Asset Pricing Notes

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Abstract

Lecture notes (literally) from Econ 712. Fall 2007. Use at your own risk.

Model

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \; c_t + x_t \leq f(k_t) + e_t$$

e_t is random and $f(k) = 0, \; \forall k$, implies

$$c_t^* = e_t. \; (\because f(k) = 0)$$

$$c_t + x_t \leq z_t f(k_t) + e_t$$

e_t is the source of randomness. $\{x_t\} \Leftrightarrow \{e_t\} = e(x_t), \; \{e_t\}$ is a Markov process

$$P(e_{t+1} \in A | e_t, e_{t-1}, \cdots) = P(e_{t+1} \in A | e_t).$$

Example 1

$$\begin{aligned} e_{t+1} &= \bar{e} + (1-\rho) e_t + \varepsilon_{t+1}, \; \varepsilon_t \overset{i.i.d.}{\sim} N \\ e_{t+1} &= g(e_t, \varepsilon_{t+1}), \; \{\varepsilon_t\} \; \textit{is a sequence of i.i.d. r.v.} \end{aligned}$$

For $f(e_t, e_{t+1})$,

$$E[e_{t+1} | e_t] = \int_E e f(e_t, e) \, de.$$

$$g(e_{t+1}) \to E[g(e_{t+1}) | e_t] = \int_E g(e) f(e_t, e') \, de'.$$

If $e_t \in \{e, e_n\}, \; f(e_t, e_j)$

$$E[g(e_{t+1}) | e_t] = \sum_{j=1}^{n} g(e_j) f(e_t, e_j).$$

Expected utility

$$u(c_t) + \beta \sum_{j=1}^{n} f(e_t, e_j) u[c_{t+1}(e_j)] + \beta^2 \cdots$$

$$c_t + b_{t+1} + \underbrace{p_t^s s_{t+1}}_{\text{Stock}} + \sum_{j=1}^{n} \underbrace{q_t(e_j) z_t(e_j)}_{\text{Arrow security (lottery)}} \leq \text{ wealth at } t \qquad (\lambda_t)$$

$$\begin{aligned} c_{t+1}(e_j) + b_{t+2}(e_j) + p_{t+1}^s s_{t+2}(e_j) \;\; \leq \;\; & y_{t+1}(e_j) + R_{t+1} b_{t+1} \\ & + s_{t+1} \left[p_{t+1}^s(e_j) + d_{t+1}(e_j) \right] + \underbrace{z_t(e_j)}_{\#\text{ of winning ticket}} . \; \left(\hat{\lambda}_{t+1}(e_j) \right) \end{aligned}$$

in a complete market. Let

$$\hat{\lambda}_{t+1}(e_j) = \beta f(e_t, e_j) \lambda_{t+1}(e_j).$$

The lagrangian

$$u(c_t) + \beta \sum_{t=1}^{n} f(e_t, e_j) \left\{ \begin{array}{c} u(c_{t+1}(e_j)) \\ +\lambda_{t+1}(e_j) \left[\begin{array}{l} y_{t+1}(e_j) + R_{t+1} b_{t+1} + \left[p^s_{t+1}(e_j) + d_{t+1}(e_j)\right] s_{t+1} \\ +z_t(e_j) - c_{t+1}(e_j) - b_{t+2}(e_j) - p^s_{t+1}(e_j) s_{t+2}(e_j) \\ -\sum_{s=1}^{n} q_{t+1}(e_s) z_{t+1}(e_s) \end{array} \right] \end{array} \right\}$$
$$+\lambda_t \left[wealth@t - c_t - b_{t+1} - p^s_t s_{t+1} - \sum_{j=1}^{n} q_t(e_j) z_t(e_j) \right].$$

