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Some Evidence on Secular Drivers of US Safe Real Rates†

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We study long-run correlations between safe real interest rates in the United States and over 30 variables that have been hypothesized to influence real rates. The list of variables is motivated by an intertermporal IS equation, by models of aggregate savings and investment, and by reduced-form studies. We use annual data, mostly from 1890 to 2016. We find that safe real interest rates are correlated as expected with demographic measures. For example, the long-run correlation with labor force hours growth is positive, which is consistent with overlapping generations models. For another example, the long-run correlation with the proportion of 40 to 64 year-olds in the population is negative. This is consistent with standard theory where middle-aged workers are high savers who drive down real interest rates. In contrast to standard theory, we do not find productivity to be positively correlated with real rates. Most other variables have a mixed relationship with the real rate, with long-run correlations that are statistically or economically large in some samples and by some measures but not in others. (*JEL* E21, E22, E24, E43, E52)

It is well known that the safe real rate in the United States has declined over the last several decades. This decline poses difficulties for monetary policy because of the effective lower bound (Yellen 2016), and may also signal secular changes in growth prospects (Summers 2016). Hence, it is vital to understand the reasons for this secular decline. In this paper, we present some reduced-form evidence on the decline via estimation of long-run correlations between the safe real interest rate in the United States and some variables that have been posited to move with safe rates.

There are broadly three approaches to thinking about long-term movements in safe real interest rates or its close cousin the natural rate of interest.[1] The approaches are not inconsistent, and can and do coexist in a given model. But they differ in

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[1] In our discussion of related literature, we will not distinguish between papers that have considered the safe real rate r (our paper) versus those that have considered the natural rate r^* (some of the papers we are about to cite). In models in which r^* appears, trend determinants of the two are generally the same. See Section IE.

which factors receive pride of place. One approach looks to secular movements in growth. Downward trends in real rates are tied to downward trends in growth (e.g., Laubach and Williams 2003, Yi and Zhang 2017). This can be motivated from an intertemporal IS equation consistent with Weil (1989) and Epstein and Zin (1991) preferences, or the constant elasticity specialization familiar from both asset pricing (e.g., Nason and Smith 2008) and New Keynesian models (e.g., Galí 2011). A second approach looks to aggregate desired savings and investment. Outward shifts in the supply of savings or inward shifts in investment demand will result in lower real rates. Factors that have been posited to be important shifters include labor force (Baker, Delong, and Krugman 2005), age distributions (e.g., Lisack, Sajediz, and Thwaites 2017), the price of investment goods relative to consumption goods (e.g., Sajedi and Thwaites 2016), flight to quality (e.g., Del Negro et al. 2017), and government saving or dissaving (e.g., Ball and Mankiw 1995). Rachel and Smith (2015) provides a recent application of the aggregate supply and demand approach. Finally, some reduced-form work has looked at some additional factors associated with the Mundell (1963)–Tobin (1965) effect. For example, Rapach and Wohar (2005) argues that regimes with higher inflation tend to have lower real rates.

We, too, take a reduced-form approach. Using the literature outlined in the previous paragraph, we construct a list of over 30 variables hypothesized to be correlated with safe real rates. We call these variables "correlates." We apply frequency and time domain techniques to estimate long-run correlations between the safe real rate and each correlate. To estimate these long-run correlations, we have collected long time samples of our correlates, most of which span 1890–2016. Because of the influence of the world wars, as well as possible drift in moments, we also present results for a 1950–2016 subsample.

Our data and econometric approach have benefits relative to much of the literature that accounts for trends in safe real rates. First, we look at an unusually broad set of correlates, allowing us to compare a comprehensive set of variables that have previously been analyzed only in various subsets. Second, our 120+ years of data has an unusually long span. For inference about long-run phenomena, a long data span is useful and perhaps essential. Finally, our econometric techniques are not tied to parametric models or even to a particular assumption about the order of integration of the data. We rely on Müller and Watson (2018) to supply confidence intervals for long-run correlations that are robust to a range of orders of integration. This range allows (but does not require) the safe rate to have a different order of integration than a correlate. The possibility of different orders of integration seems important to allow because the safe rate is very persistent, but some correlates, such as per capita GDP growth, are not.

