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# The Classical Model

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Chapter 7

Lecture notes posted online

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# The Classical Model

- Model of output determination and economy-wide equilibrium
- Can be used to study economic growth and business cycles
- Considered a more complete description of the long-run than the short-run

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# Key Building Blocks

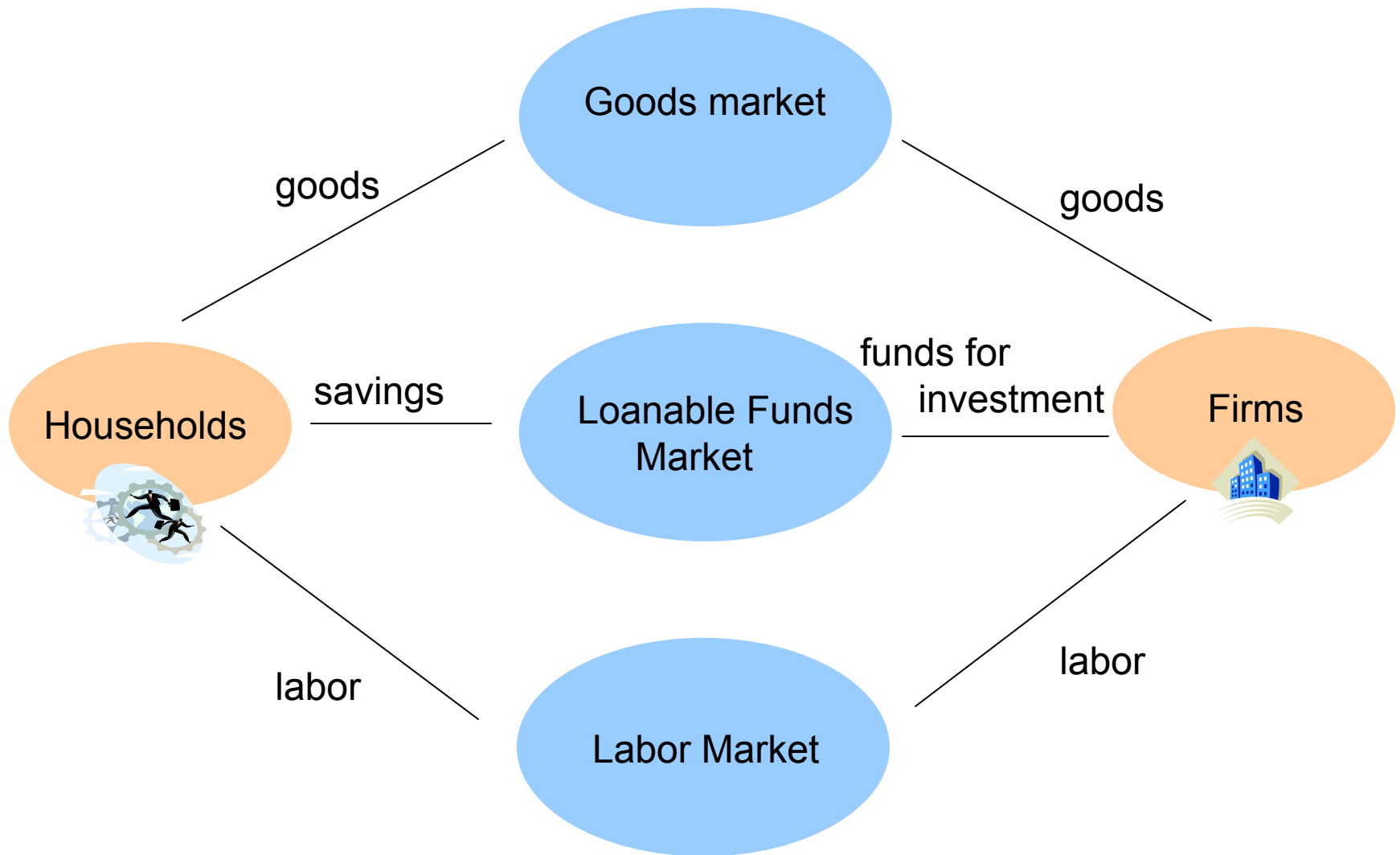
- Closed economy (no exports and imports)
- Populated by two types of agents
  - Households
  - Firms
- (Later will also add government)

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# Key Building Blocks

- Agents interact in *three* markets
  - Goods Market (GM)
  - Loanable Funds Market (LFM)
  - Labor Market (LM)
- Markets are competitive
  - Like in Econ 101, demand and supply determine the equilibrium outcome

# Agents and Markets in Classical Model



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# Markets

- **Goods market** is where firms sell their output to households (consumption goods) and other firms (investment goods)
- **Loanable funds market** is where household save funds for interest, and firms seek funds to finance investment projects
- **Labor market** is where households offer labor, and firms hire them to produce goods

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# Households

- Recipients of all factor payments by firms = income
- Make *two* important decisions:
  - Given income, decide how much to save  $S$  and consume  $C$
  - Given time endowment, decide how much labor to supply to the market, and how much leisure to consume

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# Firms

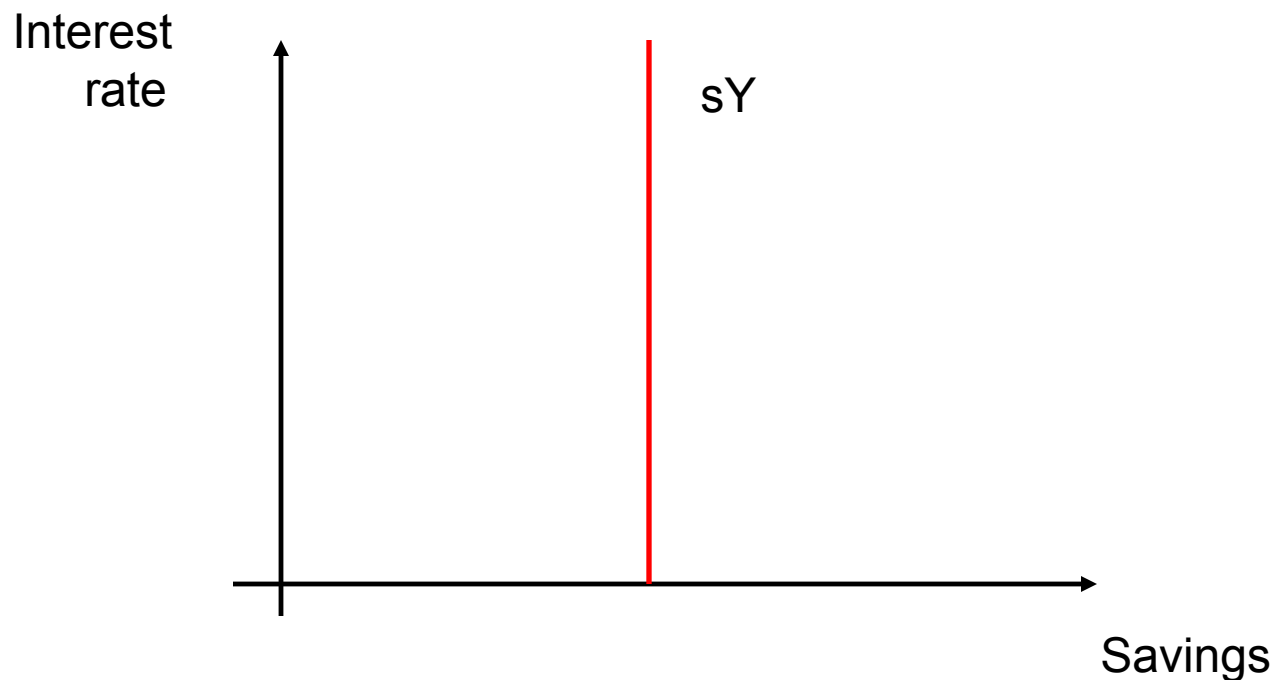
- Produce output using capital (machines) and labor (people)
- Make two important decisions:
  - How much labor to employ
  - How much to invest in new machines, that will result in more capital in the future

# Savings/Consumption Decision of HH

- Will assume households decide to save a constant fraction of their income:  $S=sY$ 
  - Implies consumption is also a constant fraction of income:  $C=Y-S=(1-s)Y$
  - Example:  $s=1/3$ , if income is 150,  $S=50$ , consumption is  $C=Y-S=(1-1/3)150=100$

# Implied Supply of Savings

- Supply of savings by households as function of interest rate (return on savings)



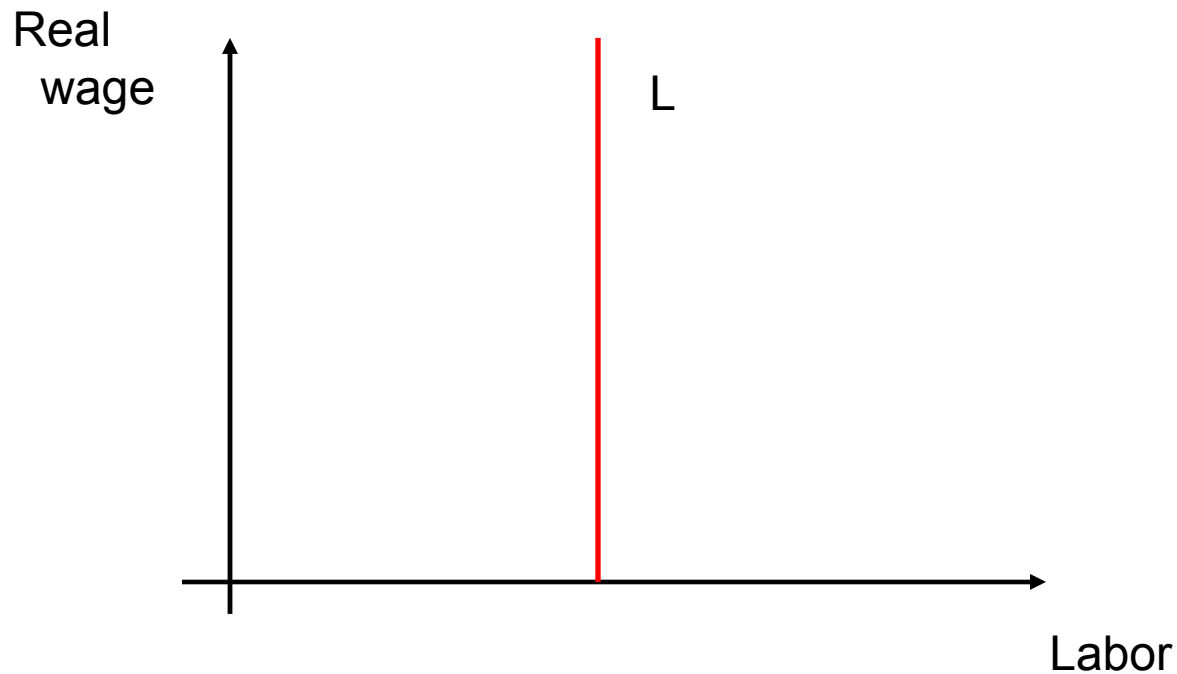
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# Labor/Leisure Choice of HH

- Assume households supply 1 unit of labor each, and so supply of labor simply equals population size =  $L$

# Supply of Labor

- Supply of labor by households as a function of real wage (real compensation of labor)



