leq (1 - \delta_k)k_t + x_{kt} \\ z_{t+1} &\leq (1 - \delta_z)z_t + x_{zt} \\ k_0 > 0, \quad z_0 > 0, \quad \text{and } b_0 &\text{ given} \end{aligned}$$

Putting Lagrange multipliers λ_t , μ_t , and γ_t on each constraint, we have the following FOCs¹:

$$\begin{aligned} u'(c_t) &= \lambda_t \\ v'(z_{t+1}) &= d\lambda_t(z_{t+1} - z_t) - d\beta\lambda_{t+1}(z_{t+2} - z_{t+1}) - \beta(1 - \delta_z)\gamma_{t+1} + \gamma_t \\ \lambda_t &= \mu_t, \quad q\lambda_t = \gamma_t \\ \mu_t &= \beta(\lambda_{t+1}zf'(k_{t+1}) + \mu_{t+1}(1 - \delta_k)) \\ \lambda_t &= \beta R^*\lambda_{t+1} \end{aligned}$$

with feasibility

$$\begin{aligned} c_t + x_{kt} + qx_{zt} + b_{t+1} + \frac{d}{2}(z_{t+1} - z_t)^2 &= zf(k_t) + R^*b_t \\ k_{t+1} &= (1 - \delta_k)k_t + x_{kt} \\ z_{t+1} &= (1 - \delta_z)z_t + x_{zt} \end{aligned}$$

¹It is ok to simplify the constraints by putting all of them together such that

$$c_t + k_{t+1} + qz_{t+1} + b_{t+1} + \frac{d}{2}(z_{t+1} - z_t)^2 = zf(k_t) + (1 - \delta_k)k_t + q(1 - \delta_z)z_t + R^*b_t.$$

Then we don't need μ_t and γ_t . Eventually, it turns out to be $\lambda_t = \mu_t$ and $q\lambda_t = \gamma_t$ by the FOCs.

and the TVCs

$$[TVC1] \quad \lim_{T \rightarrow \infty} \beta^T u'(c_T) k_{T+1} = 0$$

$$[TVC2] \quad \lim_{T \rightarrow \infty} \beta^T [v'(z_{T+1}) + u'(c_T)(d(z_{T+1} - z_T) + q)] z_{T+1} = 0$$

$$[TVC3] \quad \lim_{T \rightarrow \infty} \beta^T u'(c_T) b_{T+1} = 0.$$

2-1. Steady State

From the FOCs, we have the following system of equations for steady state solutions.

$$u'(c^*) = \lambda^* \tag{20}$$

$$v'(z^*) = q\lambda^*(1 - \beta + \beta\delta_z) \tag{21}$$

$$1 = \beta(zf'(k^*) + (1 - \delta_k)) \tag{22}$$

$$c^* = zf(k^*) - \delta_k k^* - q\delta_z z^* + (R^* - 1)b^* \tag{23}$$

As usual k^* is determined by (22) and so is $x_k^* = \delta_k k^*$. Now we are left to determine c^* and z^* . Let $c^* = A - q\delta_z z^*$ for some positive constant A (Can you figure out why $A > 0$?). Then, (20) and (21) gives

$$v'(z^*) = u'(A - q\delta_z z^*)q(1 - \beta + \beta\delta_z)$$

Note that the left hand side is strictly decreasing in z^* with $v'(0) = \infty$ and $v'(\infty) = 0$ and the right hand side is strictly increasing in z^* with $\lim_{z \rightarrow \frac{A}{q\delta}} u'(A - q\delta_z z^*) = \infty$. So, there is an unique solution $z^* \in (0, \frac{A}{q\delta})$.

Now for the comparative statics we rearrange the above equation as

$$\frac{u'(c^*)}{v'(z^*)} = \left(\frac{z^*}{c^*}\right)^\eta = \frac{1}{q(1 - \beta + \beta\delta_z)}.$$

Thus, we conclude that the ratio of durable to non-durable consumption is negatively related to q and δ_z . This is easy to figure out since we can expect that if the price of nondurable good or its depreciation get relatively higher than that of durable good, then consumption about the nondurable good become relatively smaller.

