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Econ 712: Macroeconomic Theory
Midterm Exam (10/17/98)

- Please answer all questions. If you get stuck in one section move to the next one. Do not waste a lot of time on questions that you find hard to solve.
- Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- This is a closed book exam. Students may bring two pages of notes.
- This is a two hour exam. Please hand it in at 12:00.
- Each question is worth 50 points. The point total for each section is indicated at the beginning of the section.
- Look at the point total for each section when deciding how to allocate your time!!
- Please use **one blue book** for each question and write only on the "right" page. (The odd numbered page in a newspaper.)
- Please remember to put your name in each blue book.
- Good luck !

Problem #1 **"Financial Crises and Domestic Interest Rates"** (50 Points)

Consider a representative agent once sector growth model with preferences given by,

$$\sum_{t=0}^{\infty} \beta^t u(c_t).$$

This economy is open and has access to an international bond market. The international interest rate R* satisfies $R^*\beta = 1$. The budget constraint for this economy is,

$$c_t + k_{t+1} - (1-\delta)k_t + b_{t+1} \leq f(k_t) + R^* b_t,$$

with k_0 and b_0 given. At time zero the "rest of the world" announces that from time T on $b_{T+j} \geq 0$. That is, starting at time T this country can no longer borrow.

a) (35 points). Suppose that $f(k_t) = y > 0$ and that $b_0 < 0$ (this economy is a net debtor). (Yes, this is a pure exchange economy and "all" future k's will be zero as well!)

i) Show that domestic interest rates [it is OK to measure domestic interest rates as $u'(c_t)/(\beta u'(c_{t+1}))$] will not increase immediately.

ii) Argue that relative to the pre-announcement regime, consumption will drop at t =0 and then, at time T, jump up to a level higher than before the announcement.

iii) Finally, show that domestic interest rates will exceed world interest rates between time T-1 and T.

b) (15 points) Suppose now that f(k) is productive and that $k_0 = k^*$ where k* is the steady state capital per worker [i.e., it satisfies $1 = \beta(1-\delta+f'(k^*))$]. As before assume that $b_0 < 0$.

i) Describe the "pre-announcement" level of consumption.

ii) Argue that after the announcement (at the beginning of period 0), consumption drops but it is constant until period T-1.

iii) What happens to capital and investment from t=0 to t=T-1.

iv) Show that $k_T < k^*$ if you can. If you cannot, assume that $k_T < k^*$ and argue that domestic interest rates will exceed world interest rates after period T.

Problem #1 **"Financial Crises and Domestic Interest Rates"** - Solution

To analyze the planner's problem, form the Lagrangean given by,

$$L(\mathbf{c},\mathbf{k},\mathbf{b},\boldsymbol{\lambda},\boldsymbol{\gamma}) = \sum_{t=0}^{\infty} \beta^t \{u(c_t) + \lambda_t [f(k_t) + (1-\delta)k_t - b_{t+1} + R^*b_t - c_t - k_{t+1}] + \sum_{j=0}^{\infty} \beta^j \gamma_{T+j} b_{T+j},$$

where we have substituted in the law of motion for the capital stock, and ignored the Lagrange multipliers for the non-negativity constraint on investment except, of course, for the stock of foreign bonds. (Note that, the transversality condition on the set of feasible debt values is automatically satisfied.)

The first order conditions for an interior solution are the feasibility constraints at equality and,

c_t :	(1)	$u'(c_t) - \lambda_t = 0,$	$t = 0,1,2,....$
b_{t+1} :	(2)	$\lambda_t = \beta R^* \lambda_{t+1},$	$t = 0, 1,..., T-2$
b_T :	(3)	$\lambda_{T-1} = \beta\gamma_T,$	
k_{t+1}:	(4)	$\lambda_t = \beta\lambda_{t+1} [(1-\delta) + f'(k_{t+1})]$,	$t = 0, 1, 2, ...$

and the following transversality condition,

(5) $\lim_{T\to\infty} \beta^T \lambda_T k_{T+1} = 0,$

where conditions (4) and (5) apply only to part b). That is, in part a) the "optimal" x_t is equal to zero. Since the resulting k_t is not interior it is not correct to use *any* version of (4). Thus, for part a) the first order conditions do not include (4) or (5).

a) Note that (2) implies that, since βR^* equals one, the marginal utility of consumption is constant; i.e. λ_t is constant. This, in turn (see (1)), implies that consumption is constant for t = 0, 1, ..., T-1. Thus, if there is one technology (in this case the "bond market") that gives a return equal to the discount rate, then the optimal consumption plan is constant. The feasibility constraint in case a) is given by,

(6) $b_{t+1} = y + R^*b_t - c_t$ $\quad t = 0, 1, 2, ...$

from which it follows that, after time T, $c_{T+j} = y$ for all $j \geq 0$.