F.O.Cs:

$$\begin{aligned}
c_t &: u'(c_t) = \lambda_t. \\
b_{t+1} &: -\lambda_t + R_{t+1} \beta \sum_{j=1}^{n} \lambda_{t+1}(e_j) f(e_t, e_j) = 0. \qquad \text{discounted expected MU} \\
s_{t+1} &: -\lambda_t p^s_t + \beta \sum_{j=1}^{n} f(e_t, e_j) \lambda_{t+1}(e_j) \left[p^s_{t+1}(e_j) + d_{t+1}(e_j)\right] = 0. \\
z_t(e_j) &: -\lambda_t q_t(e_j) + \beta \lambda_{t+1}(e_j) f(e_t, e_j) = 0. \\
c_{t+1}(e_j) &: \beta f(e_t, e_j) \left[u'(c_{t+1}(e_j)) - \lambda_{t+1}(e_j)\right] = 0. \\
b_{t+1} &: u'(c_t) = \beta R_{t+1} \sum_{j=1}^{n} u'(c_{t+1}(e_j)) f(e_t, e_j) \\
&: u'(c_t) = \beta R_{t+1} E_t \{u'(c_{t+1})\} \\
s_{t+1} &: u'(c_t) p^s_t = \beta E_t \left\{u'(c_{t+1}) \left(p^s_{t+1} + d_{t+1}\right)\right\}
\end{aligned}$$

$$\begin{aligned}
z_t(e_j) &= u'(c_t) q_t(e_j) = \beta u'(c_{t+1}(e_j)) f(e_t, e_j) \\
&\Rightarrow q_t(e_j) = \frac{\beta u'(c_{t+1}(e_j))}{u'(c_t)} f(e_t, e_j).
\end{aligned}$$

Non-random economies

$$\begin{aligned}
\{e_t\} &\rightarrow \left\{ \begin{array}{l} f(e_t, e_{t+1}) = 1 \\ f(e_t, e) = 0, \ e \neq e_{t+1} \end{array} \right. \\
&\Rightarrow u'(c_t) = \beta R_{t+1} u'(c_{t+1}).
\end{aligned}$$

$$\begin{aligned}
E_t(g(e_{t+1})) &= \sum_{j=1}^{n} g(e) f(e_t, e) = \int_E g(e) f(e_t, e)\, de. \\
F(e_t, e^*) &\equiv P(e_{t+1} \leq e^* | e_t) \\
E_t[g(e_{t+1})] &= \int g(e) F(e_t, de).
\end{aligned}$$

$$R^s_{t+1}(e_j) \equiv \frac{p^s_{t+1}(e_j) + d_{t+1}(e_j)}{p^s_t}.$$

$$\left.\begin{array}{l} u'(c_t) = \beta E_t\left[u'(c_{t+1}) R^s_{t+1}\right] \\ u'(c_t) = \beta E_t\left[u'(c_{t+1}) R_{t+1}\right] \end{array}\right\} = 0 = \underbrace{E_t\left\{u'(c_{t+1})\left(R^s_{t+1} - R_{t+1}\right)\right\}}_{\text{Conditional Variance}}$$

If $u' > 0 \Rightarrow \left(R^s_{t+1} - R_{t+1}\right)$ must have some state $-ve$. [Remark – Nonstochastic economy: $1 - \delta + r_{t+1} = R^s_{t+1} = R_{t+1}$].

In general, $X,\ Y$ two random variables

$$\begin{aligned} Cov(X,Y) &= E\left[(X - E(X))(Y - E(Y))\right] \\ &= E(XY) - E(X)E(Y). \end{aligned}$$

Let $X = u'(c_{t+1}),\ Y = R^s_{t+1} - R_{t+1}$,

$$E_t(XY) = 0 = E_t(X)E_t(Y) + Cov_t(XY).$$

Good time: $R_{t+1} \uparrow,\ c_{t+1} \uparrow,\ u'(c_{t+1}) \downarrow$.

Bond:

$$R^{-1}_{t+1} \stackrel{?}{=} \sum_{j=1}^{n} q_t(e_j). \qquad \text{(no arbitrage)}$$

$$\beta\frac{\sum_{j=1}^{n} u'(c_{t+1}(e_j)) f(e_t, e_j)}{u'(c_t)} = \frac{\beta E_t\{u'(c_{t+1})\}}{u'(c_t)} = R^{-1}_{t+1}\ \ (From\ b_{t+1}).$$

Assume pure discount bond:

$$\begin{aligned} & c_t + b_{t+1} + p^s_t s_{t+1} + \sum_{j=1}^{n} q_t(e_j) z_t(e_j) + \sum_{i=1}^{k} P(t, t+i) L(t, t+i) \\ \leq\ & \sum_{i=0}^{k-1} L(t-1, t+i) P(t, t+i) + p^s_t(s_t + d_t) \end{aligned}$$

$$L(t, t+i) = -\lambda_t P(t, t+i) + \beta\sum_{j=1}^{n} \lambda_{t+1}(e_j) f(e_t, e_j) P(t+1, t+i)(e_j)$$

