The Time Value of Money

Defining the Concept of Present Value

How a sure payment of $100 today compares with a sure promised payment of, say, $120 dollars in one year from now. Which one is better? Which one we should choose?

It turns out that we can give a fairly precise answer to this question. The key is to realize that when there is a market in which people borrow and lend funds at some price, this price, called (*market) nominal interest rate i,* allows us to compare payments that are spread out in time. The market nominal interest rate is determined by the condition of demand for funds = supply of funds.

So, let's assume that the market nominal interest rate is given by *i*, and let's revisit the question how a payment of $100 today compares to a sure promised payment of $120 next year.

The answer is that it depends. For example, if the interest rate *i* is 30%, and we are promised $120 next year, we can take a loan of $120/(1+ .3)=$92.3, and next year we will owe the bank (or someone else) exactly $120 that we expect to get. Thus, by this logic, when the interest rate is 30%, a sure promise of $120 next year seems equivalent to having $92.3 today – which is much less than a $100. On the other hand, when the interest rate is 10%, we should take $120 next year. In this case, we can take a loan of $120/(1+ .1)=$109.09 today – which is better than a $100. Alternatively, if we save $100 today, we will have next year only $110. As we can see, everything depends on the interest rate – but in principle, we can give a precise answer once we know the interest rate at which we can freely borrow and lend.

In the above example, we have implicitly gone through a calculation of a ***present value of a future payment.*** *The present value of a future payment $X tells us* ***the maximum loan that we can take*** *so that in the future we will have to repay $X, or alternatively, it is the* ***amount we need to save*** *so that in the future we will have exactly $X.* As we saw in the example above, the present value crucially depends on the interest rate. In general, when interest rate is high, market values having things right away and the present value tends to be low.

Present value is a very important concept in economics. In the world of business, pretty much any investment is worth to make if and only if future benefits offset the cost incurred today. The key calculation always involves some comparison of future benefits to today's cost. The concept of present value is extremely handy to do that, which we will show below.

Formula for Present Value

Here, we will translate the reasoning from the previous paragraph into symbols, and develop a general formula for the present value of any future payment.

To this end, we suppose that we are promised \$X next year (\$X=\$120 in our numerical example), and the market nominal interest rate is *i (20% in our previous example)*. According to the definition of present value given in the previous paragraph, *the present value of \$X is the maximal loan we can take today so that next period we will owe the bank exactly \$X*. According to this definition, for \$PV to be the present value of \$X, we must require that the amount we owe (1+*i*)\$PV next period is exactly equal to the payment we are promised, i.e. (1+*i*)\$PV=\$X. From this formula, it is straightforward to calculate the \$PV by dividing both sides of the equation by (1+*i*)

(1.1.) $$\$PV = \frac{\$X \text{ (payment next year)}}{1+i}.$$

This is exactly the kind of reasoning we informally went through in the previous paragraph.

Formula for Present Value of a Payment that Occurs T-period from Now

We now know how to calculate the present value of a sure promised payment occurring next period. But how do we calculate a present value of a payment in 2 periods from now, or T-periods from now?

It suffices to repeat our previous reasoning and calculate the present value of \$X occurring in *two* periods from now given that the annual nominal interest rate is expected to be *i* over the next two periods. To this end, we should note that if we take a loan of \$PV today, in one period from now we will owe the bank $(1+i)\$PV$, and in two periods from now we will

$(1+i)^2\$PV$. This is because the interest rate charged in the second year will be based on the amount we owe after the first period, which is $(1+i)\$PV$. This process is called *compounding of interest*. Having established that after 2 periods we will owe the bank $(1+i)^2\$PV$, by analogy to the previous case, to obtain present value, we should now require that the amount we owe in *two* periods from now is exactly equal to the amount $X we are promised to be paid in *two* periods from now, i.e. $(1+i)^2\$PV = \$X$. Similarly as before, by dividing both sides of the equation by $(1+i)^2$, we obtain

(1.2.) $$\$PV = \frac{\$X \text{ (payment in two periods from now)}}{(1+i)^2}.$$

Notice that the above formula differs from the previous formula by the exponent in the denominator. In fact, it is fairly easy to show that in a more general case of a promised payment in T-periods from now, we would obtain

(1.3.) $$\$PV = \frac{\$X \text{ (payment in T periods from now)}}{(1+i)^T}.$$

(If you do not see it right away, please redo the above reasoning with 3 periods instead of 2 periods, and it will become crystal clear ☺.)

Present Value If Lending Rate and Borrowing Rate Are Allowed to Differ

In the real life, the rates at which we can borrow money from the bank differ from the rates at which we can save the money (lend money to the bank). Main component of this spread is the excess risk that the individual borrower will fail to repay the loan (default) over the much lower risk that the bank will fail to accrue the interest on the saving account (by going bankrupt). However, as long as the borrower can guarantee the repayment on par with a bank, there is no reason for the borrowing rate and lending rates to be far apart (except for the profit margin of intermediation by bank that should be fairly small). Nevertheless, we should ask what happens if this is not the case? Does our reasoning break down? The answer is: not really. In our reasoning we just need to decide which rate to use, and be aware that when we calculate the PV using the lending (saving) rate, it would only be true that if we save $PV today we will have the amount equivalent to the promised payment $Y, but not when we borrow. Typically, the lending rate is used in PV calculations to reflect the sure thing aspect of the future promised payment.

Real Life Application

Suppose you are considering buying a server to set up an internet site that will intermediate sales of used textbooks for all 102 classes that I teach over the next 6 years. On each textbook sold you expect to make $1, and you expect to sell at least 200 books per year for the first 2 years, 250 for the next 4 years, and then (six years from now) you expect to resell the server at the scrap value of $300. Assume buying the site and the server will cost you $1000 today. Our goal is to answer the following questions:

- Is it worth making the investment if the market interest rate is expected to be 10% over this 6 year long period?
- Would your answer change if the interest rate was expected to be 15% instead?

Solution

The present value of expected payments can be calculated as follows

$$PV = \$200 + \frac{\$200}{1.1} + \frac{\$250}{1.1^2} + \frac{\$250}{1.1^3} + \frac{\$250}{1.1^4} + \frac{\$250}{1.1^5} + \frac{\$300}{1.1^6} = \$1271.58$$

Because the present value is larger than the cost of making the investment, it is worth making the investment at the interest rate equal to 10%.

However, if the interest rate is expected to be 15% instead, this is no longer the case as the present value is smaller than $1000 (the cost of making the investment)

$$PV = \$200 + \frac{\$200}{1.15} + \frac{\$250}{1.15^2} + \frac{\$250}{1.15^3} + \frac{\$250}{1.15^4} + \frac{\$250}{1.15^5} + \frac{\$300}{1.15^6} = \$627.16$$

As an important caveat, note that in the above calculation we have assumed that the profits are a sure thing – i.e. there is no risk involved. In reality, we would want to factor in the risk factor. So, in such case, we should rather ask if the surplus income of 272.58 we expect to make when interest rate is 10% is really enough to compensate for the risk we take.

Pricing Stocks using the Present Value

Another important application of PV calculation is to price stocks in the stock market. A stock is the ownership right to a company – or a right to a share in its future profits. The value of a stock should thus be somewhat close to the present value of future profits after factoring in risk and the number of outstanding stocks. For example, if the site selling textbooks from our sample application decided to issue 120 stocks, and attempt to sell it to students in this class, the price of its stock should be about $1271.58/120=$10.6 (assuming away the risk factor).

An exact analog of the reasoning above is used in practice to price companies in the stock market. The price calculated this way is called the **fundamental value of a stock.**