# Production in Firms

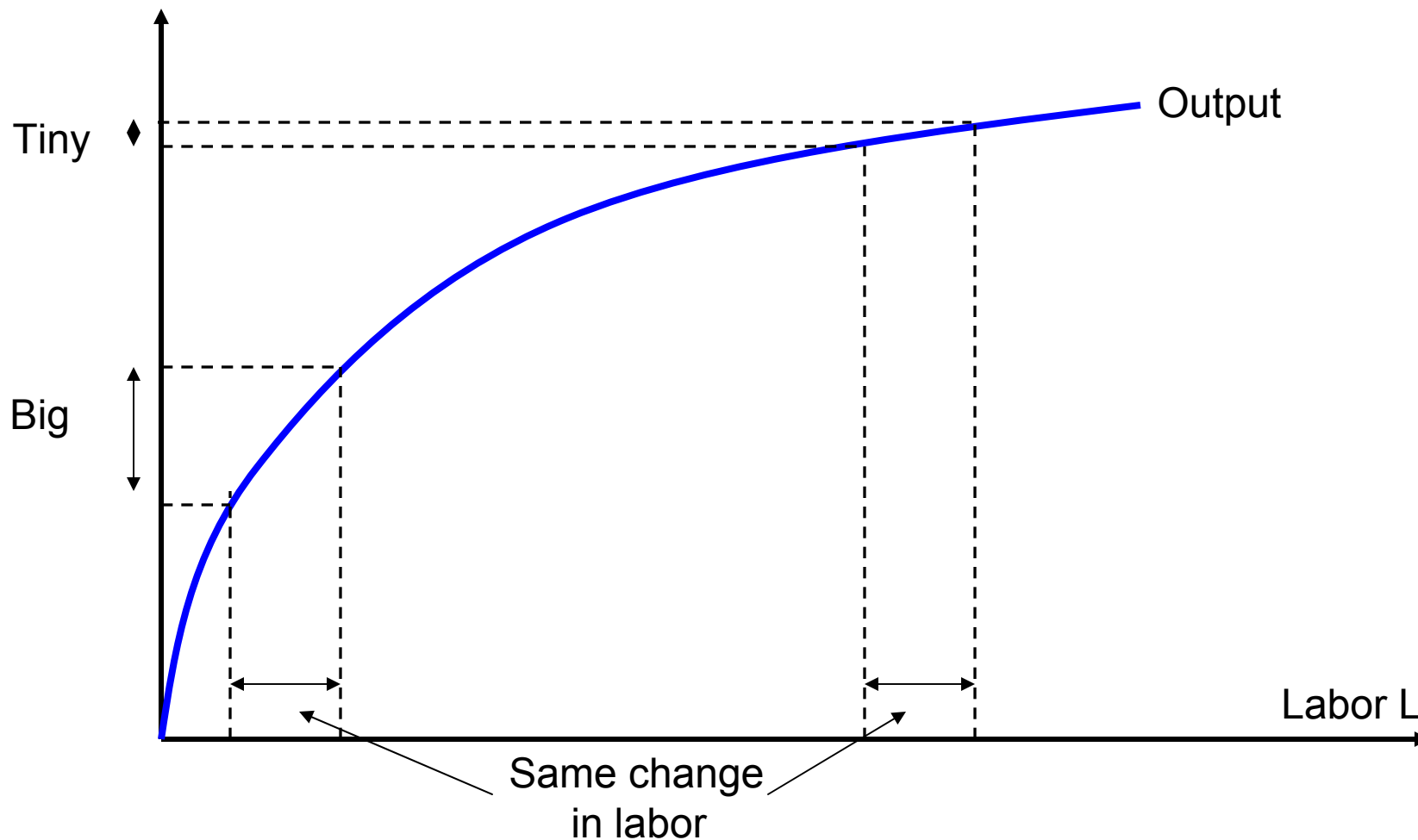
- Produce output using capital (machines) K and labor (people) L
  - Production summarized is by a **production function**
  - Example:  $Y = \sqrt{KL}$
- Assumed properties of the production function:
  - Output increasing in labor and capital
  - Diminishing returns from labor and capital
  - Returns to scale are constant

# Properties of Production Function

- **Increasing in labor and capital**
  - Add labor or capital → output will increase
- **Diminishing returns from labor and capital**
  - Add labor while keeping capital fixed → output increases, but increments smaller and smaller as you keep adding more and more labor
- **Returns to scale are constant** → production process is replicable – double labor and capital, and output will *exactly* double

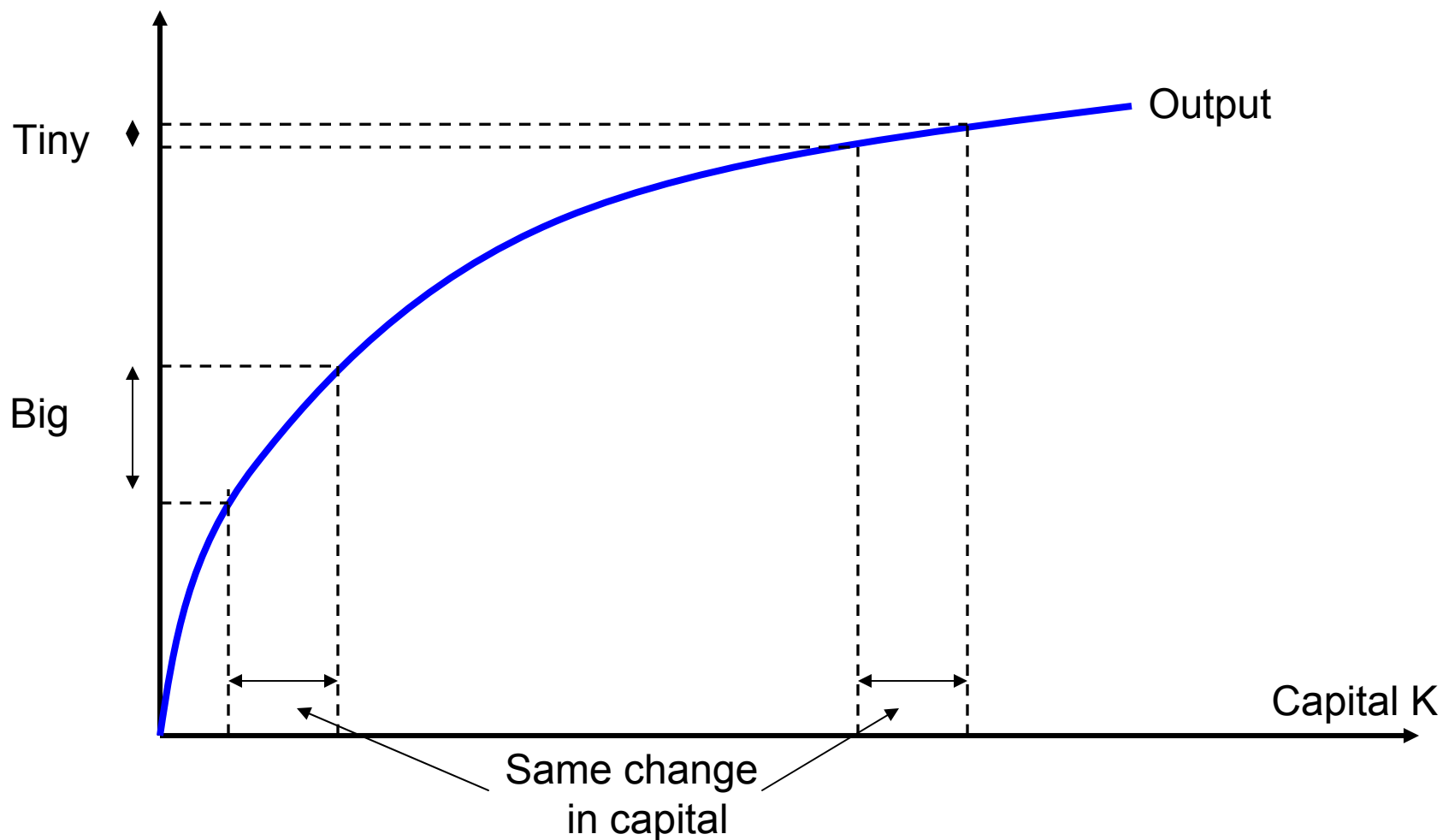
# Diminishing Returns From Labor

Fix capital:  $K=10$  (for example)



# Diminishing Returns From Capital

Fix labor:  $L=10$  (for example)



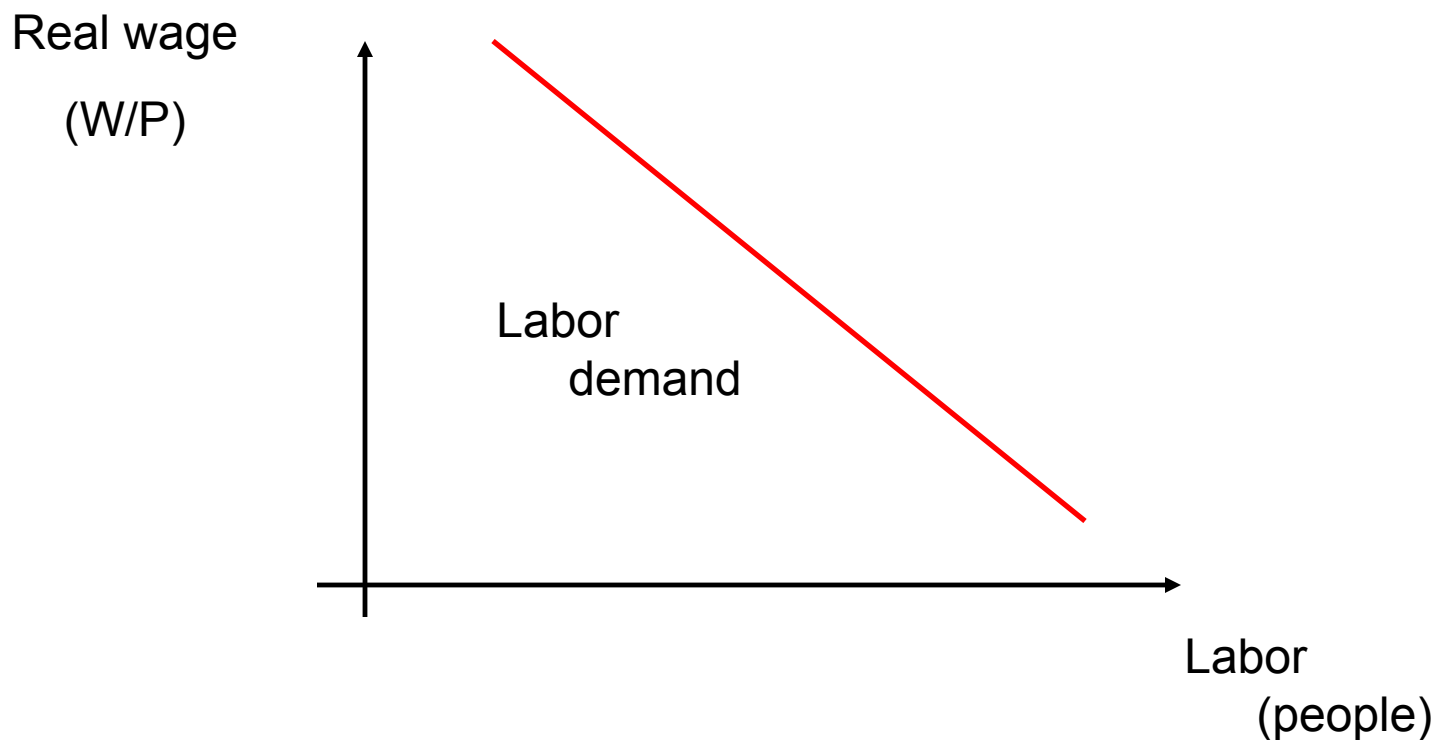
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# Demand for Labor

- Diminishing returns from labor → demand for labor decreasing function of real wage
  - Given a fixed level of capital  $K$ , each incremental worker adds *less and less* to production
    - Cost of hiring a worker increases → firms are willing to hire *fewer* workers

