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Nonstochastic Steady State of the Stochastic Growth Model

To compute values in a non-stochastic steady state, proceed as follows. The first order conditions and resource constraint may be written

(1) $$\varphi \frac{N_t}{(1-N_t)^{\gamma}} = \theta(1\text{-}\varphi)(\frac{Y_t}{C_t}),$$

(2) $$1 = \beta E_t\{\frac{C_t}{C_{t+1}}[(1\text{-}\theta)(\frac{Y_{t+1}}{K_{t+1}}) + (1\text{-}\delta)]\},$$

(3) $$\frac{C_t}{Y_t} + \frac{K_{t+1}}{Y_t} - (1\text{-}\delta)\frac{K_t}{Y_t} = 1.$$

(All notation is as in the handout "Stochastic Growth Model.")

Suppose that A_t is growing deterministically at a gross rate ≥ 1. Then the system can remain in a non-stochastic steady state in which N_t is constant and C_t, Y_t and K_t all grow at the same rate ($\Rightarrow C_t/Y_t$ and K_t/Y_t are constant). Let the common growth rate be $G\geq 1$, and let $\bar{N}$, $\overline{K/Y}$, and $\overline{C/Y}$ denote the constant values of N_t, K_t/Y_t and C_t/Y_t. Then given G, φ, β, θ, δ and γ, (1)-(3) is a system of three equations in the three unknowns $\bar{N}$, $\overline{K/Y}$, and $\overline{C/Y}$. Equation (2) may be used to solve for $\overline{K/Y}$, which then may be used with (3) to solve for $\overline{C/Y}$, which then may be used with (1) to solve for $\bar{N}$.

Specifically:

(4) $$\overline{K/Y} = (1\text{-}\theta) / [\beta^{-1}G\text{-}(1\text{-}\delta)],$$

(5) $$\overline{C/Y} = 1 - (G\text{-}1+\delta)\overline{K/Y},$$

(6) $\bar{N}$ satisfies $\bar{N}/(1\text{-}\bar{N})^{\gamma} = [\theta(1\text{-}\varphi)/\varphi]/(\overline{C/Y})$

(Restrictions on the parameters beyond those already assumed may be required to obtain $0\leq\bar{N}\leq 1$. γ=1 [utility is logarithmic in leisure] is sufficient.)

The exogenous source of growth in the model is A_t. But note that if growth in A_t causes K_t and Y_t to grow at rate G, then A_t must be growing not at rate G but at rate $G^{\theta}\leq G$. (This follows from the production function, $(Y_t/K_t)=A_tK_t^{-\theta}N_t^{\theta}$: in steady state, the left hand side and N_t^{θ} are each constant $\Rightarrow A_tK_t^{-\theta}$ is constant. Since K_t is growing at rate G, constancy of $A_t K_t^{-\theta}$ requires that A_t grow at rate G^{θ}.)

Let us solve (4)-(6) in terms of underlying parameters. For notational simplicity, let $\beta = 1/(1+r)$, let $G = 1+g$ and assume that $\gamma=1$. Then

$$\overline{K/Y} = (1-\theta)/(r+g+rg+\delta) \approx (1-\theta)/(r+g+\delta)$$

$$\overline{C/Y} = [r+rg+\theta(g+\delta)]/(r+rg+g+\delta) \approx [r+\theta(g+\delta)]/(r+g+\delta)$$

$$\bar{N}/(1-\bar{N}) = [\theta(1-\varphi)(r+g+rg+\delta)] / [\varphi(r+rg+\delta\theta+\delta g)] \Rightarrow$$

$$\bar{N} = [\theta(1-\varphi)(r+g+rg+\delta)] / \{[\theta(1-\varphi)(r+g+rg+\delta)] + [\varphi(r+rg+\delta\theta+\delta g)]\}$$