2-2. $d = 0$

First notice that $\beta R^* = 1$. Hence $\lambda_t = \lambda_{t+1}$ (say λ^*) for all $t \geq 0$. Then, the FOCs are now

$$\begin{aligned} u'(c_t) &= \lambda^* \\ v'(z_{t+1}) &= q\lambda^*(1 - \beta + \beta\delta_z) \\ 1 &= \beta(zf'(k_{t+1}) + (1 - \delta_k)) \end{aligned}$$

So, we have that $c_t = c^*$, $z_{t+1} = z^*$, and $k_{t+1} = k^*$ for $t \geq 0$. Thus,

$$\begin{aligned} x_{k0} &= k_1 - (1 - \delta_k)k_0 = k^* - (1 - \delta_k)k_0 \\ x_{z0} &= z_1 - (1 - \delta_z)z_0 = z^* - (1 - \delta_z)z_0 \end{aligned}$$

and

$$\begin{aligned} x_{kt} &= k_{t+1} - (1 - \delta_k)k_t = \delta_k k^*, \quad t = 1, 2, \dots \\ x_{zt} &= z_{t+1} - (1 - \delta_z)z_t = \delta_z z^*, \quad t = 1, 2, \dots \end{aligned}$$

We have shown that every variable except the bond holding converges to the steady state at time $t = 1$. Now, we are left to show it is also true for b_t . This is done by the TVC. Again from FOCs, we have

$$\begin{aligned} c^* + x_{k0} + qx_{z0} + b_1 + \frac{d}{2}(z^* - z_0)^2 &= zf(k_0) + R^*b_0 \\ c^* + \delta_k k^* + q\delta z^* + b_{t+1} &= zf(k^*) + R^*b_t, \quad t \geq 1. \end{aligned} \quad (24)$$

Rewriting (24) as

$$b_{t+1} = R^*b_t - B, \quad t \geq 1$$

with $B = c^* + \delta_k k^* + q\delta z^* - zf(k^*)$. Recursively we get

$$b_{t+1} = R^{*t} \left(b_1 - \frac{B}{R^* - 1} \right) + \frac{B}{R^* - 1}.$$

Applying the TVC

$$\lim_{T \rightarrow \infty} \beta^T u'(c_T) b_{T+1} = u'(c^*) \left(b_1 - \frac{B}{R^* - 1} \right) = 0,$$

we have $b_1 = \frac{B}{R^* - 1}$ and so $b_t = \frac{B}{R^* - 1}$ for all $t \geq 1$.

In conclusion, this economy reaches the steady state from time $t = 1$.

2-3. $d > 0$

Even if $d > 0$, we still have that $\lambda_{t+1} = \lambda_t$, which implies that $c_t = c^*$ and $k_{t+1} = k^*$ converges to the steady state at time $t = 1$ as given in #2-1 and #2-2. We are left to analyze the equilibrium path for z_{t+1} and b_{t+1} .

We search for z_{t+1} first. Let us consider

$$v'(z_{t+1}) = d\lambda^*(z_{t+1} - z_t) - d\beta\lambda^*(z_{t+2} - z_{t+1}) + q\lambda^*(1 - \beta + \delta_z\beta)$$

Define $w_t = z_{t+1} - z_t$. Then we have the following system of difference equations by rearranging the above equations.

$$w_{t+1} = \frac{w_t}{\beta} - \frac{v'(w_t + z_t)}{d\beta\lambda^*} + \frac{q}{d\beta}(1 - \beta + \delta_z\beta) \quad (25)$$

$$z_{t+1} = z_t + w_t \quad (26)$$

Note that z_0 is given, but $w_0 = z_1 - z_0$ is not determined since we don't know z_1 . By careful selection for w_0 (or equivalently z_1) we will argue what the equilibrium path looks like in the (w, z) -plane as we do in the (k, λ) -plane. To this end we describe two loci corresponding to the case where $w_{t+1} = w_t = w$ in (25). and $z_{t+1} = z_t = z$ in (26).

The first locus ($w_{t+1} = w_t$) consists of the pairs (z, w) or $z = z_1(w)$ satisfying

$$w = \frac{w}{\beta} - \frac{v'(w + z)}{d\beta\lambda^*} + \frac{q}{d\beta}(1 - \beta + \delta_z\beta)$$

or

$$v'(w + z_1(w)) = d\lambda^*(1 - \beta)w + q\lambda(1 - \beta + \delta_z\beta).$$

Note that $z_1(w)$ is well-defined and we have

$$v''(w + z_1(w)) \left(1 + \frac{z_1'(w)}{dw}\right) = d\lambda^*(1 - \beta).$$

Since $v'' < 0$ we have $\frac{z_1'(w)}{dw} < -1$. Thus, $z_1(w)$ is strictly decreasing function with $z_1(0) = z^*$. If $z > z_1(w)$, then $w_{t+1} > w_t$ from (25) and vice versa.

