Classical Model

The classical model is a mathematical model of output determination and economy-wide equilibrium. It is built on the most fundamental principles of microeconomics, and it is considered the frictionless benchmark of macroeconomics. What this means is that if you ask a macroeconomist what will eventually happen, she or he will most likely use a version of the classical model to answer this question.

The classical model addresses the question how various pieces of the economy, like households, markets, firms or governments, all come together to determine the aggregate outcomes. In its simplest formulation it starts off with two economic agents coexisting in a closed economy (exports and imports both equal 0): *households and firms*. This is the version that we will discuss first.[1]

Economic Agents

The economy is populated by two types of economic agents: *firms and households*. Households own firms, and firms produce output using machines and labor supplied by households. Output is either consumed by households or invested in physical capital (machines that will be used to produce future output).

The goal of our analysis is to formalize how firms and households interact in various markets to determine aggregate production, employment, investment and capital. In the classical model, this interaction happens in 3 markets: *(i) goods (commodity) market, (ii) loanable funds market,* and *(iii) labor market.* As illustrated in the figure below, the *goods market* is where households and firms purchase output produced in firms; the *loanable funds* market is where households save their income and channel these funds to firms who use them to finance investment in capital (machines); the *labor market* is where households offer their labor services, and firms hire workers to produce output. The key assumption of the classical model, which actually makes the classical model classical, is that *all these market are competitive*

[1] In the part that follows, it is important to realize that we describe a model, not the reality. A model is an abstract and simplified representation of the real world, and need not be consistent with the reality in every respect.

and clear (i.e. just like in econ 101, there is demand, there is supply, and price is such that demand=supply).

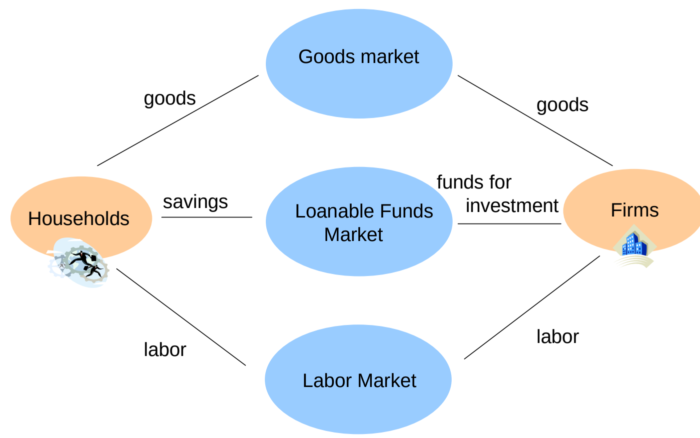

Figure 1: Markets and agents in the Classical model.

Households

Because households are owners of firms and workers, they are recipients of all factor payments made by firms. These factor payments are their income, which they save or consume.

On the economy-wide level, households make two important decisions. They decide what fraction of income to save and what fraction to consume, and they decide how much labor to supply to firm (or how much leisure to consume and how much to work). To keep things simple, we will assume very simple decision rules that govern these choices.

Consumption and Saving Decision by Households

Given income Y (=total factory payments made by firms), we assume that households *choose to save a constant fraction of this income*, and consume the rest. The key parameter that summarizes this decision is some saving rate s (s is the fraction of income that is going to be saved).

For example, if $s=1/3$ and income is $Y=150$, the households save $S=\$50$ and consume $C=\$100$ (consumption is spent on final good and services purchased in the marketplace).[2]

Clearly, the above assumption ignores many factors that may be internal to the model and influence s in general. The most important would be the reward for savings, i.e. the real interest. Another important aspect would be the wealth and expectations about the future income. To keep things simple, we will abstract from these complications, and leave it for more advanced courses.

Labor/Leisure Choice by Households

Another important decision that households make is how much labor to supply to the market (how much to work), and how much leisure to consume. Again, to keep our analysis simple, we will assume that *households supply all the labor they can supply, and thus labor supply is equal to the population size* L *(number of people).*

Production by Firms

Our next goal is to formalize the production process by firms into a mathematical model. The key simplifying assumption is that, given the level of technology, there are only two factors that matter for production: labor L, and capital (machines) K. Moreover, firms produce a homogenous good (called GDP).