The estimates of long-run correlations yield four notable results. First, growth in aggregate labor hours co-moves with the real rate as predicted by growth models or models of aggregate saving and investment. Second, demographic variables generally co-move with the real rate as predicted by overlapping generations models or models of aggregate saving and investment. For example, there is a negative long-run correlation between the safe rate and the fraction of the population aged 40–64 and a positive long-run correlation between the safe rate and the dependency ratio (percentage less than 20 or older than 65). These two results are consistent with much recent literature that points to working age population growth and age distributions as major factors in our current run of low safe real rates (Gagnon, Johannsen, and López-Salido 2016; Kara and von Thadden 2016).

Third, we find a negative rather than a positive long-run correlation between the safe rate and TFP growth and thus, presumably, between the safe rate and trend growth. Fourth, other variables suggested by the three approaches listed above deliver a mixed picture. Examples include GDP growth, the current account and interest rate spreads (correlated with the real rate as expected in the 1950–2016 subsample but not in the 1890–2016 sample as a whole) and inflation and money growth (correlated with the real rate as expected in the 1890–2016 sample but not the 1950–2016 subsample).

Much current conventional wisdom views trend GDP growth as the primary driver of the secular trend in safe real rates (Laubach and Williams 2003, 2016; Fischer 2016, 2017). However, the results in this paper and in research such as Leduc and Rudebusch (2014) and Hamilton et al. (2016) suggest that GDP growth and real rates do not show a reliably positive low-frequency correlation. Now, GDP growth is driven by both productivity and labor hours growth. We find negative low-frequency correlations between productivity growth and real rates, but positive low-frequency correlations between labor growth and real rates. That is, labor growth shows a low-frequency correlation with real rates that is reliably of the sign predicted by the economic models cited above. Hence, if forced to rely on a growth variable, labor hours growth seems preferable to GDP growth or TFP growth as a low-frequency correlate of real rates.

Beyond the previous paragraph's decomposition of GDP growth into productivity growth and labor growth, we do not attempt to tease out reasons for mixed results noted in our fourth result. We do conclude that even with over a century of data, it is difficult to eliminate all but one or two variables as especially important correlates of safe real rates. This may be because of limited data span, or regime change, or of course it may be because in fact there are many important correlates.

A semantic note: we use "low frequency," "long run," and "trend" interchangeably.

Section I outlines the models that motivate our list of correlates. Section II describes our empirical methods, Section III our data, Section IV empirical results. Section V concludes. Two online appendices contain some results omitted from the paper to save space.

I. Underlying Models

Here we outline models that motivate the variables that we examine in our reduced-form approach. We refer to these variables as potential "correlates" of the safe real interest rate r_t. The models that we are about to describe generally determine these variables jointly with r_t, and we do not attempt to rationalize causality from correlates to r_t. As well, some of these models relate r_t to the correlates not just at low frequencies but at each instant. But our interest in secular movements in r_t causes us to focus on low frequencies.

We draw on three approaches. The first relies on a first-order condition that relates consumption growth to the real interest rate—the Weil (1989) and Epstein and

Zin (1991) version of an intertemporal IS equation. The second involves an informal denumeration of factors affecting aggregate savings supply and aggregate investment demand. The third uses reduced-form VARs. The three approaches are of course not inconsistent, and determinants suggested by one are often also suggested by another.

A. *Intertemporal IS*

We begin with the Weil (1989) and Epstein and Zin (1991) intertemporal condition for optimal consumption. With some abuse of terminology, we will refer to this as "the intertemporal IS," though this term is generally confined to the constant elasticity specialization given in (5) below.

Let C_t = consumption. Preferences are

$$U_t = \left\{(1-\beta)C_t^{[(1-\gamma)/\theta]} + \beta\left[E_t U_{t+1}^{1-\gamma}\right]^{1/\theta}\right\}^{[\theta/(1-\gamma)]}, \tag{1}$$

where

$$\begin{aligned} \beta &= \text{discount factor;} \\ \gamma &= \text{relative risk aversion;} \\ \theta &= (1-\gamma)/\left(1-\psi^{-1}\right), \text{ with} \\ \psi &= \text{intertemporal elasticity of substitution.} \end{aligned} \tag{2}$$

Per period utility of the familiar form, $C_t^{1-\sigma}$ is a special case in which $\psi^{-1} = \gamma$ (so $\theta = 1$) and $\sigma \equiv \psi^{-1}$ $(= \gamma)$.