The second locus ($z_{t+1} = z_t$) is much easier to think. It is just $w = 0$ line. And we have $z_{t+1} > z_t$ if $w > 0$ and $z_{t+1} < z_t$ if $w < 0$ from (26).

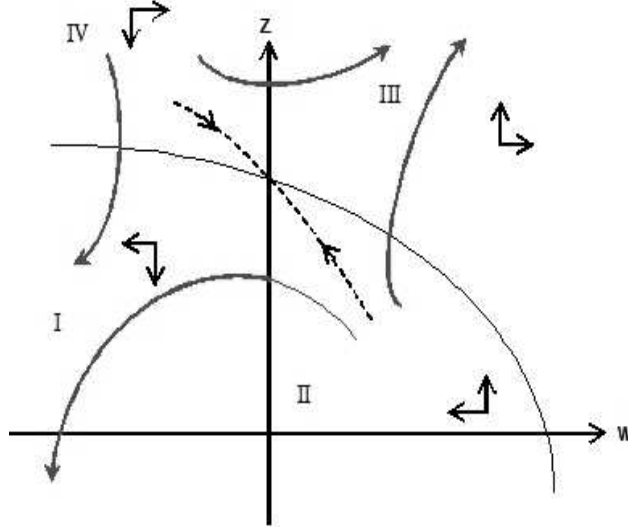


Figure 1: The dynamics of the consumption for durable good

From the figure we can check the equilibrium manifold converging to the steady state solution ($w^* = 0, z^* = z_1(0)$).

Here we will heuristically see why there is only one equilibrium path. As seen in the figure we divide the plane into 4 regions. Any path in region I eventually passes through $z = 0$ line and gets a negative value of z , which doesn't make economical sense.

At region III any trajectory blows up and thus $w_t \rightarrow \infty$ and $z_t \rightarrow \infty$. From (25), we see that

$$w_{t+1} \approx \frac{w_t}{\beta} \quad \text{for large } t \quad \text{or} \quad w_t \approx \frac{D}{\beta^{t-1}} \quad \text{for large } t$$

for some constant $D > 0$ since $v'(w_t + z_t) \rightarrow 0$ as $t \rightarrow \infty$. Now invoking TVC2, we have

$$\begin{aligned} & \lim_{T \rightarrow \infty} \beta^T [v'(z_{T+1}) + u'(c_T)(d(z_{T+1} - z_T) + q)] z_{T+1} \\ & \geq \lim_{T \rightarrow \infty} \beta^T u'(c^*) d \beta^T \frac{D}{\beta^{T-1}} z_{T+1} = \lim_{t \rightarrow \infty} u'(c^*) d D \beta z_{T+1} = \infty, \end{aligned}$$

which is a contradiction.

Thus, the equilibrium path should be in region II or in region IV and it must be as in the figure, otherwise it enters into region I or III. The uniqueness is guaranteed by the strict concavity of the utility function u and v .

Finally the dynamics of b_{t+1} is characterized by

$$b_1 = R^*b_0 + zf(k_0) + (1 - \delta_k)k_0 + q(1 - \delta_z)z_0 - c^* - k^* - qz^* - \frac{d}{2}(z_1 - z_0)^2$$

$$b_{t+1} = R^*b_t + zf(k^*) - \delta_k k^* - q\delta_z z^* - c^* - \frac{d}{2}(z_{t+1} - z_t)^2, \quad t \geq 1.$$

Note that it is no more true that $b_t = b_1, t \geq 1$ as in the case where $d = 0$, but TVC3 should be satisfied.

2-4. The Speed of Adjustment of physical capital and durable goods

As seen in #2-3, the model says that the durable consumption gets the steady state as soon as the economy opens to the world market. This is possible by borrowing or lending money from the rest of the world by using the bond market. On the other hand, it is not the case for the durable goods in the presence of the adjustment cost.

