Econ 712 Macroeconomic Theory- Second Exam. University of Wisconsin

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December 11, 2003

1 Instructions

- Please answer all questions. If you get stuck in one section move to the next one. Do not waste time on questions that you find hard to solve.
- Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- This is a closed book exam. Students may bring one page (both sides) or two pages (single sides) of notes.
- Please hand in the exam promptly at 11 AM.
- Each question is worth 50 points. The point total for each section is indicated at the beginning of the section. Look at these "prices" when deciding how to allocate your time!!
- If you believe that a question is wrong or poorly worded, please make the "minimal" necessary changes to make it "beautiful" and well posed. Of course, unnecessary changes will result in a lower grade.
- *Suggestion*: Do not try the extra credit sections until you finished the regular exam! They are more "challenging" than the regular questions, and they are there so that you can show off.
- Please use one blue book for each question and write only on the "right" page. (The odd numbered page in a newspaper.)
- Please remember to put your name in each blue book.
- Good luck !

2 Questions

Problem 1 (Aid Programs and Unemployment) *Consider a model in which unemployed workers draw wage offers from the distribution $F(w)$. Once a job is accpeted it lasts forever and the wroker cannot quit.*

In this economy, there is a program in place designed to help low income and unemployed workers. The progran works as follows: If the worker is unemployed, he receives a wage equal to b. If he is employed, the benefit is reduced at the rate k for employed individuals. This, for a worker who participates in the program, income is

$$y_p(w) = \begin{cases} b & if\ w = 0\ (unemployed) \\ b + (1-k)w & if\ w > 0\ (employed) \end{cases}.$$

Participation in the program is voluntary, and an individual may choose to quit the program after observing his wage draw. (The individual cannot quir the job.) It is clear that an individual will leave the program if $w > y_p(w)$. It follows that a worker will leave the program if $w > \bar{w}$, where $\bar{w} = b/k$, since at this point $y_p(\bar{w}) = \bar{w}$. Ignore how this program is financed (i.e. ignore the government budget constraint).

1. *(10 points) Describe the individual search problem. Argue that the optimal strategy is of the reservation wage variety. Denote the reservation wage by z.*
2. *(15 points) Consider the case in which the (endogenously determined) reservation wage, z, is such that $z \leq \bar{w}$. Go as far as you can characterizing the effect of increasing b on the reservation wage, and the average duration of unemployment. Describe the impact of an increase in b on the probability of leaving unemployment, and on the probability of leaving the program.*
3. *(15 points) Consider the case in which the reservation wage, z, is such that $z > \bar{w}$. Go as far as you can characterizing the effect of increasing b on the reservation wage, and the average duration of unemployment. Describe the impact of an increase in b on the probability of leaving unemployment, and on the probability of leaving the program.*
4. *(10 points) What accounts for the differences in the results you obtained in 2) and 3)?*
5. ***Extra Credit**: Go as far as you can describing the effects of an increase in k. (This looks harder, and the answer may depend on where in a particular interval lies the reservation wage.)*

Problem 2 (Social Security and Growth with Finite Lifetimes) *Consider a model in which individuals live for two periods. Let c_j^t be consumption of time j good by an individual of generation t. Each individual is endowed with one unit of labor when young and zero when old. Individuals inelastically supply their one unit of labor when* *young, and they save (in the form of purchases of real capital) when they are old. Thus, a representative agent of generation t solves the following problem*

$$\max \ln c_t^t + \beta \ln c_{t+1}^t$$

subject to

$$\begin{aligned} c_t^t + s_t &\leq w_t, \\ c_{t+1}^t &\leq s_t(1 + r_{t+1)}, \end{aligned}$$

where s_t is saving and $1 + r_{t+1}$ is the rate of return between periods t and $t+1$. Since there is no population growth, we normalize the size of each generation to one. The initial old simply maximize their second period consumption.

Aggregate output in this economy, Y_t satisfies

$$Y_t \leq AK_t^{\alpha} N_t^{1-\alpha}, \qquad 0 < \alpha < 1.$$

Assume that capital depreciates fully. Assume that there is one (or a large number) of firms that rent capital from households and hire labor to maximize profits.

1. *(10 points) Define an equilibrium for this economy. Go as far as you can displaying a difference equation that the equilibrium must satisfy.*

2. *(10 points) Argue that there is a unique steady state capital per worker. Is it the case that, at the equilibrium steady state capital per worker, the marginal product of capital is greater than one?*

3. *(20 points) Consider now the impact of a social security system. Each individual of generation t —including the initial old— receive b_{t+1} units of consumption when old and are taxed at the rate τ when they are young. Thus, the individual optimization problem is*

$$\max \ln c_t^t + \beta \ln c_{t+1}^t$$

subject to

$$\begin{aligned} c_t^t + s_t &\leq (1 - \tau)w_t, \\ c_{t+1}^t &\leq s_t(1 + r_{t+1)} + b_{t+1}. \end{aligned}$$

These social security payments are financed with taxes. Thus, the government budget constraint is

$$b_t = \tau w_t.$$

Argue that an increase in τ will lead to a decrease in the steady state level of capital per worker.

4. *(10 points) Are there parameterizations of this economy (basically vectors (A, α, β)) such that the introduction of a 'small' social security system (i.e. small τ) is a Pareto improvement over the laissez-faire equilibrium when comparing* steady state utilities*? Is it possible that such a policy change results in a Pareto superior allocation? (This means that individuals of all generations are better off, including the initial old).* ***Note****: You may assume that before the introduction of the social security system the economy is at the steady state.*

5. ***Extra Credit****: Consider now a social security system that invests in 'special' government issued bonds. These bonds pay a rate of return equal to $1 + q_{t+1}$, where $q_t \leq r_t$, that is, government bonds have (potentially) a lower rate of return than private assets. In this case individuals understand that their benefits depend on their contributions, i.e. each individual believes that $b_{t+1} = \tau w_t(1 + q_{t+1})$. Go as far as you can analyzing the effect of this social security regime when $q_t = r_t$. Argue that if $q_t < r_t$, a government that invests the proceeds from the social security tax purchasing physical capital can use the social security surplus to finance a policy of transfers to the young. Go as far as you can describing the equilibrium in the case $q_t = (1 - \phi)r_t,\ 0 < \phi < 1$.*