These assumptions allow us to formalize the production process by a concept called *production function*. Production function is simply a mathematical formula that tells us the level of output for all possible levels of capital and labor. For example, $Y = \sqrt{KL}$ is a production function because we can plug in numbers for K and L, and the formula will tell us what the output level is.

[2] Note that the saving rate s is sufficient to determine both savings and consumption. In fact, given income Y, consumption C is: $C=Y-S=Y-sY=(1-s)Y$.

However, to make our analysis more general, instead of assuming a concrete formula for a production function, we will find it more convenient to work with 3 key properties that a production function must obey. These properties are:

(a) *The production function is increasing in both labor and capital.* What this means is that when we add more capital or labor, output necessarily goes up.

(b) *The production function exhibits diminishing returns from both labor and capital.* What this means is that when we add more labor, while keeping capital fixed at some level, the production will increase, but the marginal increments will be smaller and smaller as we keep adding more and more labor. An analogous property holds for capital (given fixed amount of labor). Note that the production function in our example has this property, which you will show in your homework. The figure below illustrate how this assumption affects the plot of the production function – when you look at it from below, it resembles and a tilted concave surface, like a tilted umbrella…

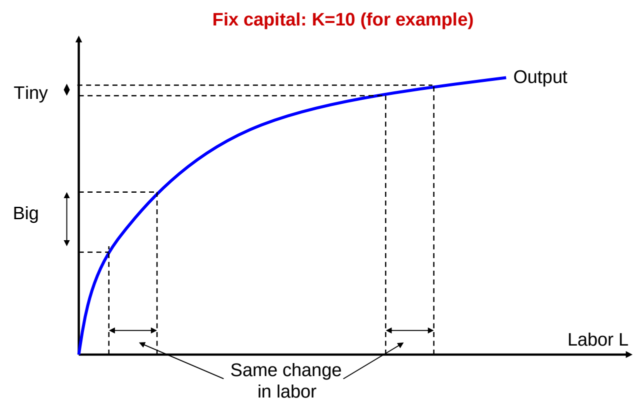

Figure 2. Shape of the production function and diminishing returns.

(c) *Production function exhibits constant returns to scale.* What this means is that the production process is fully scalable. Specifically, when we halve, double, triple or quadruple all the inputs proportionally, the production exactly halves, doubles, triples or quadruples (not more, not less). For instance, the production function we gave as an example above exhibits this property. To see this, assume that *K=10* and *L=5*, and double these inputs to 20 and 10. Initial production is given by $Y = \sqrt{10x5} = \sqrt{50}$, and as double these inputs, we get exactly twice as much output as before:[3] $Y = \sqrt{(2x10)x(2x5)} = \sqrt{4x10x5} = 2\sqrt{10x5} = 2\sqrt{50}$.

The assumption of constant returns to scale is a very important one, because t implies that we do not really have to say how many firms there are, and how production is organized to study the economy at the aggregate level; it is sufficient to specify how much capital and labor the economy employs in total to know the total production. In particular, there may be 1000 firms, or just 1 – this does not really matter. Everything is scalable, and so we may think of 1 large firm instead of 1000 small ones, and the outcome is going to be the same. In a sense, the constant return to scale assumption is what makes the concept of an aggregate production function really meaningful in macroeconomics.

Markets

The decisions that firms and households make, create demand and supply in the market. Since we assume that all markets are competitive, deriving demand and supply is sufficient to characterize the outcomes.

Equilibrium Outcome

What will be the level of output, employment and investment in our model? To answer this question, we need to find what economists call the *equilibrium* of the model, i.e *a situation in which there are no internal forces in the model that could produce a change*. To do that, we will need to take a closer look at the markets in which firms and households interact.

Labor Market

In the first step, let's take a look at the labor market. In the labor market, firms hire workers and households offer their labor for real wage that firms pay in return. We model this market in line with econ 101 demand-supply framework.

[3] What is essential in this calculation is the fact that a square root of 4 is 2, and so no matter the level of K and L our conclusion would always be true.