Remark: Risk–free is related to payoff (not price). In this model, price is endogeneous.

$$u'(c_t) P(t, t+i) = \beta E_t\{u'(c_{t+1}) P(t+1, t+i)\}$$

This is stock with no dividend (check s_{t+1} F.O.C.).

$$u'(c_{t+i-1}) \underbrace{P(t+i-1, t+i)}_{R^{-1}_{t+i-1} \to \text{Work backward}} = \beta E_{t+i-1}\{u'(c_{t+i}) \times 1\}$$

$$\begin{aligned} u'(c_t) P(t, t+i) &= \beta^i E_t\{u'(c_{t+i})\} \\ P(t, t+i) &= \frac{\beta^i E_t\{u'(c_{t+i})\}}{u'(c_t)}. \end{aligned}$$

The term structure of interest rate

$$\begin{aligned} 1 + r_t(i) &= \sqrt[i]{P(t, t+i)^{-1}} \\ 1 + r_t(i) &= \frac{1}{\beta}\left(\frac{u'(c_t)}{E_t u'(c_{t+i})}\right)^{\frac{1}{i}} \to \frac{1}{\beta}\ for\ i \to \infty \end{aligned}$$

if $u'(c_t)$ is bounded.
[Insert diagram here]

Case (I) c_t is i.i.d. $(c_t = e_t)$

$$E_t u'(c_{t+1}) = Eu'(c_{t+1})$$

$$1 + r_t(i) = \frac{1}{\beta}\left(\frac{u'(c_t)}{Eu'(c_{t+1})}\right)^{\frac{1}{i}}$$

"Bad times" c_t is low, $u'(c_t) > E[u'(c_{t+1})]$. Temporary shock – borrowing.

Case (II) Growth $\left(c_{t+1} = \gamma_{t+1} c_t \;\; \{\gamma_{t+1}\} \text{ is i.i.d.}\right)$

$$u(c) = \frac{c^{1-\theta}}{1-\theta}$$

$$\begin{aligned} P(t, t+1) &= \frac{1}{\beta}\frac{c_t^{-\theta}}{E_t\left\{c_t^{-\theta}\gamma_{t+1}^{-\theta}\right\}}, \\ P(t, t+1) &= \frac{1}{\beta}\frac{1}{E_t\gamma_{t+1}^{-\theta}},\ \gamma_t > 1 \text{ (growing)} \end{aligned}$$

Permanent shock – no borrowing. More general process will be in between permanent and temporary shock.

Lucas critique

$$\begin{aligned} c_t &= e_t. \\ y_{t+1} &= y_t^{\phi}\varepsilon_{t+1},\ \ln\varepsilon_{t+1} \sim N\left(\mu_\varepsilon, \sigma_\varepsilon^2\right), \\ g_t &= \frac{G_t}{y_t} = \theta_t, \\ (1-\theta_{t+1}) &= (1-\theta_t)^{\xi}\upsilon_{t+1},\ \ln\upsilon_{t+1} \sim N\left(\mu_\upsilon, \sigma_\upsilon^2\right). \end{aligned}$$

$$\begin{aligned} c_t^* &= (1-\theta_t)\, y_t \qquad \left[u(c) = \frac{c^{1-\eta}}{1-\eta}\right] \\ P(t,t+1) &\to u'(c_t)\, P(t,t+1) = \beta E_t \left\{u'(c_{t+1})\right\}. \end{aligned}$$

$$\begin{aligned} \Rightarrow \left[(1-\theta_t)\, y_t\right]^{-\eta} P(t,t+1) &= \beta E_t \left\{\left[(1-\theta_{t+1})\, y_{t+1}\right]^{-\eta}\right\} \\ &= \beta E_t \left\{\left[(1-\theta_t)^{\xi}\, y_t^{\phi} \varepsilon_{t+1} \upsilon_{t+1}\right]^{-\eta}\right\} \\ \Rightarrow \left[(1-\theta_t)\, y_t\right]^{-\eta} P(t,t+1) &= \beta (1-\theta_t)^{-\xi\eta}\, y_t^{-\eta\phi}\, \underbrace{E_t\left[(\varepsilon_{t+1}\upsilon_{t+1})^{-\eta}\right]}_{e^{-\eta\mu_\varepsilon + \eta^2 \frac{\sigma_\varepsilon^2}{2}} e^{-\eta\mu_\upsilon + \eta^2 \frac{\sigma_\upsilon^2}{2}}}. \end{aligned}$$