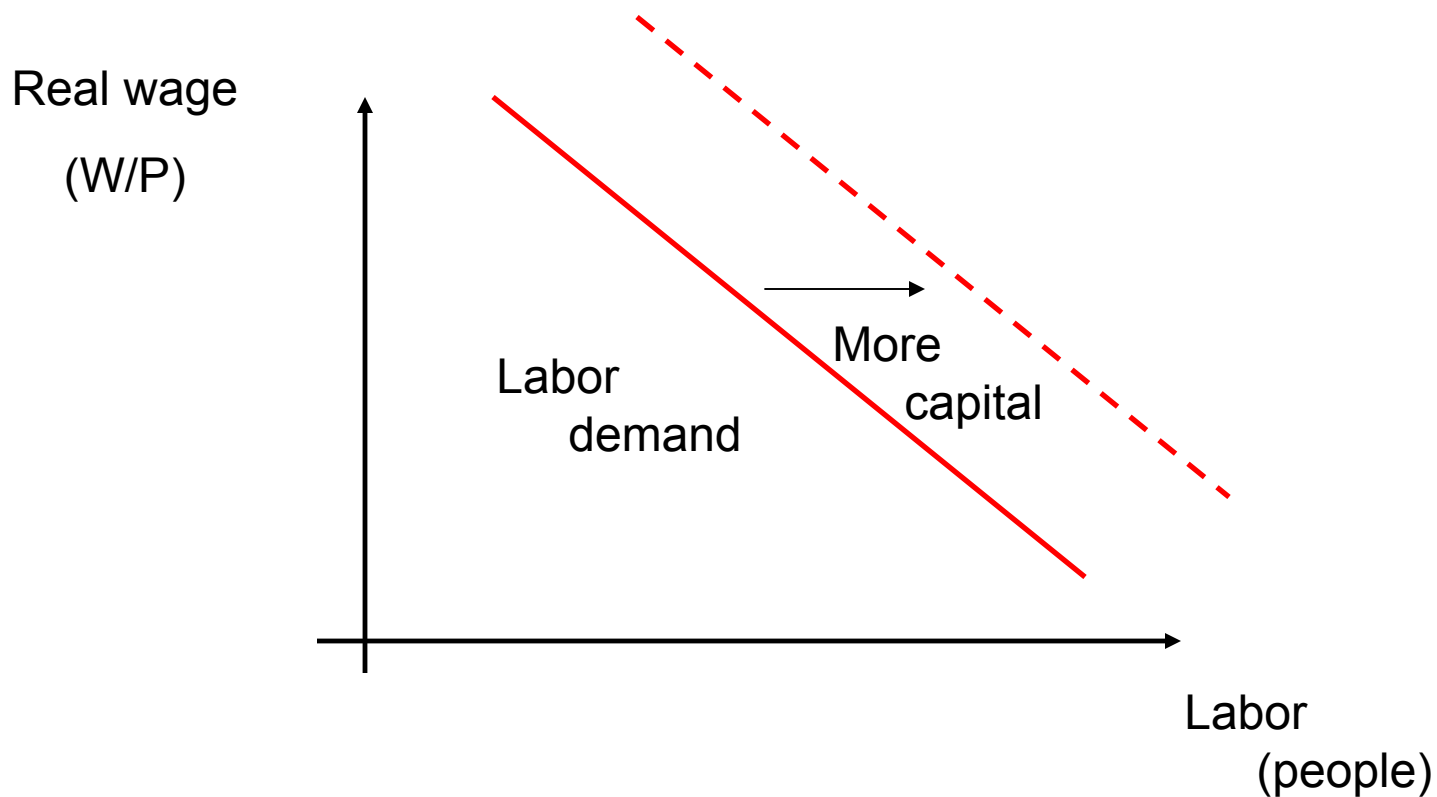
# Demand for Labor

- Demand for labor (for a fixed level of capital  $K$ )



# Demand for Labor

- Increase in capital K shifts labor demand

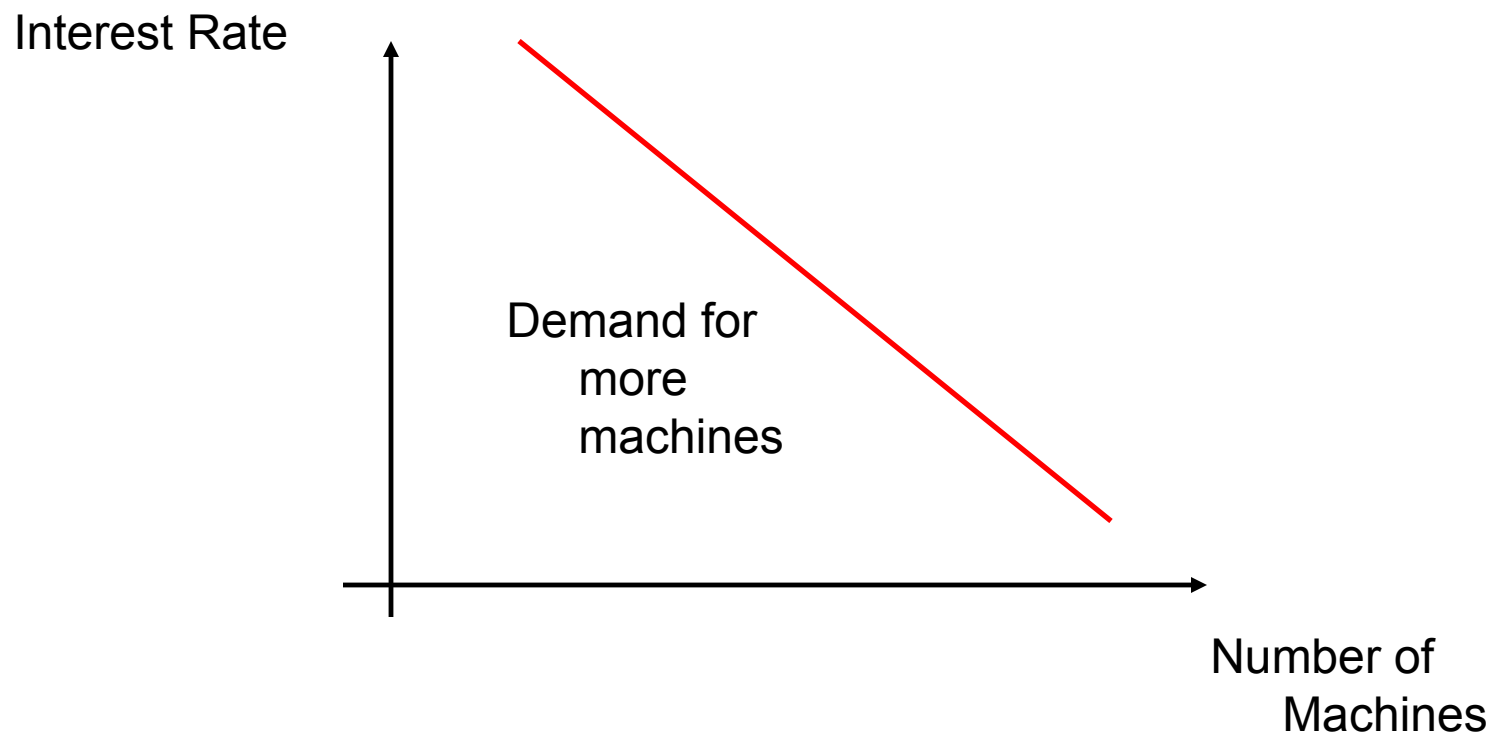


# Demand for Investment (Funds)

- Diminishing returns from capital → demand for investment decreasing function of interest rate
  - Given a fixed level of labor  $L$ , each incremental unit of capital adds less and less to production
    - Cost of funds increases → firms want to invest less

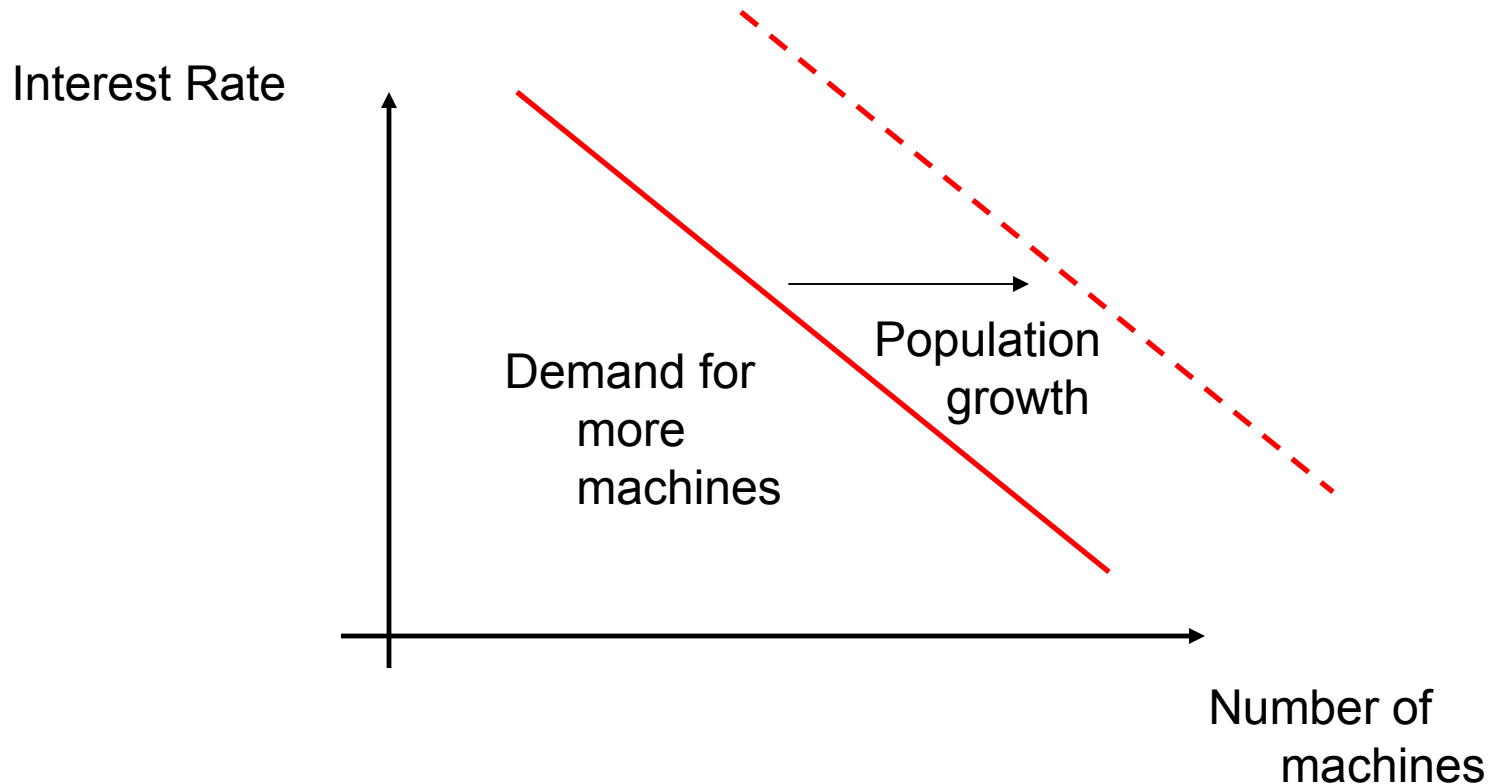
# Demand for Investment (Funds)

- Demand for funds (for a fixed level of labor)



# Demand for Investment (Funds)

- Increase in labor force (here=population) shifts demand for investment



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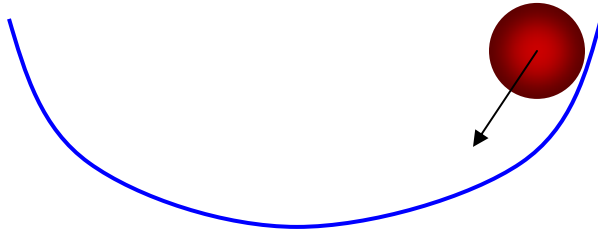
# Recap

- So far, determined demand and supply in
  - Labor market
  - Loanable funds market
- Now, need to answer what will be the outcome in the entire economy

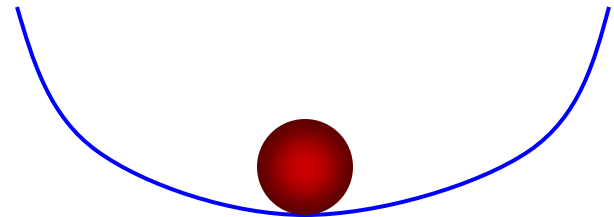
# What is Going to Happen?