To determine the equilibrium outcome in this market, we need to know the forces that determine demand and supply of labor. The supply is easy, because we have just assumed that households supply all they labor regardless the wage paid by firms. Thus, the supply curve is just a vertical line starting at L. The demand is slightly more complicated, but our assumptions (point b) imposed on the production function are sufficient to pin down the shape of the demand.

Labor Demand

Since we have assumed diminishing returns from labor, we know that as we keep adding workers, each marginal worker increases production by less and less. What this means, is that when the real wage increases (cost of hiring a marginal worker increases), firms tend to reduce employment, so that the marginal worker still produces enough to make it profitable to keep employing her or him. Thus, we can say that the demand is a decreasing function of the real wage, and everything fits into the demand and supply framework of Econ 101.

**Labor Demand: A Detailed Analysis (optional)*

To understand the above argument in the gory detail, think about a coffee shop that needs to hire workers to make coffee, and suppose the coffee shop has only 1 espresso machine. Furthermore, assume production function of this coffee shop is subject to diminishing returns from labor. To make things concrete, let's assume that the first worker that is hired increases production from 0 coffees per hour to 10 coffees p/h, the second worker increases production from 10 coffees p/h to 16 coffees per hour, the third from 16 to 20, fourth even less and so on. Now, suppose the real wage is $5 per hour, and the price of coffee is $1. How many workers should the shop employ? Well, it obviously makes sense to employ the first worker, as she produces $10 per hour and it costs to hire her only $5. It also makes sense to employ the second worker. She produces additional $6 per hour, which is again more than $5. However, it does not make sense to employ the third worker, as her marginal product per hour is only $4, and it costs $5 to hire her. The same would be true about the fourth worker, fifth and so on… This is what diminishing returns buy us. We thus can conclude that in this case labor demand is uniquely pinned down, and it is equal to 2 workers. Now, what happens if the wage rate goes up to $9 per hour? Again, how many workers should the shop employ? Well, since the first worker produces $10 per hour, it makes sense to employ her. However, it no longer makes sense to employ the second worker, and by diminishing returns, it does not make sense to employ the third worker, the fourth one and so on. As a result, labor demand is still uniquely pinned down, but falls to 1 worker. As you can see, with diminishing return things work very neatly, and the demand is nice and decreasing with respect to the wage rate.

Equilibrium in the Labor Market

We are now ready to plot demand and supply in the labor market to determine both equilibrium employment and the equilibrium real wage W/P. Clearly, it is where the two lines intersect, and employment level in equilibrium is L.

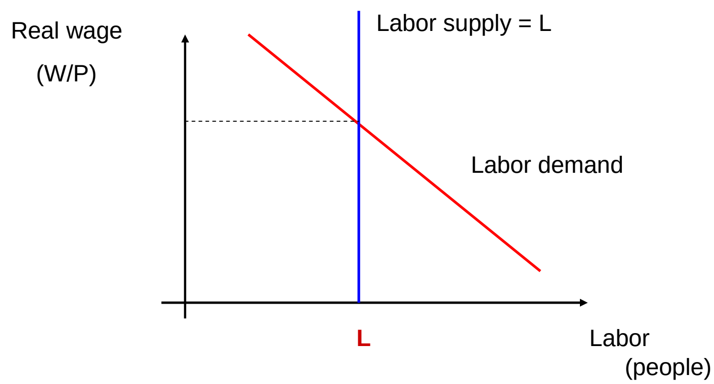

Figure 3: Labor market equilibrium.

Equilibrium Output

Given equilibrium employment L, we will try to determine equilibrium output Y. The difficulty is that we do not know the level of capital K. However, we know that capital is determined by past investment and in any given period can not be changed. Thus, it makes sense to just assume some level of capital to find equilibrium output. This is what we are going to do.

Given assumed (initial level of capital) level of capital denoted by K, the following diagram shows how using the production function we can determine output Y that the economy will produce in this period.

