Econ 712 - Homework # 2
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Problem 1 (Capital Utilization) *Consider an economy in which each individual ranks consumption sequences according to the functional*

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1$$

where the function u is increasing, differentiable and strictly concave. This economy imports investment goods. Assume that the international price of the investment good is $p_k \geq 1$. Thus, the feasibility constraint faced by the planner is

$$c_t + p_k x_t \leq f(k_t^e),$$

where k_t^e denotes effective units of capital per worker (to be defined). The planner chooses that intensity of use of the capital stocks. Let intensity of use be denoted υ, where $0 \leq \upsilon \leq 1$. If the economy has available k units of capital and it is used at intensity υ, the effective supply of capital is $k^e = \upsilon k$. In this case the depreciation rate is $\delta(\upsilon) = \upsilon^{1+\lambda}$, $\lambda > 0$, and $0 \leq \upsilon \leq 1$.

In this economy there are installation costs. If x units of investment goods are allocated to the production of new capital, they produce $G(x, a)$ units of new capital goods. Thus, the aggregate law of motion of capital is

$$k_{t+1} = (1 - \delta(\upsilon_t))k_t + G(x_t, a),$$

where a is a factor in fixed supply. The function G is assumed increasing in each argument, differentiable and concave. One interpretation of G is that it represents installation costs that are necessary to make capital goods productive.

Assume that the production function is given by

$$f(k_t^e) = z(k_t^e)^{\alpha}, \qquad 0 < \alpha < 1.$$

Assume that the economy is at the steady state

1. *Assume that $G(x, a) = x$, and that λ is sufficiently high so that the solution is interior. Argue that the steady state value is independent of the price of investment goods.*

2. *Let $G(x,a) = x^{1-\theta}a^{\theta}$, with $0 < \theta < 1$. Go as far as you can describing how the price of investment goods (p_k) affects capital utilization. Describe how changes in this price affect the capital-output ratio and the investment-output ratio. Be explicit about the assumptions that guarantee an interior solution.*

3. *Let $G(x,a) = x^{1-\theta}a^{\theta}$, with $0 < \theta < 1$. Given that the economy is at the steady state, is it possible to describe the dynamics associated with a once and for all permanent increase in the international price of the investment good? If it is possible, describe the dynamics. If it is not, explain why.*

Problem 2 (Open Economy with Durable Goods) *Consider the following simple economy where preferences and technology are given by*

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(z_{t+1})] \\ c_t + x_{kt} + qx_{zt} + b_{t+1} + \frac{d}{2}(z_{t+1} - z_t)^2 &\leq zf(k_t) + R^* b_t \\ k_{t+1} &\leq (1-\delta_k)k_t + x_{kt}, \\ z_{t+1} &\leq (1-\delta_z)z_t + x_{zt}, \\ k_0 &> 0, z_0 > 0 \text{ and } b_0 \text{ given.} \end{aligned}$$

In this setting, z_{t+1} is the amount of durable goods available for consumption during period t, while c_t is now interpreted as nondurable consumption. Given the law of motion for the evolution of the stock of capital goods, it follows that new goods produced today, x_{zt}, can be enjoyed in the same period. The term $\frac{d}{2}(z_{t+1} - z_t)^2$ captures the cost of installing new capital goods. The term b_t corresponds to the stock of foreign bonds (issued by the rest of the world) held by this planner. Note that access to an international bond market is equivalent, from a formal point of view, to the existence of a second, linear, technology to shift wealth over time.

Assume that u an v are strictly increasing, strictly concave and C^2. Let $d \geq 0$, and, as usual, $0 < (\beta, \delta_k, \delta_z) < 1$. Let the "world" interest rate be such that $R^* = \beta^{-1} = 1 + \rho$.

To prevent the planner from borrowing too much (running a Ponzi game), we impose the following condition on the planner's problem:

***Condition (A)**: Any solution must satisfy $\lim_{T\to\infty} \beta^T u'(c_T) b_{T+1} = 0$.*

***Remark** : The previous condition —a sort of transversality condition— must be satisfy by any feasible program. However, there may be other —similarly looking— conditions that optimal programs must also satisfy.*

1. *Go as far as you can describing the steady state (assume that $b_t \to b^* < \infty$). In particular, describe the impact of the installation cost on the steady state and the effect of the rate of depreciation of the durable good, δ_z, and its cost parameter, q, on the ratio of durable to non-durable consumption if both u an v are isoelastic (i.e. of the form $u(x) = x^{1-\eta}/(1-\eta)$ for some η)*

2. *Assume that $d = 0$. Go as far as you can describing the optimal path.*

3. *Assume that $d > 0$. Take the steady sate level of consumption as given (i.e. c^* is given) and assume that z_0 is small. Go as far as you can describing the dynamics of capital and durable goods in this economy.*

4. *What does the model you analyzed in 3) say about the speed of adjustment of different types of capital goods (i.e. physical capital and durable goods)?*

Problem 3 (Excess Capacity and Dynamics) *In the standard growth model there is no room for varying the rate of utilization of capital. In this problem you explore how the nature of the solution is changed when variable rates of capital utilization are allowed. As in the standard model there is a representative agent with preferences given by*

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad 0 < \beta < 1,$$

where u is strictly increasing, concave, and twice differentiable. Output depends on the actual number of machines used at time t, κ_t. Thus, the aggregate resource constraint is,

$$c_t + x_t + g \leq z f(\kappa_t)$$

where the function f is strictly increasing, concave, and twice differentiable. In addition, f is such that the marginal product of capital converges to zero as the stock goes to infinity. Assume that g is "small."

Capital that is not used does not depreciate. Thus, capital accumulation satisfies the following,

$$k_{t+1} \leq (1-\delta)\kappa_t + (k_t - \kappa_t) + x_t$$

where we require that the number of machines used, κ_t, is no greater than the number of machines available, k_t, or $k_t \geq \kappa_t$. This specification captures the idea that if some machines are not used, $k_t - \kappa_t$ in our notation, they do not depreciate.

1. *Describe as precisely as you can the development path of this economy for an arbitrary (but small) k_0.*

2. *What does the model imply about the differences in capital utilization between poor and rich economies? (You may assume that all economies are identical except for their initial conditions.)*

3. *Consider an economy that has been running from $t = -N$, under the belief that $g_t = g$, for all t. In the morning of $t = 0$, the planner discovers that $g_t = 0$, for $t = 0, 1, ..., T-1$. Go as far as you can describing the dynamic behavior of consumption and the capital stock.*

4. *Use your results in 3) to discuss whether temporary decreases in government spending result in low rates of capacity utilization.*