Let $1 + i_t$ = known gross return on a nominally safe asset (issued in period t, payoff in period $t+1$), P_t = price level, P_{t+1}/P_t = stochastic gross inflation, R_{mt+1} = gross stochastic return on the market portfolio. Then the first-order condition for purchase of a nominal one period bond (Campbell, Lo, and MacKinlay 1997, 319) is

$$1 = \beta^\theta E_t\left[\frac{1+i_t}{(P_{t+1}/P_t)}\left(\frac{C_{t+1}}{C_t}\right)^{-\theta/\psi}(R_{mt+1})^{\theta-1}\right]. \tag{3}$$

Let $\pi_{t+1} = \ln(P_{t+1}/P_t)$, $\Delta c_{t+1} = \ln(C_{t+1}/C_t)$, $r_{mt+1} = \ln(R_{mt+1})$. The online Appendix shows how a second-order log linearization of (3), in conjunction with a second-order log linearization of the comparable first-order condition for purchase of a share in the market portfolio, leads to

$$\begin{aligned} &\ln(1+i_t) - E_t\pi_{t+1} \\ &\approx -\ln\beta + \frac{1}{\psi}E_t\Delta c_{t+1} - \frac{1}{2}\left[\left(\frac{\theta}{\psi^2}\right)\mathrm{var}_t\Delta c_{t+1} + \mathrm{var}_t\pi_{t+1} + 2\left(\frac{\theta}{\psi}\right)\mathrm{cov}_t(\Delta c_{t+1}, \pi_{t+1})\right] \\ &\quad + \frac{1}{2}(\theta-1)\left[\mathrm{var}_t r_{mt+1} + 2\mathrm{cov}_t(r_{mt+1}, \pi_{t+1})\right]. \end{aligned} \tag{4}$$

Here, "var_t" and "cov_t" denote conditional variance and covariance. In the isoelastic case, with $\gamma = \psi^{-1}$ and $\sigma \equiv \psi^{-1}$, (4) is the familiar intertemporal IS equation

$$(5)\quad \ln(1 + i_t) - E_t\pi_{t+1}$$
$$\approx -\ln(\beta) + \sigma E_t \Delta c_{t+1} - \frac{1}{2}\left[\sigma^2 \text{var}_t \Delta c_{t+1} + \text{var}_t \pi_{t+1} + 2\sigma \text{cov}_t(\pi_{t+1}, \Delta c_{t+1})\right].$$

The left-hand side of (4) and (5)—the real rate of interest on a nominally safe security—is our variable of interest. After approximating $\ln(1 + i_t) \approx i_t$, our empirical counterpart to the left-hand side, which we call r_t, is constructed via

$$(6)\qquad r_t = i_t - E_t\pi_{t+1}.$$

We use rolling regressions to construct $E_t\pi_{t+1}$ (details below).

The well known Laubach and Williams (2003, 2016) model for the natural rate of interest relies in large measure on (5). That model focuses on trend output growth as a determinant, with trend growth motivated by the $E_t\Delta c_{t+1}$ term. Trend output growth in turn suggests TFP growth as a determinant. Other research that has pointed to TFP growth as a long-run determinant of real rates includes Yi and Zhang (2017). The second-order terms in (5) have received attention in Nason and Smith (2008).[2] The additional second-order terms in (4) have been an element in models that solve Weil's (1989) "risk free rate" puzzle (e.g., Bansal and Yaron 2004). We measure the return on the market portfolio as the real return on the S&P 500.

From this literature, we are motivated to consider the set of potential correlates listed in rows (1)–(3) in Table 1. "Aggregate growth" will be measured by per capita consumption growth (per (3)), per capita GDP growth, and TFP growth. Apart from the two terms in (4) involving r_{mt+1}, the entries in the "expected sign" column come directly from (4): positive for aggregate growth, negative for second moments. The terms involving r_{mt+1} in principle have ambiguous expected signs, since $\theta - 1$ may be either positive or negative. We take both calibration (Bansal and Yaron 2004) and estimation (Rapach and Tan 2018) studies as arguing that $\theta < 1$. Hence, we give "$-$" as the expected sign for these two variables.

Of course, (4) is an equilibrium relationship and the entries under "expected sign" unambiguously follow from (4) only if we hold all other variables constant. But here and throughout Table 1, we present the sign relevant if the variable is a dominant determinant of r and thus displays an unconditional correlation whose sign is consistent with the conditional correlation delivered when other variables are held constant.

B. *Aggregate Savings and Investment*

Barro and Sala-i-Martin (1990) is an early example and Rachel and Smith (2015) is a recent example of research that considers trends in r when r is determined by the

[2] Some literature has extended the utility function to allow habit persistence. As explained in Hamilton et al. (2016), such an extension does not have important implications for trend real rates.