- **Equilibrium** of a system is a state of the system in which there are no internal forces in the system to produce a change

This is *not* an equilibrium



This is an equilibrium



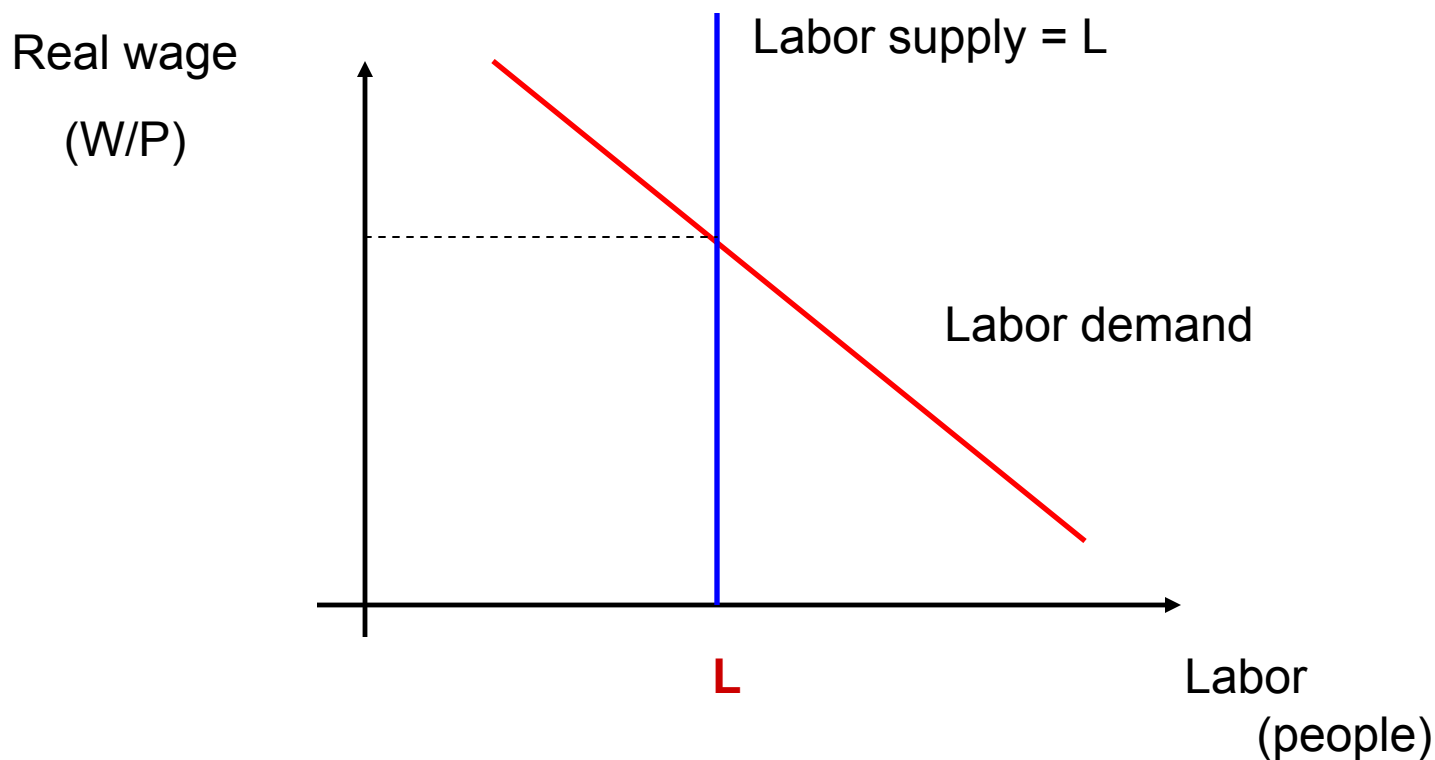
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# What is Going to Happen?

- **Equilibrium in this model** is when all markets clear
  - Demand for labor = supply of labor
  - Demand for investment = supply of funds
  - All output gets sold, i.e. planned consumption and planned investment equals output ( $C+I=Y$ )

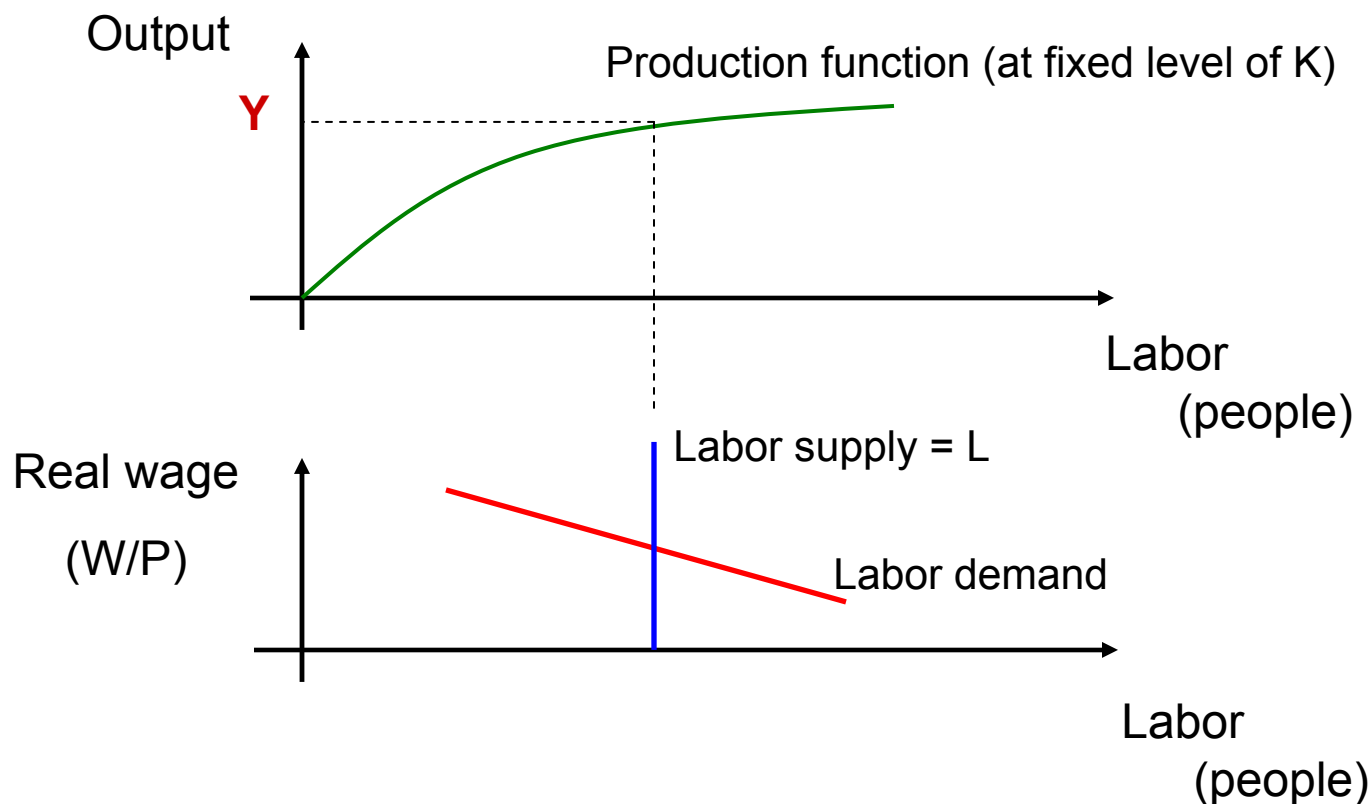
# Equilibrium Employment $L$

- Labor market equilibrium determines  $L$



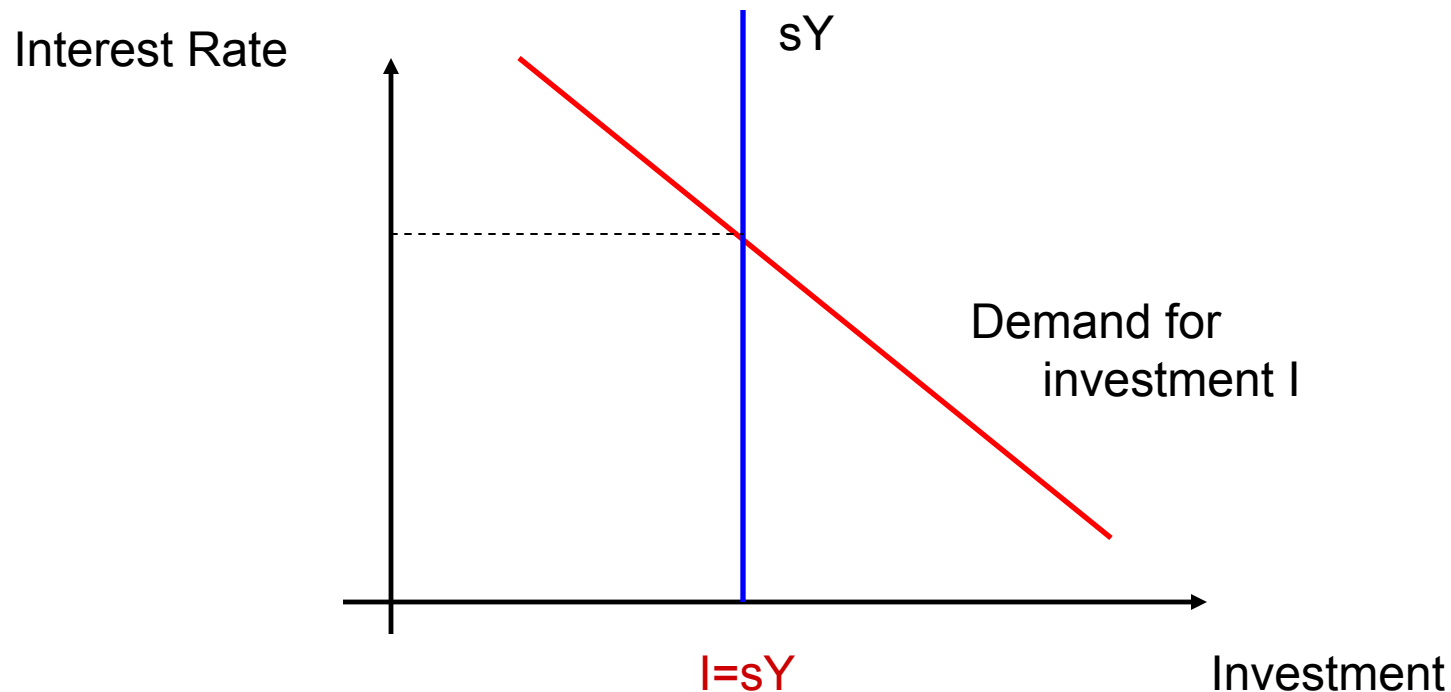
# Equilibrium Output $Y$

- $Y$  determined by  $L$  (capital  $K$  predetermined)