(Assume ε_{t+1} and υ_{t+1} are independent)

$$\frac{1}{P(t,t+1)} = (1-\theta_t)^{\eta(\xi-1)}\, y_t^{\xi(\phi-1)} \frac{1}{\beta} e^{\eta\mu_\varepsilon - \eta^2\frac{\sigma_\varepsilon^2}{2}} e^{\eta\mu_\upsilon - \eta^2\frac{\sigma_\upsilon^2}{2}}.$$

$$\begin{aligned} \ln(R_{t+1}) &= \ln\left(\frac{1}{\beta}\right) + \eta(\phi-1)\ln y_t + \eta(\xi-1)\ln(1-\theta_t) + \eta\mu_\varepsilon - \eta^2\frac{\sigma_\varepsilon^2}{2} + \eta\mu_\upsilon - \eta^2\frac{\sigma_\upsilon^2}{2}, \\ &= \ln\left(\frac{1}{\beta}\right) + \eta(\phi-1)\ln y_t + \eta(1-\xi)\frac{G_t}{y_t} + \eta\mu_\varepsilon - \eta^2\frac{\sigma_\varepsilon^2}{2} + \eta\mu_\upsilon - \eta^2\frac{\sigma_\upsilon^2}{2}. \\ &\quad (\because \ln(1-x) \approx -x) \end{aligned}$$

Now suppose $(1-\theta_{t+1}) = (1-\theta_t)^{\xi}\, \upsilon_{t+1},\ \upsilon_{t+1} = \upsilon,\ \sigma_\upsilon^2 = 0,\ \xi = 1,$

$$\ln R_{t+1} = \beta_0 + \beta_1 \ln y_t + \beta_2 \ln \frac{G_t}{y_t},\ R^2 = 1.$$

Lucas critique: $\beta_0,\ \beta_1,\ \beta_2$ no use after policy change (regime change, no use in policy analysis).

Lucas "Tree" economy

$$u'(c_t) = \beta R_{t+1} E_t \left\{u'(c_{t+1})\right\} \tag{1}$$

$$P(t,t+j) = \beta^j E_t \left\{\frac{u'(c_{t+1})}{u'(c_t)}\right\} \text{ } (j-\text{period bond}) \tag{2}$$

$$u'(c_t)\, p_t^s = \beta E_t \left\{u'(c_{t+1})\left(p_{t+1}^s + d_{t+1}\right)\right\} \text{ (Price of stock)} \tag{3}$$

$$q(c_t, c') = \beta \frac{u'(c')}{u'(c_t)} f(c_t, c') \text{ (Arrow's securities)} \tag{4}$$

Yield $\left(\frac{1}{P(t,t+j)}\right)^{\frac{1}{j}}$. Rewrite (3) → Optimal capital structure and price earning's ratio → pricing options.

$$(d_{1t}, \cdots, d_{kt}) = X_t,\ c_t = \sum_{k=1}^{K} d_{kt}.$$

From (4),

$$q(c(X_t, X')) = \beta \frac{u'(c(X'))}{u'(c(X_t))} \hat{f}(X_t, X').$$

From (3) and get rid of p_{t+1}

$$\begin{aligned} u'(c_t)\,p_t^s &= \beta E_t\left\{u'(c_{t+1})\left[d_{t+1}+\frac{\beta E_{t+1}\left(u'(c_{t+2})\left(d_{t+2}+p_{t+2}^s\right)\right)}{u'(c_{t+1})}\right]\right\} \\ &= \sum \beta^j E_t\left[E_{t+j}\left(u'(c_{t+j})\,d_{t+j}\right)\right]+\beta^T E_t\left[E_{T-1}\left(u'(c_T)\right)p_T^s\right] \\ &= \sum_{j=1}^{T}\beta^j E_t\left[u'(c_{t+j})\,d_{t+j}\right]+\underbrace{E_t\left[\beta^T u'(c_T)\,p_T^s\right]}_{\to 0 \text{ by transversality condition}}. \end{aligned}$$