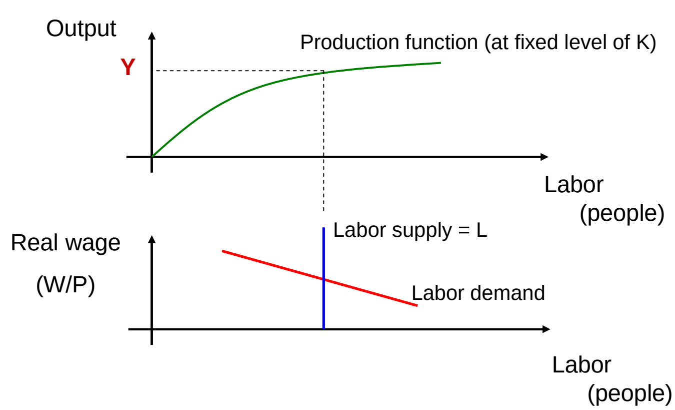

Figure 4: Output determination given equilibrium in the labor market and initial capital K.

Loanable-Funds Market

Loanable-funds market determines (planned) investment *I* in future capital and the price of funds (ex-ante real interest rate=price of investment and reward for saving). The key components are the supply of funds (savings by households) and demand for funds by firms to finance investment projects. In the loanable funds market, households seek return from their savings, and firms use these funds to invest in future capital that requires initial investment in return for future benefits. Clearly, if the present value of the future benefits outweighs the cost, the investment project is profitable and a firm will seek funds, if not, the project is not profitable, and the firm will not seek funds. This is a very important market, because investment today is capital tomorrow, and thus investment affects output in the future, and thereby economic growth.

From the previous set of lecture notes, we know that the present value of future benefits is a decreasing function of the interest rate. From this, we can infer that the demand for funds is a decreasing function of the interest rate: *When the interest rate is high, only best investment projects are worth financing, and the number of them is limited due to diminishing returns from capital*[4]. In addition, from our simple assumptions about the saving/consumption behavior of households, we can infer that the supply of funds is vertical. Figure 4 illustrates the resulting equilibrium in this market.

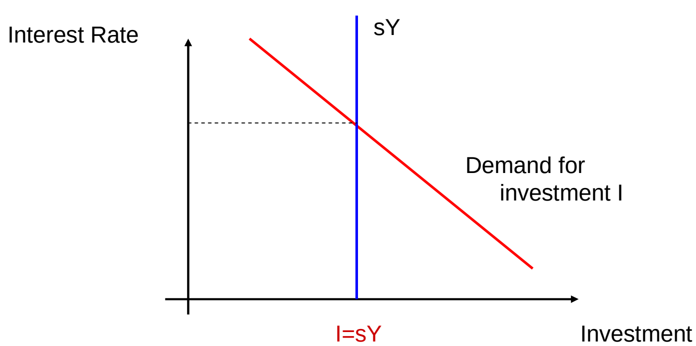

Figure 5: Equilibrium in the loanable-funds market.

[4] We can show it similarly as we did in the case of labor demand.

(Note that when the population and labor supply increases in the economy, capital become more productive. This increases the demand for investment, and shifts the demand line outwards.)

Equilibrium Revisited…

The output level, investment level and employment level we have found above is an equilibrium because there are no forces that will take us out of this situation. Of course, more generally, investment today will change capital in the future, and so it may produce a change in the future. Therefore, we will refer to this equilibrium as a *static equilibrium*, and extend our analysis later to a more general dynamic equilibrium that includes capital. We call it dynamic equilibrium because it is an equilibrium within a period, but also *across periods*. Before we do so, let's talk about the goods (commodity) market first.

Goods (commodity) market

At this point, you probably wonder why we have ignored the goods market. How do we know that produced output matches aggregate expenditures $C+I$? In other words: Will everything that is produced be sold? Will everything that is demanded in expenditures be produced? It turns out that the answer to both of these questions is yes. As long as the loanable funds market is in equilibrium, we do not have to worry about the goods market at all. This property is referred to as *Say's law*.