Table 1—Possible Low Frequency Correlates of Real Rates

	Variable	Expected sign of low frequency correlation with r
(1)	Aggregate growth	+
(2)	Volatility of aggregate growth of the market return	–
(3)	Covariance between inflation and aggregate growth or between inflation and the market return	–
(4a)	Labor hours	+
(4b)	Capital deepening	–
(5a)	Dependency ratio (percent population <20 or >64)	+
(5b)	Percent of population 40–64	–
(5c)	Δ percent of population 40–64	+
(5d)	Δ life expectancy	–
(6)	Government dissaving	+
(7)	Current account	+
(8)	Relative price of investment goods	+
(9)	Inequality	–
(10)	Spread between public and private rates	–
(11)	Inflation	–
(12)	Money growth	–

Notes: Variables in rows 1–3 are suggested by the intertemporal IS implied by Weil (1989) and Epstein and Zin (1991) preferences, in rows 4–10 by models of aggregate desired savings and investment, and in rows 11–12 by reduced-form studies. See text for references.

intersection of aggregate desired savings and aggregate desired investment. Factors that might shift the aggregate savings schedule include: demographics, such as the dependency ratio; inequality; government savings or dissavings; the emerging market savings glut; the spread between safe and risky rates. Factors that might shift the aggregate investment schedule include labor force growth and the falling relative price of capital goods.

Rows 4–10 in Table 1 list the correlates we consider. For brevity, we limit ourselves to one or two cites for each of our assertions.

- Baker, Delong, and Krugman (2005) observes that in the steady state of certain overlapping generations models (and in the Solow model) interest rates are positively related to the rate of labor force growth.[3] Kara and von Thadden's (2016) numerical results illustrate that the positive relationship also obtains in Blanchard's (1985) and Gertler's (1999) multi-period finite lived model. Hence, the "+" in row 4a. Those models typically have labor inelastically supplied. We use labor hours to allow for fluctuations in labor hours per individual. In a standard production function, an increase in the capital labor ratio is associated with

[3] This is in contrast to the infinitely lived model underlying the intertemporal IS. In overlapping generations models, each generation faces the usual intertemporal condition (equation (3)) trading off first- versus second-period consumption. But higher labor force growth leads to lower capital per worker and a higher marginal product of capital (higher interest rate). See Romer (2012, chapters 1 and 2).

a lower return to capital (all else equal). Hence, the "−" in row 4b. We measure capital deepening as growth in capital per hour.

- Define the dependency ratio as the percentage of the population younger than 20 or older than 64. An increase in the dependency ratio will shift the savings schedule in, thus raising r (Gagnon, Johannsen, and López-Salido 2016); an increase of the fraction middle aged will work in the opposite direction. Geanakoplos, Magill, and Quinzii (2004) argues that changes in what they call the middle to young ratio will be positively correlated with r; we conclude from their study that the change in the fraction of middle aged will also be positively correlated with r. Carvalho, Ferrero, and Nechio (2016) argues that an increase in life expectancy will lower r because workers will save more expecting a longer retirement period.
- Transitory decreases in government saving—i.e., increases in government purchases or decreases in taxes financed by borrowing—have been argued to push up real rates (Ball and Mankiw 1995). If those transitory decreases happen every couple of decades, say because of wars, then there will be a low-frequency link between government saving or dissaving and real rates. And even with lump sum taxes, in non-Ricardian models there can be a long-run relation between government debt and real variables, with higher debt/GDP associated with higher interest rates (Gertler 1999). Hence, the "+" in row 6 of Table 1 (higher deficits and higher debt mean higher r).
- US current account deficits have an ambiguous effect. Bernanke (2005) argues that these deficits reflect an inflow of savings to the United States so that increased deficits are associated with lower real rates. In contrast, if current account deficits are driven by increased local demand, then an increased deficit is associated with higher real rates.[4] We view Bernanke's (2005) global savings glut hypothesis as more widely endorsed, so there is a "+" in the "current account" row.
- A falling relative price of investment also has ambiguous effects—a smaller expenditure on capital is needed to produce a given amount of output, but firms have an incentive to shift into capital. Eichengreen (2015), citing the International Monetary Fund (IMF 2014), argues that the empirical evidence indicates that the sign is positive.
- Since higher income families have lower marginal propensities to consume (Dynan, Skinner, and Zeldes 2004), an increase in inequality will shift the saving schedule out, lowering r.
- Finally, an increase in spreads that results from a flight to quality will depress safe rates as savings are shifted from risky investment to Treasury debt (Del Negro et al. 2017).