$$p_t^s=\sum_{j=1}^{\infty}\beta^j E_t\left\{\underbrace{\frac{u'(c_{t+j})}{u'(c_t)}}_{\uparrow X_{t+j}}\underbrace{d_{t+j}}_{\uparrow Y_{t+j}}\right\}. \tag{5}$$

$$p_t^s \overset{?}{=} \sum_{j=1}^{\infty} E_t(d_{t+1})\,P(t,t+j) \qquad \text{(Expectational theory)}$$

From (5)

$$\begin{aligned} p_t^s &= \sum_{j=1}^{\infty}\beta^j E_t\left[\frac{u'(c_{t+j})}{u'(c_t)}\right]E_t(d_{t+j})+\beta^j Cov_t\left(\frac{u'(c_{t+j})}{u'(c_t)},d_{t+j}\right) \\ &= \sum_{j=1}^{\infty}P(t,t+j)\,E_t(d_{t+j})+\underbrace{\sum_{j=1}^{\infty}\beta^j Cov_t\left(\frac{u'(c_{t+j})}{u'(c_t)},d_{t+j}\right)}_{\text{(Market price of risk)}} \end{aligned}$$

If $u(c_t)=\bar{u}_0+\bar{u}_1 c_t$ (Risk neutral), $Cov_t=0$.

<u>Two trees</u>

$$\begin{array}{ccc} & d_{t+j}^A & d_{t+j}^B \\ E_t() & E\left(d_{t+j}^A\right) & E\left(d_{t+j}^B\right) \\ Cov_t() & Cov_t\left(c_{t+1},d_{t+1}^A\right)>0 & Cov_t\left(c_{t+1},d_{t+1}^B\right)<0 \end{array}$$

Expectational theory (only concern 1st moment) $\Rightarrow$

$$\sum_{j=1}^{\infty}P(t,t+j)\left[\frac{E_t\left(d_{t+j}^A\right)}{p_{At}^s}-\frac{E_t\left(d_{t+j}^B\right)}{P_t^B}\right]=0.$$

(2nd moment)

$$Cov_t\left(\frac{u'(c_{t+j})}{u'(c_t)},d_{t+j}^A\right)<0,\ Cov_t\left(\frac{u'(c_{t+j})}{u'(c_t)},d_{t+j}^B\right)>0.$$

$$\begin{aligned} \left\{d_t^k\right\} &= \left(d_{t+1}^k,d_{t+2}^k,\cdots\right) \\ \text{One period bond} &= (1,0,0,\cdots) & -P(t,t+1)=R_{t+1}^{-1} \\ j-\text{period bond} &= \Big(0,\cdots,0,\underset{\uparrow t+j}{1},0,\cdots\Big) & -P(t,t+j) \\ \text{Constant coupon bond} &= (n,n,n,\cdots) \end{aligned}$$

If $X_{t+2} = X''$ and $X_{t+1} = X'$ (1 unit @$t+2$, component effect)

$$\int_{\mathbb{X}} \beta \frac{u'\left(c\left(X'\right)\right)}{u'\left(c\left(X_t\right)\right)} \beta \frac{u'\left(c\left(X''\right)\right)}{u'\left(c\left(X'\right)\right)} \hat{f}\left(X_t, X'\right) \hat{f}\left(X', X''\right) dX'$$

Two period Arrow–Debreu price

$$\begin{aligned} q^2\left(X_t, X''\right) &= \beta^2 \frac{u'\left(c\left(X''\right)\right)}{u'\left(c\left(X_t\right)\right)} \int_{\mathbb{X}} \hat{f}\left(X_t, X'\right) \hat{f}\left(X', X''\right) dX \\ &= \beta^2 \frac{u'\left(c\left(X''\right)\right)}{u'\left(c\left(X_t\right)\right)} \hat{f}^{(2)}\left(X_t, X''\right). \\ q^j\left(X_t, X''\right) &= \beta^j \frac{u'\left(c\left(X''\right)\right)}{u'\left(c\left(X_t\right)\right)} \hat{f}^{(j)}\left(X_t, X''\right). \end{aligned}$$