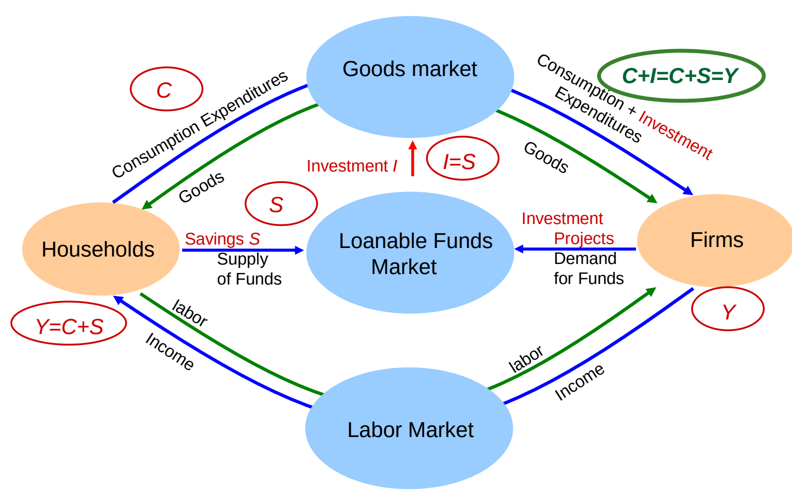

Figure 6: Say's law and circular flows in the classical model.

To understand what makes Say's law hold, we should look at the circular flows in this economy (see figure above). So, in the context of the figure above, let's start with firms paying out all output Y to households as factor payments (circled Y on the lower right-hand-side). When this flow hits households, it is split into consumption expenditures C and savings S. Only consumption expenditures enter the expenditure (spending) side, and savings are an outflow from the system. However, savings are not lost in vacuum. As indicated in the figure, they enter the loanable funds market, and in this market the real interest rate is such that savings are exactly equal to investment projects that firms find profitable to fund. Investment, in turn, is an inflow to the system on the expenditure side. Since inflows = outflows, total expenditures C+I exactly equal output Y.

We can easily extend this setup to additionally include the government (see slides and the textbook). The key modifications include: (i) households' income in such case is factor payment less net taxes T, (ii) the government demands or supplies funds to the loanable funds market depending whether it has budget deficit G-T>0 or budget surplus T-G>0, and (iii) the government expenditures on goods and services G enter final expenditures (which are C+I+G). It is straightforward to show that Say's law still holds, and qualitatively the model works in exactly the same way.

Evolution of Capital

So far, we have been silent about capital. We have simply assumed that it is predetermined (inherited from the past), and given this predetermined level, we found equilibrium level of employment, output and investment in future capital. In what follows, we will refer to this equilibrium as *static equilibrium*, and extent it to make capital part of our analysis.

To include capital in the analysis, we need to make some assumptions how investment in new machines translates into capital in the future. To this end, we will postulate a very simple law of motion that assumes that a constant fraction of capital (which is a stock) depletes or gets worn out every period, and whatever is left, augmented by investment (new machines), turns into capital next period. Assuming fraction δ of capital depreciates (depletes every period), here is how mathematically capital transitions from one period to next in our model:

(1.1) $$K_{tomorrow} = (1-\delta)K_{today} + I.$$

To see how this formula works, suppose capital today is 5, investment is 1 and depreciation rate is $\delta = 1/5$. According to the above formula, capital tomorrow is then given by:

(1.2) $$K_{tomorrow} = \frac{4}{5} * 5 + 1 = 5.$$

Numerical Example

1. Suppose, *G=0, T=0, s=0.2, Population=10, K=10*, and $Y=K^{1/2}L^{1/2}$, calculate output and investment in the static equilibrium of the classical model.
2. Assuming depreciation rate of capital = 10%, calculate the level of capital next period.
3. Is it more or less than today?

Solution: 1. From equilibrium in the labor market, we obtain L=population size=10. Plugging in capital and labor to the production function, we have $Y= K^{1/2}L^{1/2}=10$. Using the decision rule for savings, we have: *S=s(Y-T)=.25x8=2*. The loanable funds market equilibrium implies *I=S=2*. 2. Thus, $K_{tomorrow} = .9K_{today} + 2 = .9x10+2=11$. 3. More. $K_{tomorrow} > K_{today}$ (see 2 above).

