

Example of research:

Robert Solow, “Technical Change and the Aggregate Production function”  
(1957)

Recall the four steps in a typical research paper:

1. Definition of the topic
2. Specification of the theoretical framework.
3. Estimation or simulation.
4. Discussion of results.

## 1. Topic (p312)

Output in the U.S. has grown over time. Recent data:

	(1)	(2)	(2)/(1)	annual growth rate
GDP (billion \$)	1949 267	2004 10734	43.9	6.9%

Two sources of growth in nominal GDP that Solow is *not* concerned with:

- inflation
- population and employment

	(1)	(2)	(2)/(1)	annual growth rate
Price level (2000=100)	1949 16.4	2004 109.1	6.7	3.5%
Real GDP (billion \$)	1635	10756	6.6	3.4%
Population (millions)	149	294	2.0	1.2%
Real GDP per capita	10970	36590	3.3	2.2%

(Source: <http://www.bea.doc.gov>)

Thus, even after adjusting for inflation and population growth/employment, output has grown:

- it doubled during Solow's 1909-1949 time period
- it tripled during the 1949-04 time period

Question: how much of the growth is due to growth in capital stock, how much to improved technology?

## 2. Specification the theoretical framework (pp 312-313)

Two factors of production

capital stock:  $K(t)$  (machines, buildings, ...)

labor:  $N(t)$  (hours worked, or number of workers)

$t$ =time (year), 1909, 1910, ... , 1949.

Technological progress is represented by

level of technology:  $A(t)$

The amount of output produced by given  $K(t)$  and  $L(t)$  increases over time as  $A(t)$  grows.

Problem: there is no data on  $A(t)$ . Solow shows that growth in technology can nonetheless be inferred from growth in output, capital and hours worked.

Illustrate with Cobb-Douglas production function:

$$Y(t) = A(t)K(t)^\alpha N(t)^{1-\alpha}.$$

where:

$Y(t)$ =output (GDP or GNP)

$A(t)$ =level of technology

$A(t) \uparrow$  means technological progress

$A(t)$  not directly observable

$\alpha$ =parameter,  $0 < \alpha < 1$

We will work with data in terms of percentage growth. This will involve a data transformation called “log differences”.

Reminder: “ln” is “natural logarithm”. And for two numbers “x” and “z”

$$\ln(xz) = \ln(x) + \ln(z).$$

Apply to Cobb-Douglas production function:

$$\ln[Y(t)] = \ln[A(t)] + \alpha \ln[K(t)] + (1-\alpha) \ln[N(t)]$$

Letting lower case letters denote logarithms, this is

$$y(t) = a(t) + \alpha k(t) + (1-\alpha)n(t).$$

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Subtracting successive observations (“first differencing”) gives

$$y(t)-y(t-1) = a(t)-a(t-1) + \alpha[k(t)-k(t-1)] + (1-\alpha)[n(t)-n(t-1)]$$

or

$$\Delta y(t) = \Delta a(t) + \alpha \Delta k(t) + (1-\alpha) \Delta n(t).$$

where  $\Delta y(t) = y(t) - y(t-1)$ , etc.

This says: growth in output = growth in the level of technology, plus a weighted sum of growth in capital and labor.

Numerical example, with  $\alpha=1/3$ , illustrating that log differences are approximately percentage growth:

	A	K	N	$Y=AK^{1/3}N^{2/3}$
t	10	200	100	1260
t+1	10.2	206	106	1349
ratio t+1/t	1.02	1.03	1.06	1.071
% growth	2%	3%	6%	7.1%
$\Delta \ln$	.020	.030	.058	.068

Apart from rounding,

$$.068 = .020 + (1/3)(.03) + (2/3)(.058)$$

(Mathematical reason log difference is approximately the same as percentage change: a mean value expansion can be used to show that  $\ln(1+x) \approx x$  for  $x$  near zero. So for growth in A, for example,  $\ln(1.02) \approx .02$ .)

### 3. Estimation of parameters and testing of hypotheses (pp314-316)

Given data on output  $Y$ , capital stock  $K$ , and labor force  $N$ , as well as the value of the parameter  $\alpha$ , the growth rate of  $A$  can be computed as a residual that is left over after the effects of capital and labor growth are accounted for:

$$\Delta a(t) = \Delta y(t) - \alpha \Delta k(t) - (1-\alpha) \Delta n(t).$$

$A(t)$  is called the “Solow residual” or “multifactor productivity” or “total factor productivity”.

To get a value for  $\alpha$ , one can use data on labor or capital income, as a share of total income, as follows. If factor markets are competitive, factors are paid their marginal products. The share of labor income in national income will be  $(1-\alpha)$ , that of property income will be  $\alpha$ .

Specifics:

Marginal product of labor =

$$\frac{\partial Y}{\partial N} = (1-\alpha)AK^\alpha N^{-\alpha} = (1-\alpha)\frac{Y}{N}$$

Then total income to labor =  $N \frac{\partial Y}{\partial N} = (1-\alpha)Y$ .

Share of labor income in national income =  $1-\alpha$ .

Similar algebra says

$$\frac{\partial Y}{\partial K} = \alpha \frac{Y}{K},$$

share of capital income in national income =  $\alpha$ .

Solow works in more general framework (arbitrary constant returns to scale production function rather than Cobb-Douglas, with the factor shares possibly changing from year to year).

What he finds is that most – about  $7/8$  – of the growth in output per worker is due to technological progress. Output per worker grew at a rate of 1.7% per year; had there been no growth in the level of technical knowledge, the rate would have been .3%.

#### 4. Discussion of results (pp316-320)

Solow notes that the rate of growth of  $A(t)$  is about constant, compares his conclusions to those of others, estimates some production functions (including Cobb-Douglas, finding [in my notation]  $\alpha=.35$ ).

FYI, U.S. Bureau of Labor Statistics estimates (<http://stats.bls.gov>) for recent period (1948-2001) are:

- Y/N tripled
    - Y grew by a factor of about 6, N grew by a factor of about 2
    - annual rate of growth of Y/N was about 2.5%
  - Growth in A accounted for over half of the growth in Y/N
    - annual rate of growth of A was about 1.4%
    - subperiod variation in rate of growth A was enormous
- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1948-73 | 1979-90 | 1990-95 | 1995-00 | 2000-02 |
| 2.1%    | 0.5%    | 0.6%    | 1.3%    | 1.0     |

## Recent data on productivity (private business)

	1948-2	1948-73	1973-90	1990-95	1995-0	2000-2	2001-2
Output per hour of all persons	2.5	3.3	1.5	1.5	2.7	3.6	4.5
Contribution of capital intensity	0.9	0.9	0.8	0.4	1.1	1.8	1.7
Contrib. of information processing equipment and software	0.3	0.1	0.4	0.4	0.9	0.9	0.8
Contrib. of all other cap services	0.5	0.8	0.3	0.0	0.2	0.9	0.8
Contribution of labor composition	0.2	0.2	0.2	0.4	0.3	0.7	0.8
Multifactor productivity	1.4	2.1	0.5	0.7	1.3	1.0	1.9

Source: "Multifactor productivity trends, 2002"  
<http://www.bls.gov/news.release/prod3.nr0.htm>

## Extensions:

- multiple factors (distinguish between different types of capital, different types of workers)
- drop assumption of perfect competition
- distinguish between different types of technological change
- apply to disaggregate data
- apply to data from other countries