# Equilibrium (Planned) Investment /

- Loanable funds market determines investment  $I$

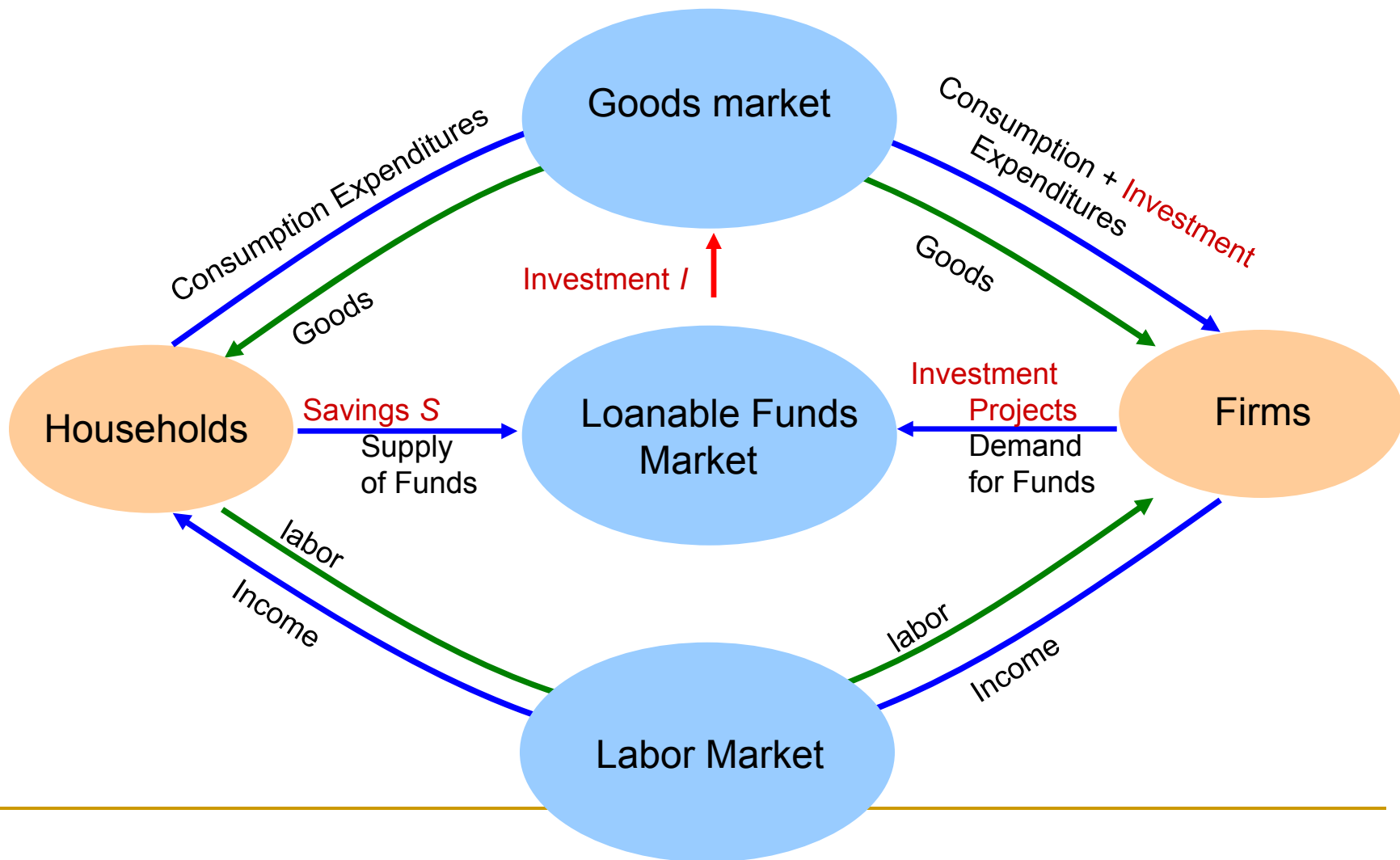


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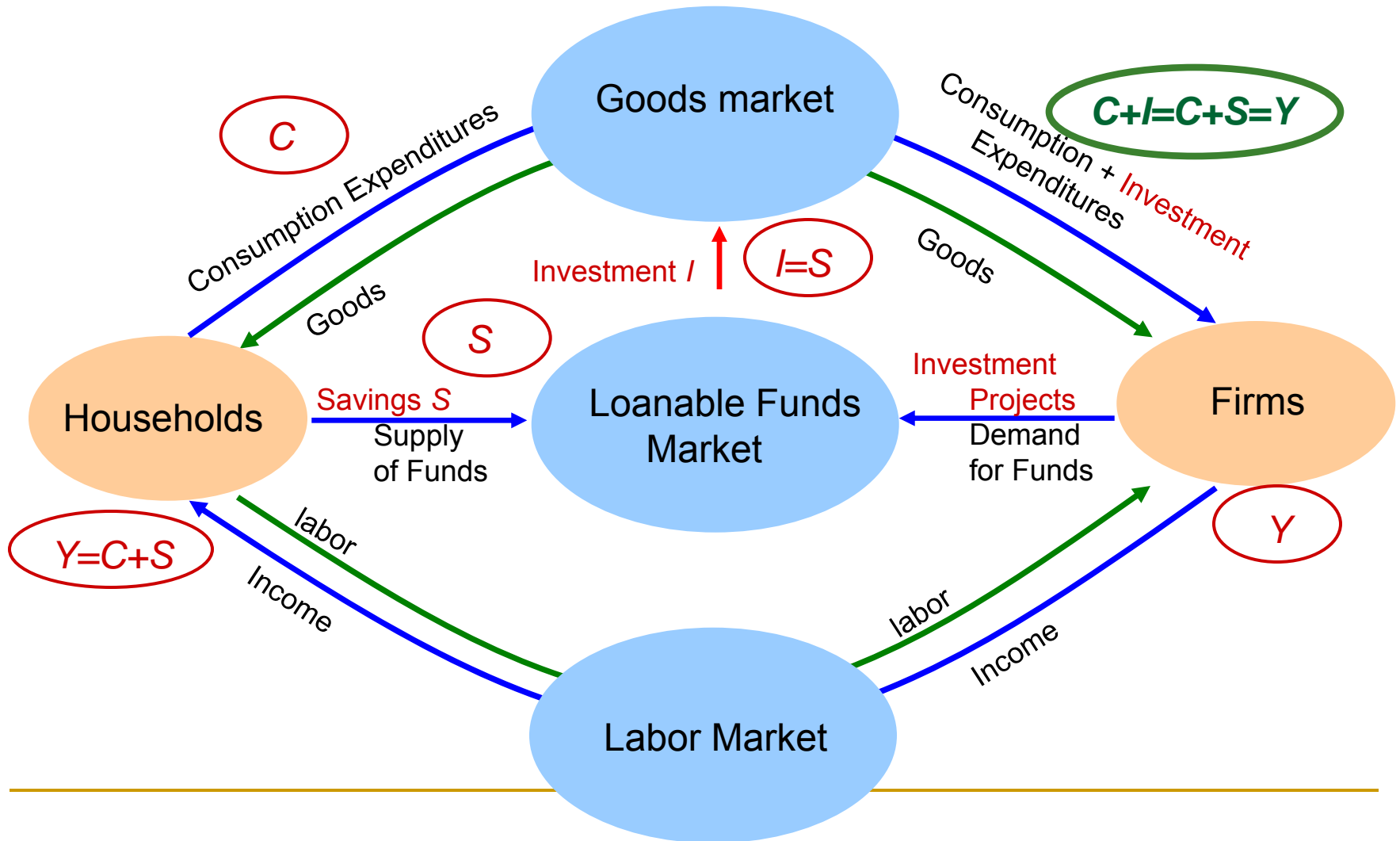
# Say's Law: Spending Purchases Output

- In the classical model, all output gets sold automatically, and so  $C+I=Y$
- We do not need to worry about the goods market
  - Follows from a simple accounting identity implied by the circular flows

# Circular Flows in the Classical Model



# Say's Law: Outflows = Inflows



# Say's Law: Outflows = Inflows

- $Y$  flows from firms to households (by definition  $Y=C+S$ )
- On the expenditure side
  - $S$  flows out
  - $I$  flows in
  - Since in equilibrium in the loanable funds market  $S=I$ , outflows = inflows, and so  $C+I = Y$

# What Happens to Capital K?

- Today's K predetermined, but future K evolves with investment
  - Capital tomorrow = capital today – depreciation of capital + investment

$$K_{tomorrow} = (1 - \delta)K_{today} + I$$

- Where  $\delta$  is depreciation rate of capital (fraction of capital worn out in production)

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# What Happens to Capital K?

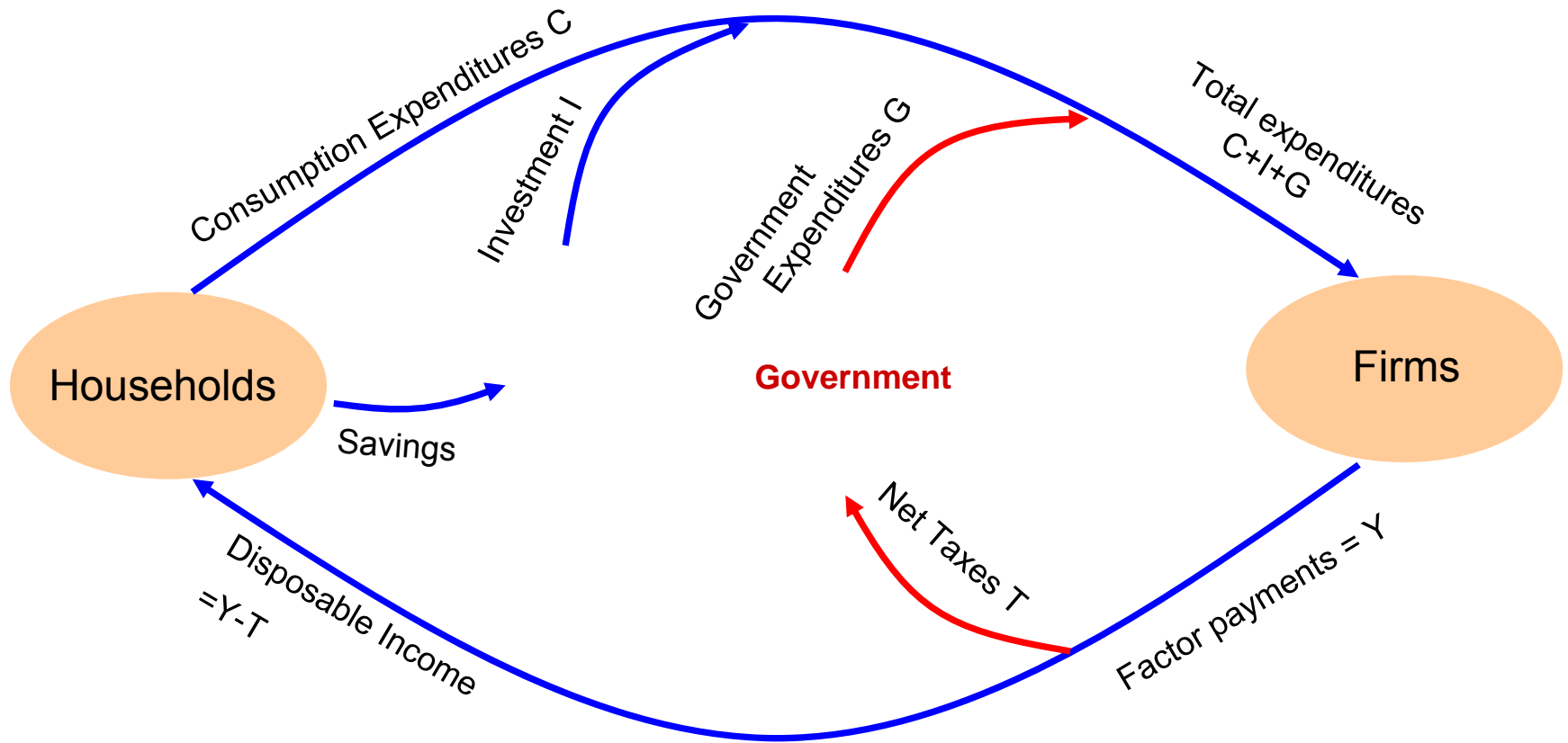
- Our equilibrium is a **static equilibrium**
  - Given capital, we pin down investment, employment and output
- It raises questions where the economy is heading in the future
  - Will make capital part of our analysis soon

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# Adding Government to the Model

- Key assumptions:
  - Government takes away net taxes  $T$  from households factor income  $Y$  and spends  $G$  in the goods market on goods and services

# Adding Government to the Model



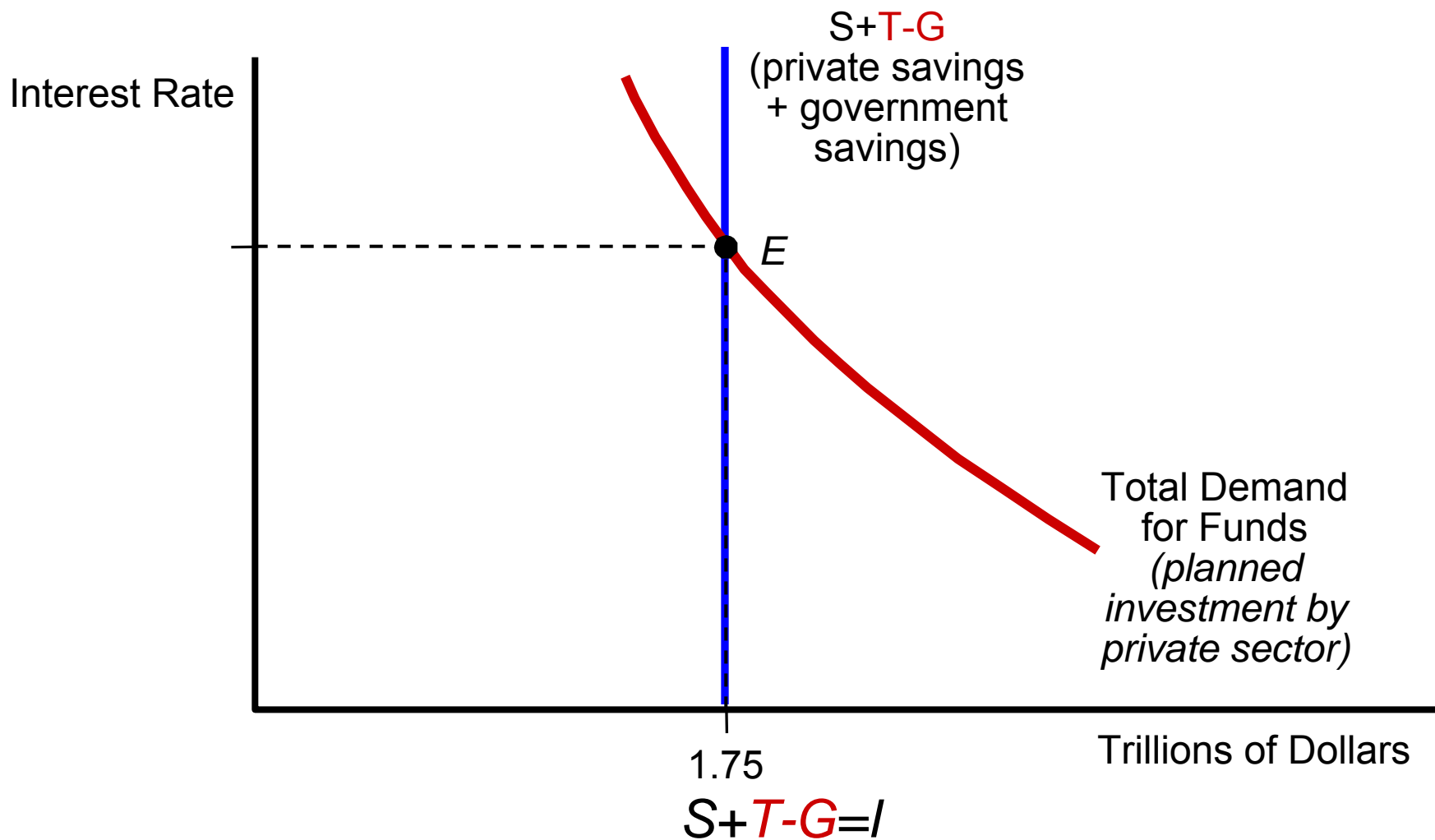
# Key Modifications

- Government savings  $T-G$  (or deficit if negative) *affects* the loanable funds market
  - When  $T-G > 0$ , governments saves and supplies funds to the LF market
  - When  $T-G < 0$ , government borrows from the private sector and takes away funds from LF market
- Households' after tax income is  $Y-T$