Problem 3. Excess Capacity and Dynamics

The problem is

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1$$

subject to

$$c_t + x_t + g \leq z f(\kappa_t)$$

$$k_{t+1} \leq (1 - \delta)k_t + (k_t - \kappa_t) + x_t$$

$$\kappa_t \leq k_t$$

By putting Lagrange multipliers λ_t , μ_t , γ_t to each constraint we have the following FOCs (easy to see $\lambda_t = \mu_t$):

$$[c_t] \quad u'(c_t) = \lambda_t$$

$$[k_{t+1}] \quad \beta \lambda_{t+1} - \lambda_t + \beta \gamma_{t+1} = 0$$

$$[\kappa_t] \quad \lambda_t z f'(\kappa_t) - \lambda_t \delta - \gamma_t = 0$$

with

$$[feasibility] \quad k_{t+1} = z f(\kappa_t) - \delta k_t + k_t - g - c_t$$

$$[CS] \quad k_t - \kappa_t \geq 0, \quad \gamma_t \geq 0, \quad (k_t - \kappa_t)\gamma_t = 0$$

and

$$[TVC] \quad \lim_{T \rightarrow \infty} \beta^T u'(c_T) k_{T+1} = 0.$$

First of all, it is not very hard to verify that the steady state solution is uniquely determined by

$$k^* = \kappa^*, \quad z f'(\kappa^*) = 1/\beta - 1 + \delta.$$

(Can you check this out?²) The steady state has no difference from that of the model in the lecture note.

²Hint: The complementary slackness [CS] implies $(k^* - \kappa^*)(z f'(\kappa^*) - \delta) = 0$.

3-1. Development Path

If $k_t > \kappa_t$, then $\gamma_t = 0$ by [CS]. From $[k_{t+1}]$ and $[\kappa_t]$ we have

$$zf'(\kappa_t) = \delta,$$

which implies $k_t > \kappa_t = k_a > k^*$ where k_a and k^* are defined by

$$zf'(\kappa_t) = \delta \quad zf'(k^*) = \frac{1}{\beta} - 1 + \delta.$$

But, this contradicts $k_0 < k^*$. We are looking for the case where the development from the low capital status. Hence, we have $k_t = \kappa_t$, so the development path in this economy must be the same as the usual development path of the model in the lecture, i.e.,

$$\begin{aligned} k_{t+1} &= (1 - \delta)k_t + zf(k_t) - g - c(\lambda_t) \\ \lambda_{t+1} &= 1/(\beta[(1 - \delta) + f'(k_{t+1})]) \end{aligned}$$

3-2. The Difference between Poor and Rich Countries

As long as $k_0 < k^*$ (in the increasing economy), the capital is fully utilized $k_t = \kappa_t$ independent of the initial amount of the capital.

3-3. The Unanticipated Temporary Shock

As seen in the previous analysis, the capital stock is fully utilized on the optimal path. We don't have to care about κ_t here and this question is ask the case of the unanticipated temporary shock. We can analyze this problem in two stages.

- Stage 1. At time 0, the planner discovers the news that will happen in the future. Thus, there must be a consumption jump at time 0 (It is an upward jump).
- Stage 2. As soon as the information (for $g_t = 0, t = 0, T - 1$) is released at time 0 our problem is the same as the case where the anticipated permanent shock that will occur at time T , i.e., it is just like that the planner hear the news that there will be a permanent shock (for $g_t = g, t \geq T$ forever afterwards) at time T .

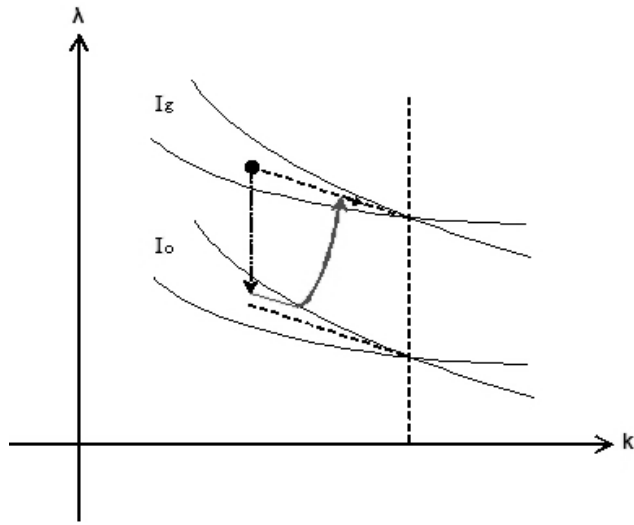


Figure 2: The effect of unanticipated temporary shock: Trajectory 1

At Stage 1, the planner changes his consumption plan from the manifold for $g_t = 0$ to the manifold for $g_t = g$, which is a downward jump in the marginal utility λ .