[4] We thank a referee for pointing this out.

C. *Mundell-Tobin Effect*

Here we add variables not directly suggested by the previous two literatures. In particular, some reduced-form studies find that inflation regimes or inflation expectations regimes are correlated with r (Koedijk, Kool, and Kroes 1994; Rapach and Wohar 2005). So we add the rate of inflation and the rate of money growth to our list of potential correlates. Consistent with the Mundell (1963)-Tobin (1965) effect, the studies just cited find that higher inflation is associated with lower r. Hence, we posit money growth and inflation will show a negative correlation with r.

D. *But What about DSGE Models?*

Our list of variables includes ones consistent with the logic of DSGE models that dominate monetary economics today. For example, a first-order condition similar to (1) is ubiquitous in such models; even when (1) is generalized to allow features such as habit persistence, there is still a link between trend growth and low-frequency movement in r (see Hamilton et al. 2016). Some such models include technology shocks that lead to a trend in the relative price of investment (e.g., Justiniano, Primiceri, and Tambalotti 2011). However, such models typically do not have a life-cycle component, nor, so far as we know, do they tie secular movements in real rates to movements in inflation or money. Hence, our decision not to motivate our list from a DSGE model.

E. *r versus r^**

We are interested in long-run determinants of the safe interest rate r_t. A related though distinct object is the "equilibrium" or "natural rate" of interest, call it r_t^*. A question raised by early readers of this paper is the implications of our study for r_t^*. Of course an answer to that depends on exactly how one defines r_t^*. For the purposes of this subsection, let us understand r_t^* as the real rate consistent with output being at potential. In New Keynesian models, this is the level of output consistent with flexible goods prices and wages and constant markups in goods and labor markets.

Now, some of the models cited above, such as overlapping generations models, are real models with competitive markets. Hence, the interest rate in those models is the natural rate, and those models have no need to separately define and reference r_t and r_t^*. Insofar as one interprets the steady state of such models as being compatible with the steady state of related New Keynesian models, one could reasonably suppose that low-frequency movements in correlates suggested by those models—labor hours growth, for example—will be associated with low-frequency movements in r_t and r_t^* in the same direction. This is because in the typical New Keynesian model, the steady-state values of r_t and r_t^*, which are determined by purely real forces, are the same (see Galí 2011). Of course, there may be New Keynesian models in which a primitive shock causes labor hours growth or another correlate to have low-frequency associations with r_t and r_t^* of opposite sign—hence, our use of the phrase "reasonably suppose."

In other models cited above (e.g., Laubach and Williams 2003, 2016), r_t and r_t^* do both appear. As just noted, in the typical New Keynesian model, the steady-state values of r_t and r_t^* are the same; moreover, both respond similarly to growth in technology. So one can state unambiguously that for low-frequency movements in TFP growth, predictions for r_t are also applicable to r_t^* in those models. That need not be true for all correlates that appear in New Keynesian models. For example, in the basic new Keynesian model, monetary shocks affect r_t (transitorily) but not r_t^* (even transitorily). Thus, if there is a low-frequency component to monetary shocks or to inflation caused by money shocks, that model predicts a low-frequency association with r_t but not r_t^*.[5]

In sum, the reader should understand that while our study will shed light on the correlates of trend r_t^*, that is a byproduct of our study of the correlates of trend r_t.[6] But it is not the central purpose of our study. Indeed, we will have no further occasion to discuss r_t^*.

II. Empirical Methods

Let x_t be one of our potential correlates of r_t. We do not perform tests for stationarity. In some respects, we rely on the literature cited above to decide whether to difference a variable before relating it to r_t. For example, we use growth rates of TFP but levels of the relative price of investment goods. As described below, we also present results robust to r and a given correlate being $I(0)$ or $I(1)$ or even fractionally integrated.

For a given correlate, we measure the strength of the long-run correlation with r_t via both frequency and time domain techniques. The frequency domain technique produces an estimate of the low-frequency correlation between x_t and r_t. The time domain technique simply averages x_t and r_t over long periods (with ten years as our window) and computes a correlation using the averages as observations.

In the end, the two approaches yielded qualitatively similar results. Hence, we will sometimes use "low-frequency correlation" or "long-run correlation" to encompass both types of correlations.