# Budget Deficit and Surplus

- Budget **deficit** is a situation when government spending exceeds net tax receipts  $G-T>0$ 
  - Government **borrow**s in the loanable funds market
- Budget **surplus** is a situation when government net tax receipts exceeds spending  $T-G>0$ 
  - Government **saves** in the loanable funds market

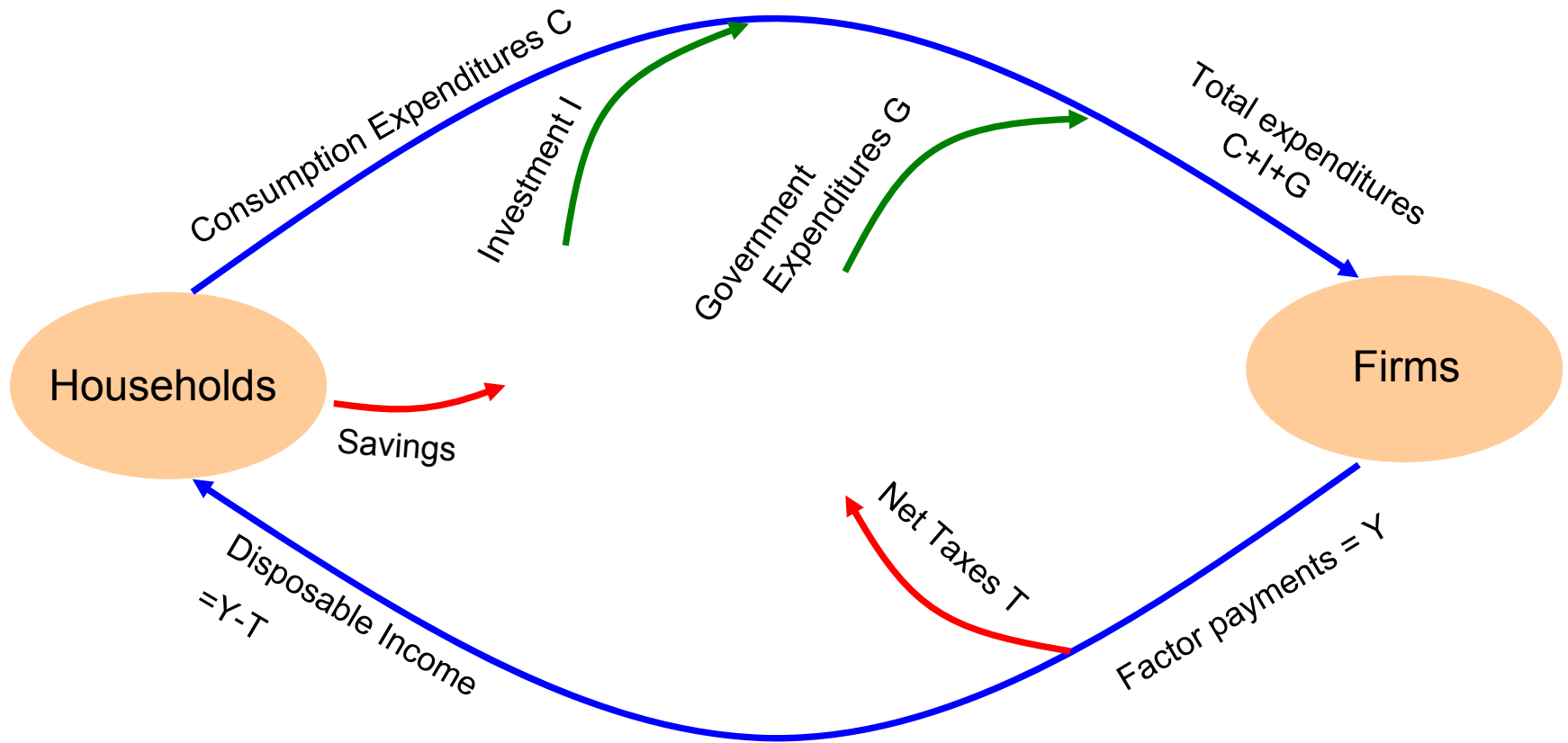
# Modified Loanable Funds Market



# Say's Law Holds with Government

- Again, think in terms of inflows and outflows
  - Y flows from firms to households
  - S and T flows out
  - I and G flows in
  - $S+T-G=I$  in equilibrium in loanable funds market
- So, again, what flows out  $S+T$ , flows in as  $I+G$ , and so  $C+I+G=Y$ 
  - Algebraically,  $C+I+G=C+S+T-G+G=C+S+T=Y$

# Say's Law Holds: Red = Green



# Numerical Example

- Suppose,  $G=0$ ,  $T=0$ ,  $s=.2$ ,  $L=10$ ,  $K=10$ , and  $Y=K^{1/2}L^{1/2}$ , calculate output and investment in the equilibrium of the classical model
- Assume depreciation rate of capital = 10% (10% of capital stock wears out each period due to aging). What will be the level of capital next period?
- Is it more or less than today?

# Solution

- $Y = K^{1/2}L^{1/2} = 10$
- $S = s(Y - T) = .2 \times 10 = 2$ 
  - In equilibrium,  $I = S + T - G$ , and so  $I = 2$
- $K \text{ tomorrow} = .9K \text{ today} + 2 = .9 \times 10 + 2 = 11$
- $11 > 10$ ,  $K$  is growing
  - $K \text{ today} = 10$
  - $K \text{ tomorrow} = 11$

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# Dynamic (Long-run) Equilibrium

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# Dynamic Equilibrium

- So far, silent about  $K$ 
  - Given predetermined capital  $K$ , determined: output, investment and employment
  - In other words, determined static equilibrium (within a period)...but not dynamic (across periods)
- Our goal: Find out where the economy is heading in the future?

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# Basic Idea

- Future output, investment and employment depends on today's investment
- Evolution of capital from one period to the next critical to determine where the economy is heading

# Evolution of Capital

- Recall our assumption:

$$K_{tomorrow} = (1 - \delta)K_{today} + I$$

- IMPLIES: Future capital depends how investment  $I$  compares to depreciation  $\delta K$