At Stage 2, we have two possible scenarios. If T is sufficiently large, then the downward jump of λ is fairly high so λ_0 start from the inside of region I_0 . For $t = 0, \dots, T - 1$, the dynamics is governed by lower two loci. Thus, it follows trajectory 1 in the figure. The consumption is temporarily increasing for a while and it will start to decrease until $t = T$. After T periods it again will increase.

Another case is when T is relatively small. Then, λ_0 is located at the outside the lower two loci and it follows trajectory 2. The consumption is only decreasing until it reaches the stable manifold for $g_t = g$.

3-4. The Unanticipated Temporary Shock

In this model the the capital is fully utilized. Thus the shock in the government spending has no effect on the rate of capacity utilization.

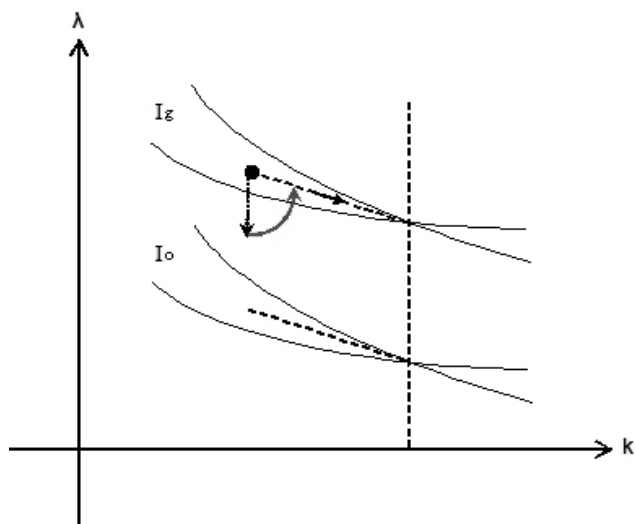


Figure 3: The effect of unanticipated temporary shock: Trajectory 2

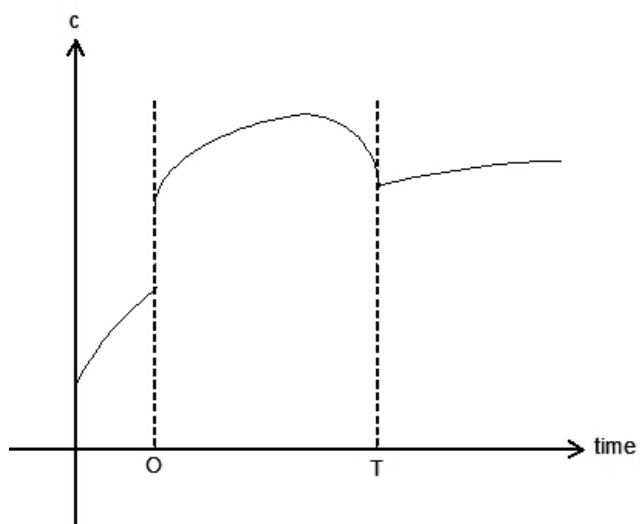


Figure 4: Time path of the consumption: Trajectory 1

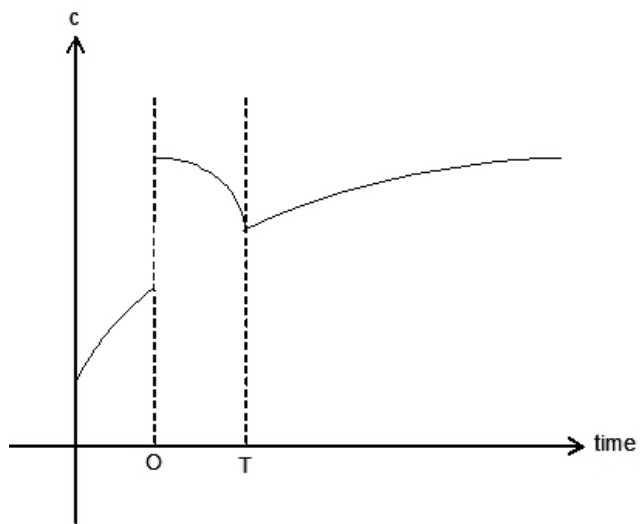


Figure 5: Time path of the consumption: Trajectory 2