Our next task will be to include capital in the analysis by defining an extended equilibrium concept in which formation of capital is part of it. We call it *dynamic equilibrium (or steady state equilibrium).*

Dynamic Equilibrium (Steady State Equilibrium)

Dynamic or steady state equilibrium of the model is a particular *static equilibrium* in which investment is such that capital level does not change from one period to the next – just like in the example underlying formula (1.2).

RECAP:

Static Equilibrium

Given K, static equilibrium is employment *L*, output *Y*, and investment *I* such that labor market, loanable funds market, and goods market are all in equilibrium.

Dynamic Equilibrium (Steady State Equilibrium)

Level of K such that in the underlying *static equilibrium* investment *I* is exactly equal to depreciation of capital δK.

Solving for Dynamic Equilibrium

To find a condition that determines dynamic equilibrium, we should take a closer look what really makes equation (1.1) behave as it did in the example underlying the calculation in (1.2). Closer analysis reveals that the critical condition is that the number of machines that depreciates (we retire due to aging) is equal to the number of machines we add up due to investment, i.e.

(1.3) $$\delta K = I.$$

Our goal is to find the level of capital K so that (1.3) holds.

So, let's revisit our previous numerical example, and see what pins down the dynamic equilibrium. In the example, we notice that both capital depreciation $D = .1K$, and equilibrium investment $I=.2Y=.2K^{1/2}10^{1/2}$ depend on K. Thus, a natural thing to do is to plot them both on one diagram, and see how they compare. From the law of motion for capital, (1.1), we know that when $.2K^{1/2}10^{1/2} > .1K$ then capital will increase over time, and when $.2K^{1/2}10^{1/2} < .1K$, it will fall.

Since depreciation is a constant fraction of existing capital stock, we should plot a straight depreciation line (red). Since investment is a constant fraction of production (exhibiting diminishing returns to scale), we should plot a squeezed down output line (black) as investment line (blue). The figure below illustrates these two lines.

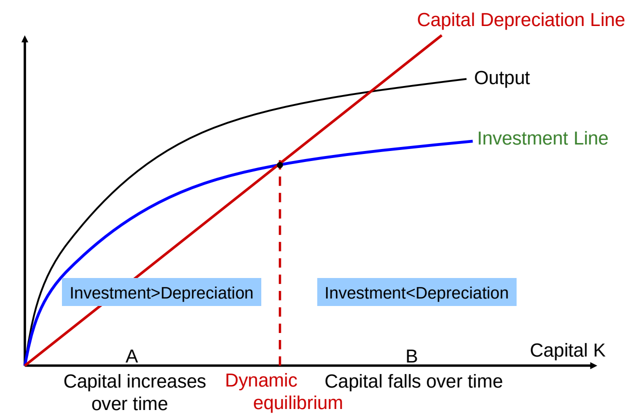

Figure 7: Dynamic equilibrium.

From Figure 7, we notice what follows. Due to diminishing returns from capital, at low levels of capital (like point A), investment line is above the depreciation line, and so capital we will grow over time if we start there. On the other hand, the opposite is true at high levels of capital (like point B). If we start at point B, capital will fall over time. The crossing point is where things exactly balance out – and so if we start there, we stay there. This is our dynamic (or steady state) equilibrium.

Now, an interesting feature here is that this equilibrium is a stable one. What this means is that not only if we start there, we will stay there, but also if we start somewhere else, we will eventually end up there. This is nice, because our model turns out to have a very sharp prediction where the economy is going to head.

Numerically, we can calculate the dynamic equilibrium point by solving the following equation:

$$.1K = .2\sqrt{10K}.$$

Dividing both sides by the square root of K, we have

$$.1\sqrt{K} = .2\sqrt{10}.$$

Furthermore, dividing both sides by .1, we derive

$$\sqrt{K} = 2\sqrt{10},$$

and finally, raising both sides to the power of 2, we obtain K=40.

Let's verify that K=40 really works. If K=40, and L=10, output is Y=20, savings are .2*20=4. Thus, savings are equal to depreciation of capital = .1K=.1x40=4.