$$K_{tomorrow} = (1 - \delta)K_{today} + I = K_{today} - \delta K_{today} + I,$$

and so,  $K_{tomorrow} - K_{today} = I - \delta K_{today}$

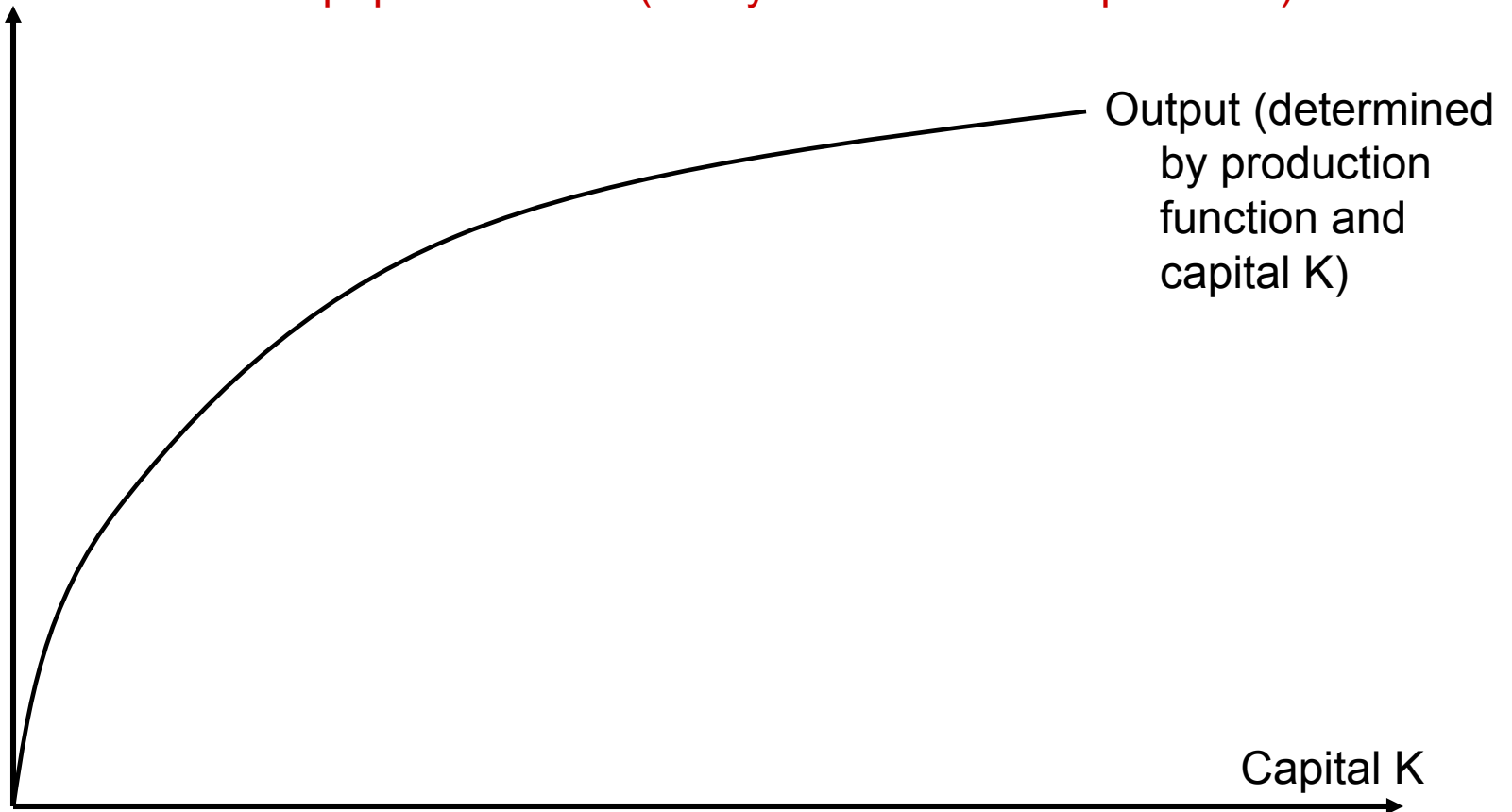
# Evolution of Capital

$$K_{tomorrow} - K_{today} = I - \delta K$$

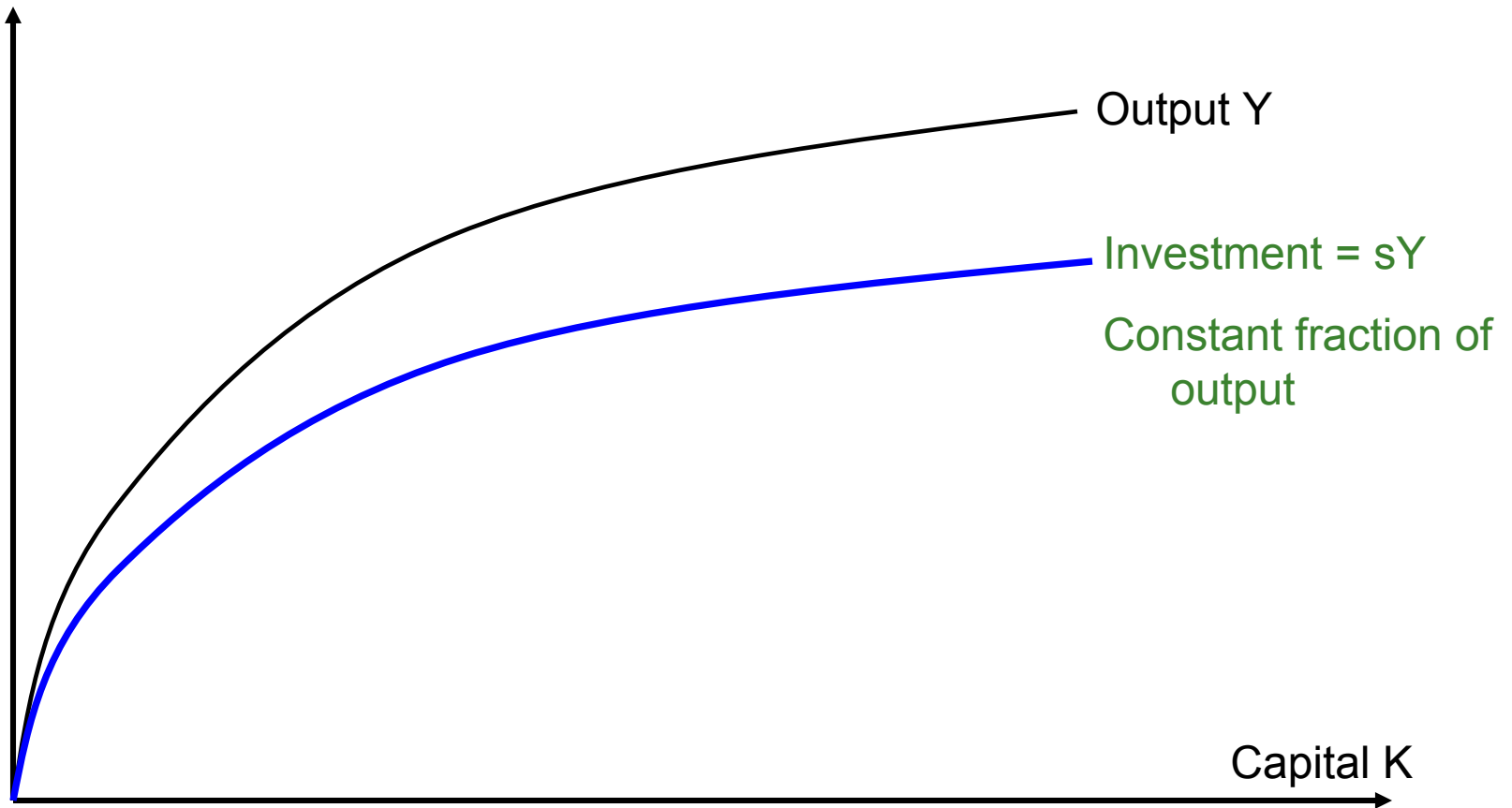
- The above equation implies:
  - $I > \delta K_{today} \rightarrow$  capital K *grows*
  - $I < \delta K_{today} \rightarrow$  capital K *falls*
  - $I = \delta K_{today} \rightarrow$  capital K remains *unchanged*

# Finding Out Where Economy Is Heading

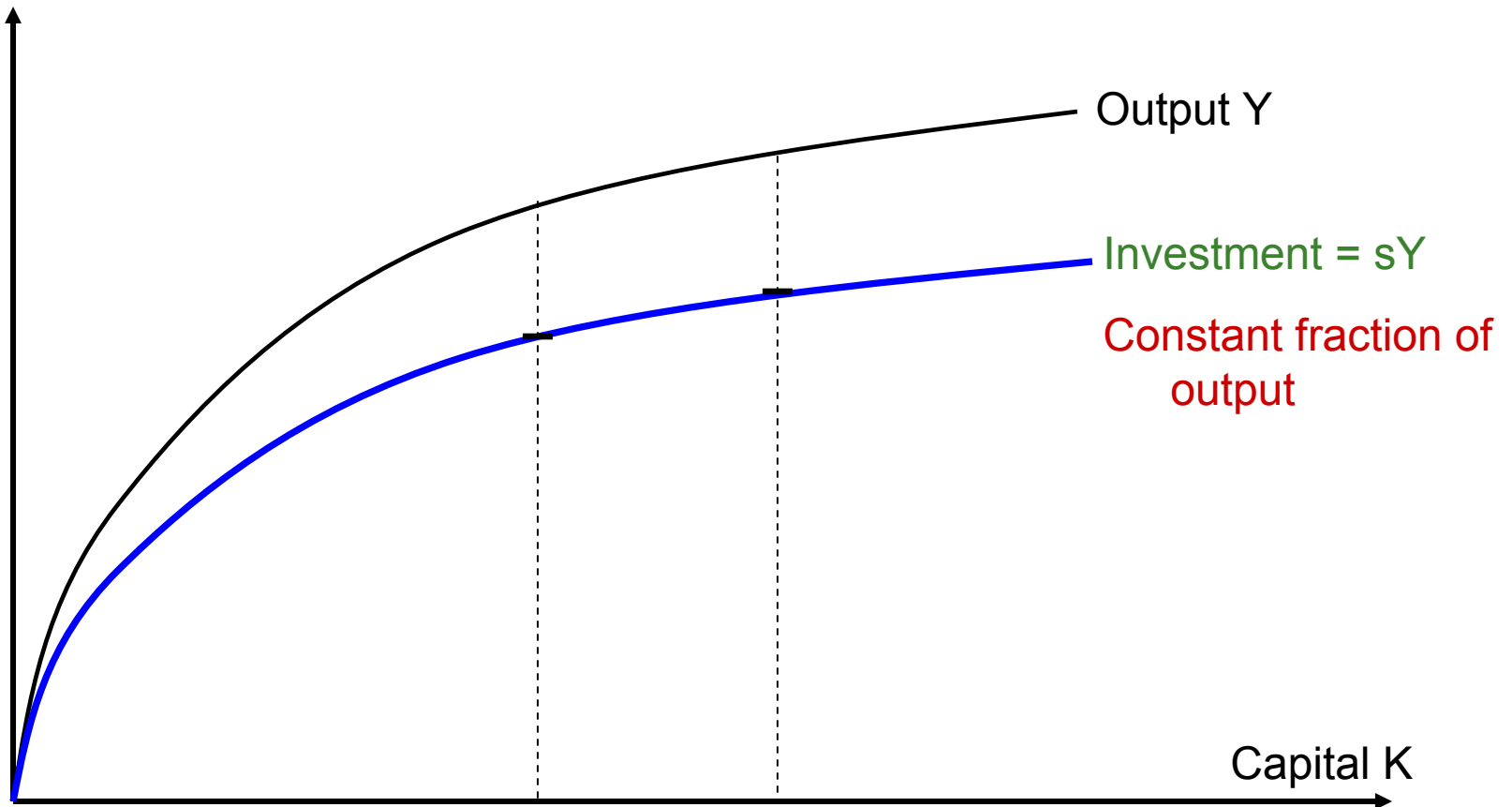
L fixed = population size (always true in static equilibrium)