A. *Low-Frequency Correlation*

For r_t and a given correlate x_t, we rely on Müller and Watson (2018) to filter out frequencies higher than ten years and compute the low pass correlation between the resulting low-frequency series. One can think of this as a band spectral regression applied to frequencies lower than ten years (Engle 1974), with the regression coefficient renormalized to be expressed as a correlation rather than a regression coefficient. The confidence interval that comes with the point estimate relies on

[5] More generally, some research motivated by New Keynesian literature assumes that $r_t - r_t^*$ is $I(0)$ (e.g., Del Negro et al. 2017, who, more generally, work under the presumption that r_t might be similar to r_t^*). Post-World War II evidence consistent with this assumption may be found in Justiniano and Primiceri (2010). Our work is also consistent with, but does not require, that $r_t - r_t^*$ is $I(0)$.

[6] See Del Negro et al. (2017) for a paper that uses trends in r_t to learn about trends in r_t^*.

asymptotic arguments and procedures for inference quite different from preceding literature on band spectral regression (see Müller and Watson 2018 and below).

We compute the low-pass correlation first under the assumption that r_t and x_t are $I(0)$, letting "$\rho^{LP} - I(0)$" denote the resulting correlation. Here, "*LP*" stands for low pass.

We also compute the low-pass correlation under Müller and Watson's (2018) procedure that produces a confidence interval robust to r_t and x_t each being $I(d)$ for any value of d between -0.4 and 1.[7] They refer to this as the (A, B, c, d) model, and supply code (used by us). Using a uniform prior on d, the code first executes Bayesian estimation of the low-pass correlation from this model. We report the posterior mean correlation, denoted as "$\rho^{LP} - I(d)$." Posterior median correlations are reported in the Appendix. The code then adjusts the equal-tailed Bayes credible set to produce a confidence interval that enforces coverage over the entire set of values of d considered, according to what Müller and Watson call an approximate least favorable distribution.[8] For additional details, see Müller and Watson (2018).

As well, for a band spectral regression of r_t on the correlate, we report the R^2 for $\rho^{LP} - I(0)$ and the posterior mean R^2 for $\rho^{LP} - I(d)$. The R^2 for $\rho^{LP} - I(0)$ is merely the square of the estimate of the long-run correlation. The posterior mean R^2 for $\rho^{LP} - I(d)$ is not the square of the posterior mean of $\rho^{LP} - I(d)$ but instead is a weighted average of the posterior squared correlations where the weights are the posterior probabilities. We do not necessarily endorse R^2 as a measure of how much of r_t is explained by a given correlate. Rather, and recalling that in a bivariate regression such as ours R^2 is a monotonic function of the t-statistic on the correlate, R^2 supplements the confidence interval as an indicator of the statistical strength of the relationship.

B. *Correlations of Ten-Year Moving Averages*

Suppose we have annual data on x_t and r_t running from say 1890 to 2016. We compute ten-year moving averages 1890–1899, 1891–1900, …, 2007–2016. We date these 1899, 1900, …, 2016. Let x_t^{MA} and r_t^{MA} be the resulting series of 118 observations, 1899–2016, where "MA" is short for "moving average." We use these observations to estimate the correlation between x_t^{MA} and r_t^{MA},

$$\text{corr}(x_t^{MA}, r_t^{MA}) \equiv \rho^{MA}. \tag{7}$$

In initial work, all our computations were repeated with non-overlapping ten-year samples, with very little change in point estimates. (That is, if we only use the 12

[7] To clarify what we are estimating, some additional notation is required. Let d_r and d_x be differencing parameters between -0.4 and 1.0, so that $(1-L)^{dr} r_t \sim I(0)$ and $(1-L)^{dx} x_t \sim I(0)$. The object estimated is the same for all d_r and d_x: as we vary d_r and d_x, we are always estimating the low-pass correlation between r_t and x_t and not the low-pass correlation between $(1-L)^{dr} r_t$ and $(1-L)^{dx} x_t$.

[8] In any given sample, the enforcement of coverage over all values of d may not produce a confidence interval that is larger than the confidence interval for $\rho^{LP} - I(0)$, even though $d = 0$ is accounted for in the construction of the robust confidence interval.

observations dated 1906, 1916, . . . , 2016 to compute the correlation, the value is very close to that computed from all 118 observations.)

Confidence intervals are constructed from standard asymptotic theory, using Newey and West (1994) to construct the relevant asymptotic variance-covariance matrix. See the online Appendix for details.

III. Data

We describe construction of real rates and briefly describe other sources of data. We use annual data throughout. We aim to use samples that start in 1890, though some samples start later because of limited data. We use long samples because of the limited information available about low-frequency patterns or trends. For example, our ten-year moving averages yield only 12 non-overlapping observations between 1890 to 2016. Hence, even though we use up to 127 years of data, which is long by typical time-series standards, the number of observations of the trend is limited.