What happens before the announcement? Since $\beta R^*=1$, consumption was expected to be constant and, to satisfy the transversality condition on the stock of bonds, foreign debt had to be constant as well. Then, the pre-announcement level of consumption, denoted c_i (i stands for access to international markets) is just,

(7) $c_i = y + (R^*-1)b_0 < y,$

as b_0 is negative.

What happens during periods 0 and T-1? We have already argued that consumption will be constant, and let's denote this constant level by c_f (f for financial crisis). It must satisfy the appropriate version of (6) for a given initial condition, b_0, *and* a terminal condition given by b_T =

0. Thus, during the transition, we have,

(8) $b_{t+1} = y + R^*b_t - c_f$.

At this point it is easy to see that $c_f < c_i$ because c_i keeps the debt level constant, while c_f brings it down to zero in finite time; and also that $c_f < c_a = y$, where c_a is the "autarky" level of consumption. Can we calculate the value of c_f? Yes, just iterate on (8) and impose that $b_T = 0$. Formally, b_T, is given by,

(9) $b_T = \sum_{t=0}^{T-1} (R^*)^t (y - c_f) + (R^*)^T b_0$,

and imposing that $b_T = 0$, we get,

(10) $c_f = y + [(R^*)^T/(1-(R^*)^T)](R^*-1)b_0$.

Thus, comparing (10) and (7) it follows that $c_f < c_i$. This is the answer to part ii). To answer part i) note that the domestic interest rate, R_{t+1}, satisfies,

(11) $R_{t+1} = u'(c_t)/(\beta u'(c_{t+1}))$.

Since consumption is constant between t = 0, and t= T-1, $R_{t+1} = R^*$, t = 0, 1, ..., T-2. At T-1 we get $R_{T-1} = u'(c_f)/(\beta u'(c_a)) > \beta^{-1}$ since $c_a > c_f$. This establishes iii).

b) In this section we allow capital to be productive. First note that the economy starts at the steady state because with full access to international markets, $[(1-\delta) + f'(k_{t+1})] = R^*$ which implies $k_t = k^*$ where k^* is the solution to $\beta[(1-\delta) + f'(k^*)] = 1$. Next, observe that (2) implies that $k_t = k^*$ for t = 0, 1,..., T-1. Also (2) implies that consumption is constant. These two results imply that investment does not change from t = 0 to t = T-1: it is just $x^* = \delta k^*$.

The pre-announcement level of consumption was,

(12) $\hat{c}_i = f(k^*) - \delta k^* + (R^*-1)b_0$,

that is, consumption is equal to steady state income minus (because b_0 is negative) interest payments to the rest of the world. After the announcement, consumption is given by $\hat{c}_f$ (some constant), and foreign debt evolves according to,

(13) $b_{t+1} = f(k^*) - \delta k^* + R^*b_t - \hat{c}_f$,

with the added requirement that b_T be equal to zero. Inspection of (12) and (13) shows that $\hat{c}_f < \hat{c}_i$.

In this answer (to "publish it" promptly) I will assume $k_T < k^*$. In this case the economy is a closed economy after period T. Standard arguments show that for any initial capital stock less than k^*, the economy will converge (monotonically) to k^*. Thus, the capital stock increases over time. The domestic interest rates satisfy,

$$R_{t+1} = [(1-\delta) + f'(k_{t+1})], \tag{14}$$

and, hence, decrease over time.

Problem #2 **"Unemployment Compensation and Employment Probability"** (50 Points)

Consider the behavior of a risk neutral worker that seeks to maximize the expected present discounted value of wage income. Assume that the discount factor fixed and equal to β, with $0<\beta<1$. In this economy, jobs are forever. Once the worker has accepted a job, he/she never quits and the job is never destroyed. If an individual is unemployed he/she receives $c > 0$ units of consumption if he/she has been unemployed for at most n-1 periods. From the nth period on, an unemployed worker receives $c = 0$.

a) (10 points) Consider the situation of an unemployed worker who is in his/her nth period of unemployment (that is, he/she is not collecting unemployment benefits). Argue that the optimal strategy is of the reservation wage variety, and display a formula for the reservation wage. Denote this reservation wage by $w^*(0)$.

b) (30 points) Consider next the situation of an unemployed worker who is in his/her (n-1)th period of unemployment (that is, this is the last period that the worker will collect unemployment compensation). Let the reservation wage of such a worker be $w^*(1)$. Argue that $w^*(1) > w^*(0)$. What does this say about the probability of leaving unemployment as a function of the duration of the unemployment spell?

c) (10 points) Let $w^*(2)$ be the reservation wage of a worker who can collect unemployment compensation for two periods before his/her eligibility expires. Show that $w^*(2) > w^*(1)$.

