Econ 712 - Homework # 1
(Version 1 - Issued 9/04/07)

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Due on September 13, 2007

Problem 1 (Reaching a Target) *Consider the problem faced by an entrepreneur in period* 0 *who has agreed to deliver* y^* *units of output in period* T*. Output produced at time* t *can be accumulated without depreciation. Let the flow of output at time* t *be denoted* x_t*. The law of motion of cumulative output is*

$$y_{t+1} = y_t + x_t, \qquad t \geq 0.$$

Assume that the cost of producing x_t *is given by*

$$c(x) = \frac{x^2}{2}.$$

There is no discounting. Let the initial level of output satisfy $y^* > y_0 \geq 0$.

1. *Describe the problem faced by an entrepreneur that wants to minimize the cost of delivering* y^* *units of output in period* T.
2. *Go as far as you can analyzing the solution to the problem. Is it unique?*
3. *How would your answer change if the entrepreneur discounted future costs at rate* β $(0 < \beta < 1)$*, and output depreciated. In this case, the objective function (to be minimized) is*

$$\sum_{t=0}^{T-1} \beta^t \frac{x_t^2}{2},$$

and the law of motion is

$$y_{t+1} = (1-\delta) y_t + x_t, \qquad t \geq 0.$$

Assume that $0 < \delta < 1$.

Problem 2 (Growth) *Consider the problem faced by Robinson Crusoe.*

$$\max \sum_{t=o}^{T} \beta^t c_t, \qquad 0 < \beta < 1,$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq Ak_t, \qquad k_0 > 0,\ A > 0, \\ c_t &\geq 0,\ k_{t+1} \geq 0 \qquad t \geq 0 \end{aligned}$$

1. *Does the problem satisfy the conditions of the Kuhn-Tucker theorem?*
2. *Go as far as you can describing the solution(s). Discuss whether the solution(s) is unique.*

Note: Make sure that describe the solution for all feasible combinations (A, β).

Problem 3 (Random Math) 1. *Consider the following problem*

$$\max_{x \geq 0, y \geq 0} x^2 + y^2,$$

subject to

$$x + y \leq K, \qquad K > 0.$$

   (a) *Go as far as you can characterizing the solution. Is it unique? Justify your answer.*

2. *Consider the following problem*

$$\max_{x \geq 0, y \geq 0} \sqrt{y} + \sin(x),$$

subject to

$$mx + y \leq 1, \qquad m > 2.$$

   (a) *Go as far as you can characterizing the solution. Is it unique? Justify your answer.*

   (b) *How would your answer and argument change if the constraint had been*

$$mx + y \leq 1, \qquad m < 1.$$

   Be explicit about what is different in this case (i.e. what type of argument is not available).