We note that using long samples comes with a cost since longer samples have greater possibility of breaks and regime shifts. In particular, as we discuss below, the world wars have a large influence on the behavior of real rates. But the world wars may not be informative about the movement in real rates in the post-World War II decades. As well, unconditional moments and hence long-run correlations may have drifted over time. Hence, we also estimate long-run correlations on a 1950–2016 sample.

A. *Real Rates*

For a nominally safe rate we use call money rates for 1870–1917, the discount rate for 1918–1919, 3 to 6 month Treasury notes and certificates for 1920–1933, and the three-month T-bill for 1934–2016. These were obtained from the NBER Macro History Database and FRED. In the early years, we use call rates rather than commercial paper because Homer and Sylla (2005) indicates that call loans became more liquid in the late nineteenth century and because the call rates had lower average rates (suggesting greater safety). In each case, the year t value was the monthly average of January rates in year $t+1$. We chose January in $t+1$ rather than December in t because of pronounced seasonality in call money rates. For inflation, we use the GNP/GDP deflator from Romer (1989) for 1870–1929 and from the BEA (line 1 of NIPA table 1.1.4) for 1930–2016. We set expected inflation to zero through 1914. (See Barsky 1987). For 1914-present, we compute expected inflation from an AR(1) using rolling samples of 20 years, setting the AR(1) coefficient to 0.999 if it is estimated to exceed 1. In some initial work, we experimented with constructing expected inflation from an AR(1) for 1890–1913. Results were hardly changed.

Figure 1A plots the resulting real rate. One can see that r_t is quite trendy, broadly trending down until the mid-1940s, then trending up until around 1980, and then trending down again. There is a very large negative spike in 1917 and another, not quite as large, in 1946. One can see in the plot in Figure 1B that this reflects