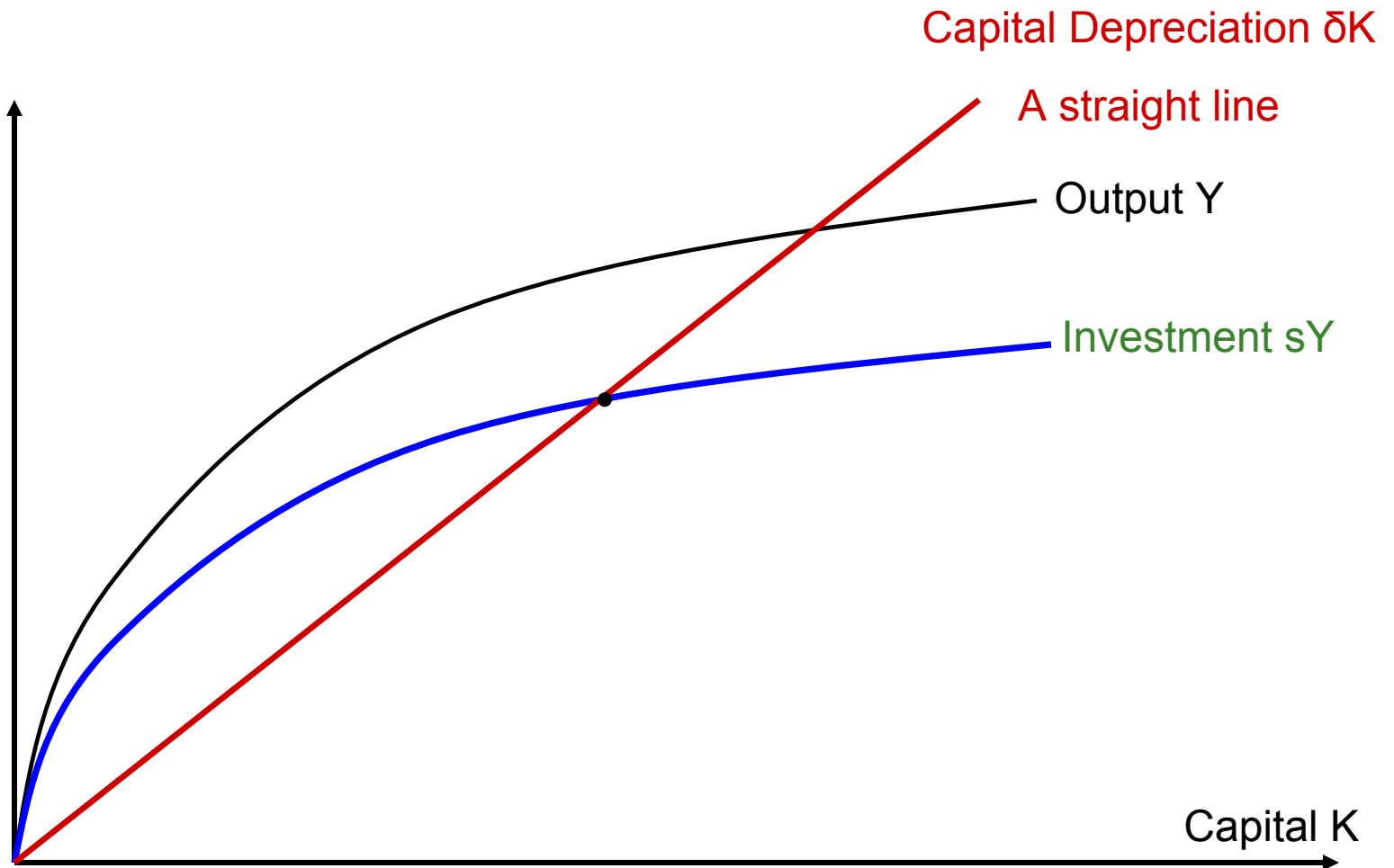
# Finding Out Where Economy Is Heading



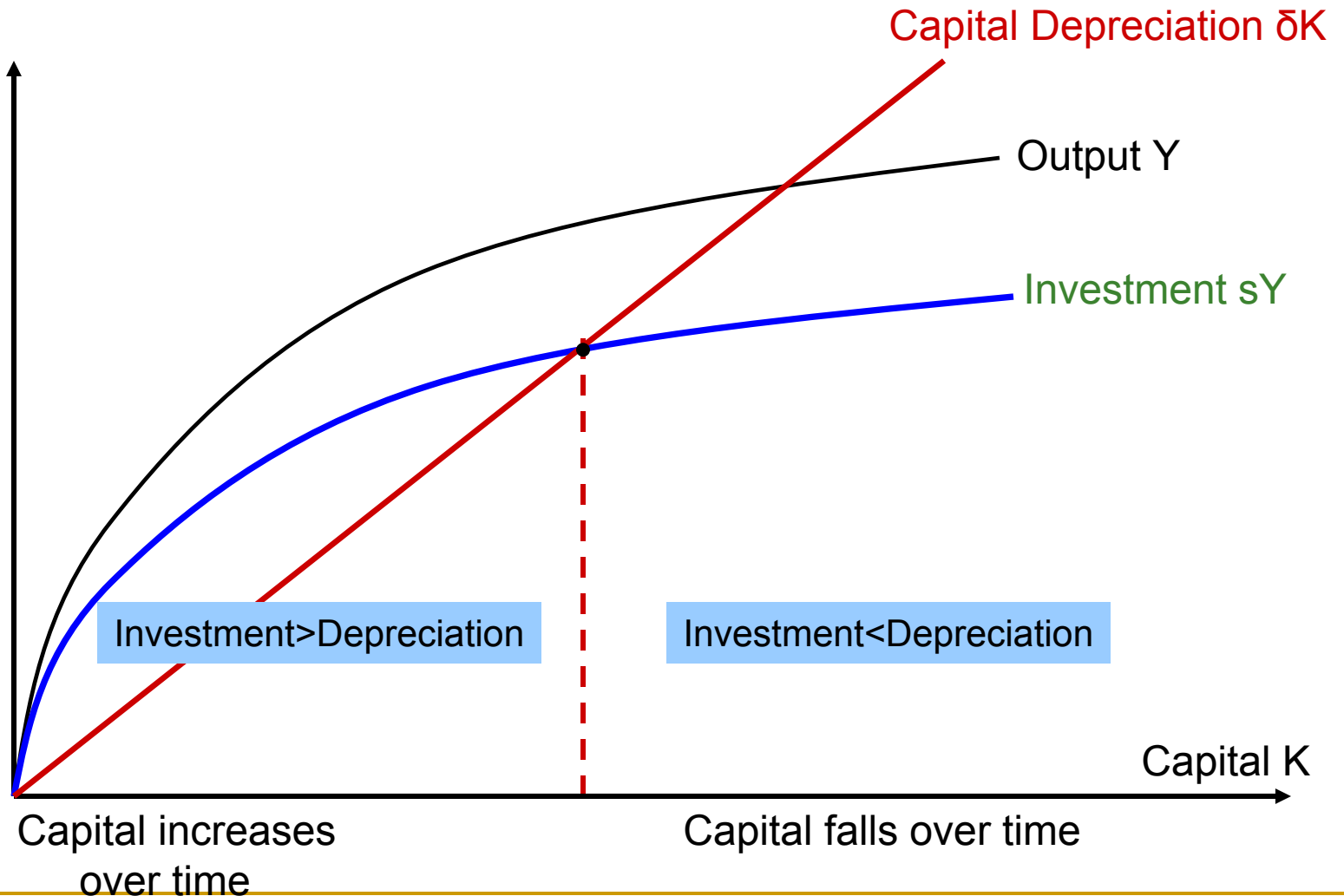
# Finding Out Where Economy Is Heading



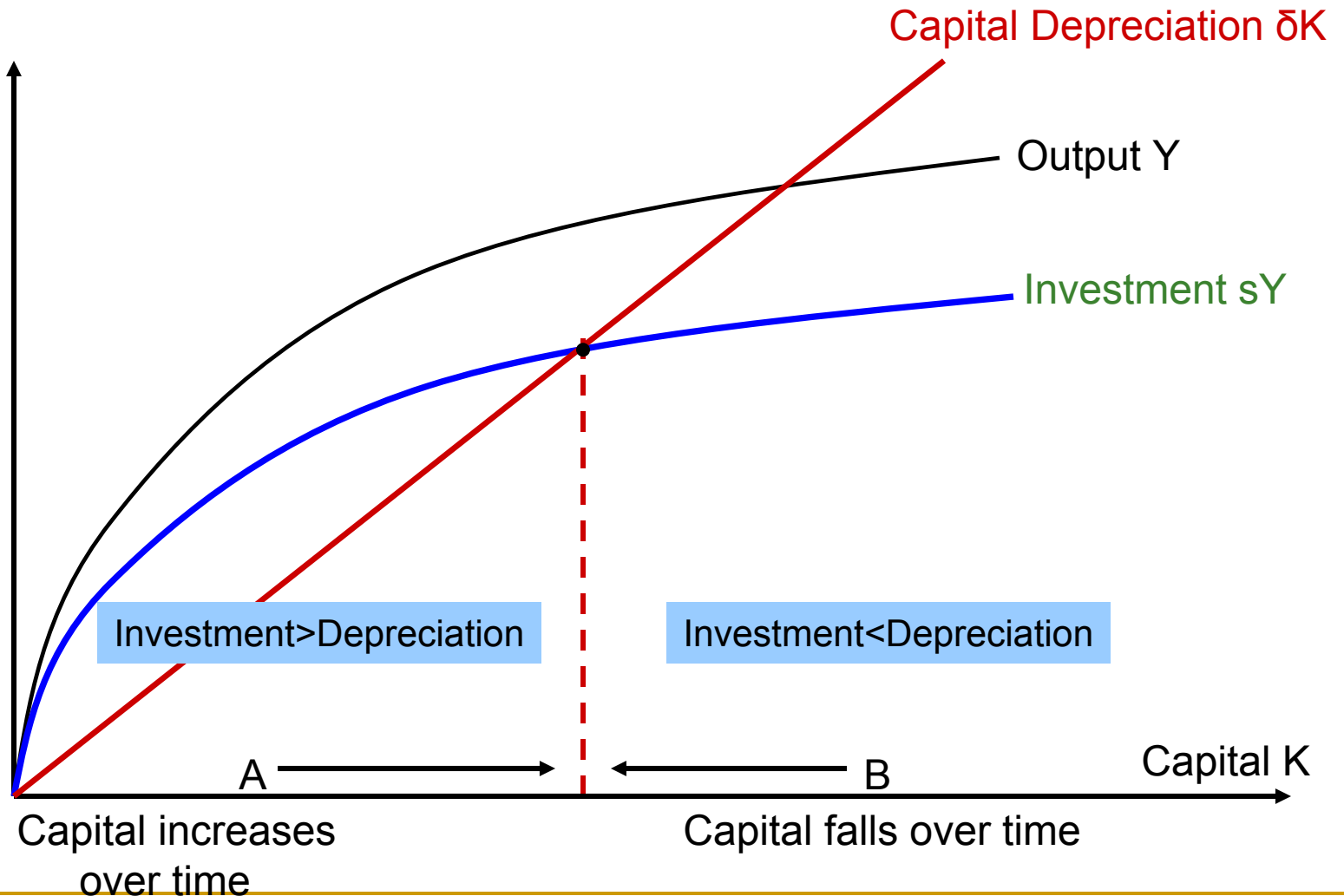
# Finding Out Where Economy Is Heading



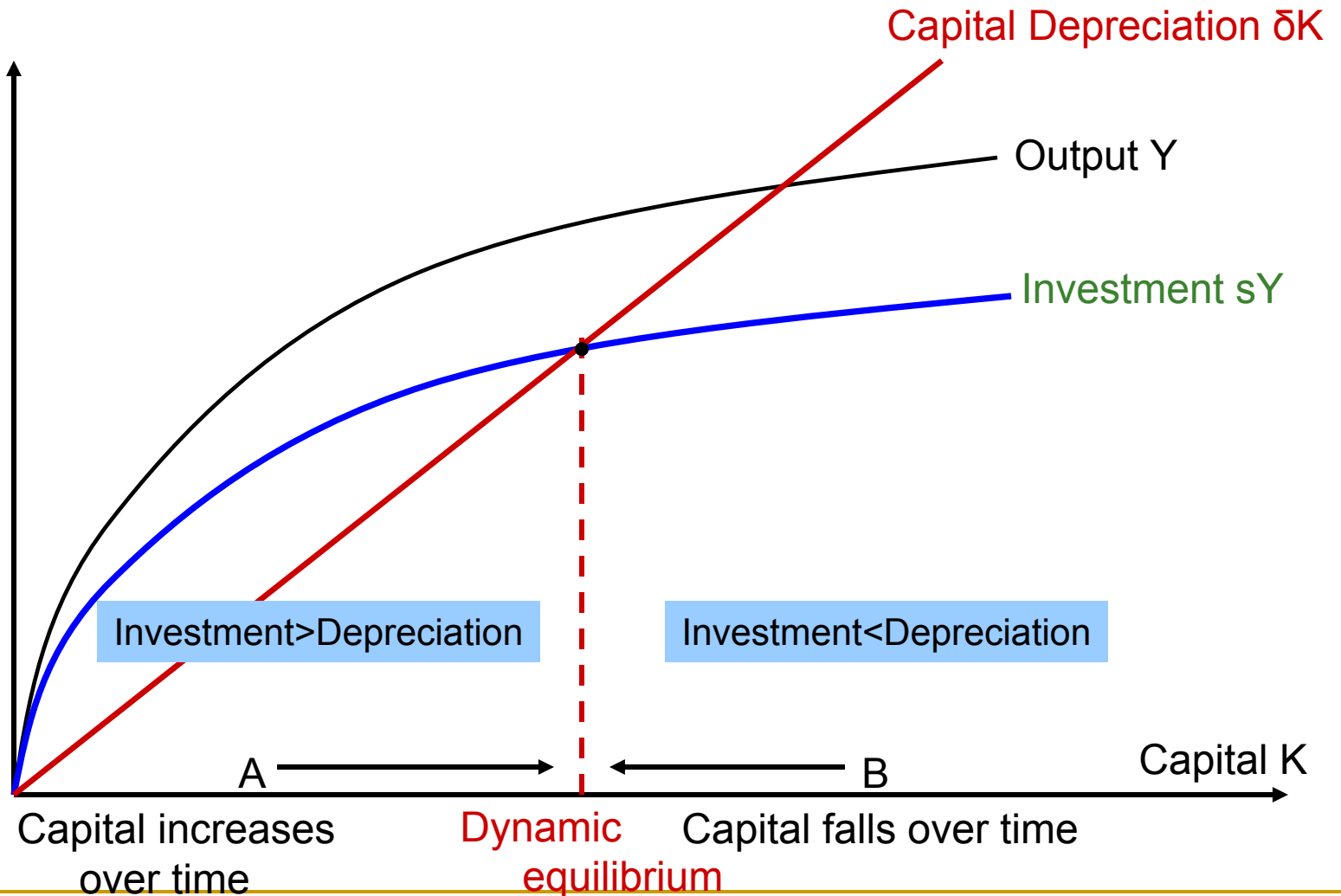
# Finding Out Where Economy Is Heading



# Finding Out Where Economy Is Heading



# Finding Out Where Economy Is Heading



# What Is Dynamic Equilibrium?

- Recall: Given predetermined level of capital  $K$ , **static equilibrium** is employment, output and investment such that all three market are in equilibrium
- **Dynamic equilibrium** is the level of capital  $K$  such that in the underlying static equilibrium investment  $I =$  depreciation of capital  $\delta K$

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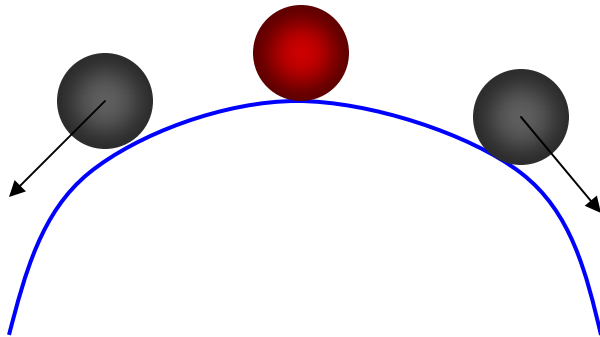
# Key Properties of Dynamic Equilibrium

- Dynamic equilibrium is an **equilibrium** as there are no internal forces to produce a change
  - You start there, you stay there forever
- But, it is also a **stable equilibrium**
  - No matter where you start, internal forces bring you back to this this point

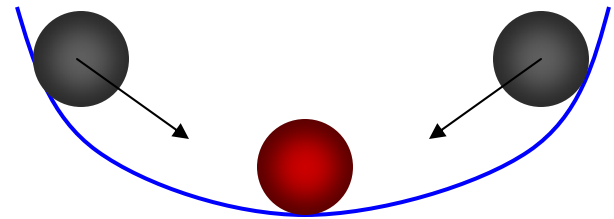
# Stable Equilibrium

- **Stable equilibrium** of a system is an equilibrium such that the system automatically returns to it, if disturbed

This is *not* a stable equilibrium (the ball falls, if disturbed)



This is a stable equilibrium (the ball automatically returns, if disturbed)



# Numerical Example Continued

- Suppose,  $G=0$ ,  $T=0$ ,  $s=.2$ ,  $L=10$ ,  $K=10$ , and  $Y=K^{1/2}L^{1/2}$ 
  - Assuming depreciation rate of capital = 10%, find the dynamic equilibrium

# Solution

- Need to find  $K$  such that in the underlying static equilibrium  $I = \delta K$
- In any static equilibrium,  $L = 10$ ,  $I = s(Y - T) + T - G = .2K^{1/2}L^{1/2} = .2K^{1/2}10^{1/2}$ 
  - Thus, need  $K$  such that  $.2K^{1/2}10^{1/2} = \delta K$  (\*)
- Calculating, we obtain  $K = 40$ 
  - *HINT: Divide both sides of (\*) by the square root of  $K$ , and then raise both sides to the square. Compute  $K$ .*