Now

$$\begin{aligned} p_t^s &= \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} d_{t+j} \right\} \\ &= \sum_{j=1}^{\infty} \int_{\mathbb{X}} \beta^j \frac{u'\left(c\left(X_{t+j}\right)\right)}{u'\left(c\left(X_t\right)\right)} d_{t+j}\left(X_{t+j}\right) \hat{f}^{(j)}\left(X_t, X_{t+j}\right) dX_{t+j} \\ &= \sum_{j=1}^{\infty} \int d_{t+j}\left(X_{t+j}\right) q^j\left(X_t, X_{t+j}\right) dX_{t+j} \end{aligned}$$

(All the price can be expressed in terms of Arrow security)

$$r_{t+j}\left(X_{t+j}\right) \text{ arbitrary payout}$$

$$p_t^B = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} r_{t+j}\left(X_{t+j}\right) \right\}$$

If B bonds are issued,

$$\hat{d}_{t+j} = d_{t+j} - r_{t+j}\left(X_{t+j}\right) B$$

$$\hat{p}_t^s = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} \hat{d}_{t+j} \right\}$$

$$p_t^s = \hat{p}_t^s + p_t^B B$$

$$p_t^s = \sum_{j=1}^{\infty} \beta^j E_t \left\{ \frac{u'\left(c_{t+j}\right)}{u'\left(c_t\right)} \left[\underbrace{\hat{d}_{t+j} + r_{t+j}\left(X_{t+j}\right) B}_{d_{t+j}} \right] \right\}$$

Option: Gives the holder the right to buy (before devidends are paid out) a share at price $\bar{p}$ (strike price)

$$\begin{aligned} V\left(X_t, \bar{p}\right) &= \int_{\mathbb{X}} r^v\left(X_{t+1}\right) q\left(X_t, X_{t+1}\right) dX_{t+1} \\ r^v &= \begin{cases} p^s\left(X_{t+1}\right) + d_{t+1}\left(X_{t+1}\right) - \bar{p} \geq 0 & \text{if } X_{t+1} \in A \\ 0 & \text{if } X_{t+1} \in A^c \end{cases} \end{aligned}$$

$$V\left(X_t,\bar{p}\right)+\bar{p}P\left(t,t+1\right)\overset{?}{\gtreqless}p_t^s$$

$$\begin{aligned}
V\left(X_t,\bar{p}\right) &= \int_A\left[p_{t+1}^s\left(X_{t+1}\right)+d_{t+1}\left(X_{t+1}\right)-\bar{p}\right]q\left(X_t,X_{t+1}\right)dX_{t+1}\\
&= \int_{\mathbb{X}}\left[p_{t+1}^s\left(X_{t+1}\right)+d_{t+1}\left(X_{t+1}\right)-\bar{p}\right]q\left(X_t,X_{t+1}\right)dX_{t+1}\\
&\quad -\int_{A^c}\left[p_{t+1}\left(X_{t+1}\right)+d_{t+1}\left(X_{t+1}\right)-\bar{p}\right]q\left(X_t,X_{t+1}\right)dX_{t+1}
\end{aligned}$$

$$\begin{aligned}
V &= \int_{\mathbb{X}}\underbrace{\left[p_{t+1}^s+d_{t+1}\right]q\left(X_t,X_{t+1}\right)dX_{t+1}}_{p_t^s}-\bar{p}\int_{\mathbb{X}}\underbrace{q\left(X_t,X_{t+1}\right)dX_{t+1}}_{P(t,t+1)}\\
&\quad -\int_{A^c}\left[p_{t+1}+d_{t+1}-\bar{p}\right]q\left(X_t,X_{t+1}\right)dX_{t+1}
\end{aligned}$$

$$\underbrace{\bar{p}P\left(t,t+1\right)+V\left(X_t,\bar{p}\right)}_{\text{For sure get }\bar{p}}=p_t^s-\underbrace{\int_{A^c}\left[\overbrace{p_{t+1}+d_{t+1}-\bar{p}}^{-ve}\right]q\left(X_t,X_{t+1}\right)dX_{t+1}}_{+ve}$$