Extra Credit: Show that $w^*(k) > w^*(k-1)$

Problem #2 **"Unemployment Compensation and Employment Probability"**- Solution

a) Consider the problem faced by a worker who has exhausted his/her unemployment benefits and has offer w in hand. The value of such an offer, denoted $V(w,0)$ is just,

(1) $$V(w,0) = \max \{ w/(1-\beta), \beta Q(0)\},$$

where $Q(0)$ is the expected value of being unemployed tomorrow, and it satisfies,

(2) $$Q(0) = E[V(w,0)].$$

To calculate the expected value note that (1) implies that there exists a value of w, denoted $w^*(0)$, such that $w^*(0)/(1-\beta) = \beta Q(0)$, and, hence, that $V(w,0)$ is given by,

(3) $$V(w,0) = \begin{cases} w^*(0)/(1-\beta) & \text{if } w \leq w^*(0) \\ w/(1-\beta) & \text{if } w \geq w^*(0). \end{cases}$$

It then follows that (2) is just,

$$(4) \qquad Q(0) = \int_0^{w^*(0)} w^*(0)/(1-\beta)F(dw') + \int_{w^*(0)}^{B} w'/(1-\beta)F(dw')$$
$$= w^*(0)/(1-\beta) + \int_{w^*(0)}^{B} [w' - w^*(0)]/(1-\beta)F(dw').$$

Using this expression in $w^*(0)/(1-\beta) = \beta Q(0)$ we get,

$$(5) \qquad w^*(0) = \beta(1-\beta)Q(0) = [\beta/(1-\beta)] \int_{w^*(0)}^{B} [w' - w^*(0)]F(dw').$$

b) Consider next the situation of a worker who has a wage offer w in hand and has one period of unemployment benefits left. Let the value of such an offer be denoted V(w,1). It follows that optimal behavior requires,

$$(6) \qquad V(w,1) = \max \{ w/(1-\beta), c+\beta Q(0)\}.$$

The reason why Q(0) --and not something that one might want to call Q(1)-- appears on this equation is that, starting tomorrow, this worker will have zero periods of benefits left. Thus, the expected value of being unemployed *tomorrow* is E[V(w,0)] = Q(0). Inspection of (6) reveals that the optimal strategy is of the reservation wage variety. Let the reservation wage be w*(1). Then, since w*(1) makes the worker indifferent between accepting or rejecting an offer, it must satisfy,

$$(7) \qquad w^*(1)/(1-\beta) = c+\beta Q(0) = c + w^*(0)/(1-\beta),$$

or,

$$(8) \qquad w^*(1) - w^*(0) = (1-\beta)c > 0.$$

Note that the probability of accepting a job when there are n periods of unemployment benefits left is just 1 - F(w*(n)). Since w*(1) > w*(0), workers with one period left of unemployment benefits reject offers that workers who have exhausted their eligibility accept and hence, have a lower probability of leaving unemployment.

c) Here I use a general approach (a more specific answer is equally acceptable) to show that reservation wages decrease over time (increase in the number of periods left). Note that for any k,

$$(9) \qquad V(w,k) = \begin{cases} w^*(k)/(1-\beta) & \text{if } w \leq w^*(k) \\ w/(1-\beta) & \text{if } w \geq w^*(k), \end{cases}$$

and that,

$$(10) \qquad V(w,k+1) = \max \{ w/(1-\beta), c+\beta Q(k)\}.$$

Parts a) and b) have established that $w^*(1) > w^*(0)$, and this implies $Q(1) > Q(0)$. The argument is by induction. Assume that $Q(k) > Q(k-1)$. Then, (10) implies that $V(w,k+1) > V(w,k)$ and this, in turn, implies that $Q(k+1) > Q(k)$. Thus, the sequence of expected value of being unemployed increases with k. However, the reservation wage, $w^*(k)$, satisfies,

$$w^*(k) = \beta(1-\beta)Q(k-1), \qquad \text{for } k \geq 1. \tag{11}$$

Since the sequence Q(k) is increasing so is the sequence $w^*(k)$.