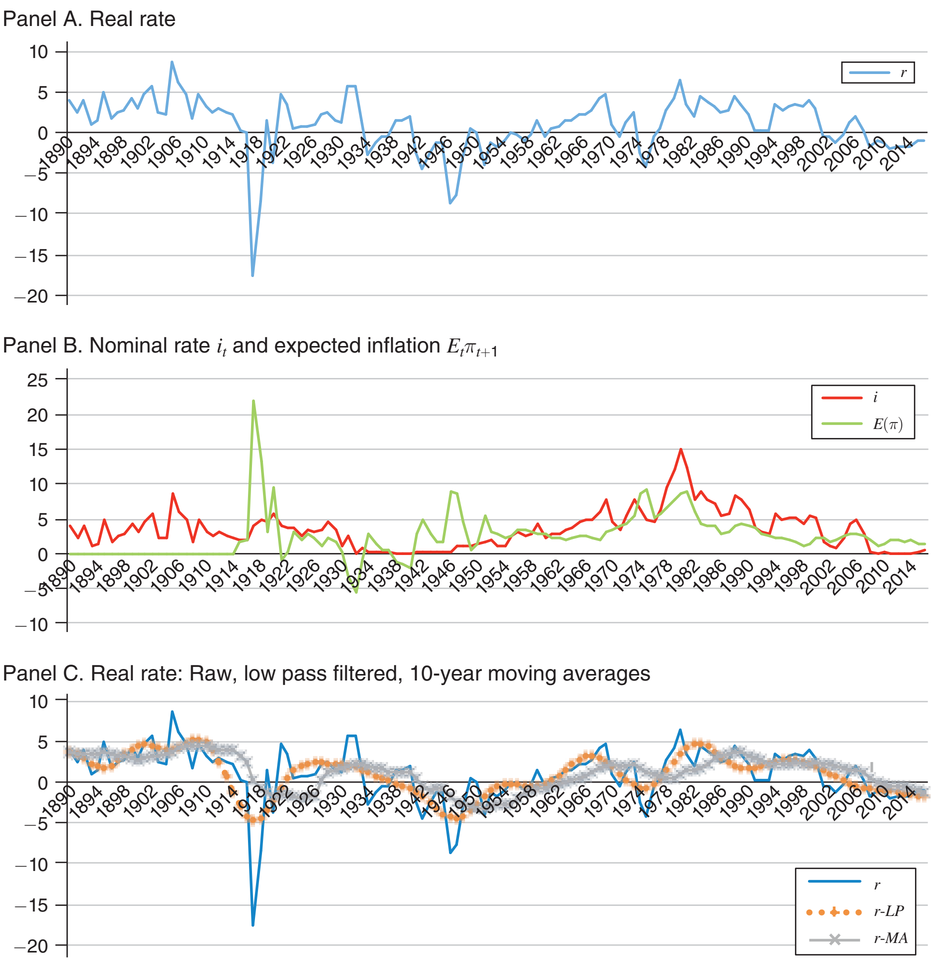

Figure 1

sharp positive spikes in expected inflation. Actual inflation (not plotted) rose from 1 percent (1914) to 20 percent (1917) and was still in double digits (13 percent) in 1920; it rose from 1 percent (1940) to 12 percent (1946) and remained elevated (5 percent) in 1948. (We comment on implications for trend real rates below.)

Figure 1C repeats the Figure 1A plot of r_t along with both of our trend measures of r_t. Because the ten-year moving average series (labeled r-MA in the figure) is a backward average and the low pass filtered series (r-LP) is two-sided, the moving average series is shifted forward relative to the low-pass filtered series. But, that point aside, here and throughout almost all of our analysis, the two trend measures are very similar. The two slide downward together through the mid-1940s. They then move upward until the 1980s, with the Müller and Watson (2018) filtered series (labeled r-LP in the figure) peaking a little earlier than does the ten-year moving average (r-MA). Finally, both move downward from the mid-1980s to the present.

Table 2—Basic Statistics on the Annual Real Rate

		1890–2016	1950–2016
(1)	Mean	0.97	0.97
(2)	Standard deviation	3.28	2.25
(3)	Median	1.38	0.71
(4)	First-order autocorrelation	0.60	0.73
(5)	Maximum	8.65	6.37
(6)	Minimum	−17.7	−4.21

Notes: Annual data, computed as nominal rate minus expected inflation: $r_t = i_t - E_t\pi_{t+1}$. The nominal rate i_t is the average of January rates in $t+1$: call rates 1890–1917, the discount rate 1918–1919, and three-month Treasury bills 1920–2016. Inflation π_t is measured by the GNP/GDP deflator, using Romer (1989) prior to 1929 and BEA data 1930–present. Expected inflation $E_t\pi_{t+1}$ is set to zero 1890–1913. For 1914–2016, $E_t\pi_{t+1}$ is computed from an AR(1) in inflation using rolling samples of 20 years.

Table 2 has basic statistics on the real rate, for a sample starting in 1890 along with the familiar postwar period (1950–). The real rate is volatile, though it became less volatile in the postwar period.

In the 1950–2016 column, the mean of 0.97 percent is below the conventionally presumed value of 2 percent. This partly reflects our choice of nominal interest rate, the three-month T-bill. Over the period of overlap with the federal funds rate (1954–2016), the three-month T-bill rate was 0.44 percent below the federal funds rate on average. As well, the beginning part of our 1950–2016 sample perhaps reflects financial repression that lingered on after the 1951 Treasury-Fed accord, and the end of our sample of course includes the period in which rates well below 2 percent inspired research such as ours. A shift in the interest rate measure and a focus on the 1954–2007 period would yield a figure of about 2 percent.

A similar comment applies to the longer sample period in Table 2. Our choice of nominal interest rate (the call rate) was generally below a possible alternative, the commercial paper rate.

B. *Correlates*

For the most part, our correlates are constructed from US data. When correlates were expressed per capita, the data source for population was Carter et al. (2006).

- Real GNP/GDP growth, per capita. (a) US: Romer (1989) prior to 1929, BEA (line 1 of NIPA table 1.1.3) 1930–2016. Romer and the BEA were also the sources for nominal GDP used in the denominator of series described below that are expressed relative to GDP. (b) World: 23 countries, GDP measured at purchasing power parity rates.[9] Maddison-Project 1890–2010, the IMF (GDP) and the UN (population) 2011–2015.

[9] The countries are: Australia, Austria, Belgium, Brazil, Canada, Chile, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sri Lanka, Sweden, Switzerland, United Kingdom, United States, and Uruguay. These are the countries for which the Maddison data goes back to 1890.

- Growth of real per capita consumption spending. (a) In the 1890– sample, we used total consumption spending: Kuznets (1961) 1890–1929, (line 2 of NIPA table 1.1.3) BEA 1930–2016. (b) In the 1950– sample, nondurables and services spending was available and hence was used (lines 5 and 6 of NIPA table 1.1.5).
- Growth in total factor productivity. Gordon (2016) 1890–1948, FRED series MFPNFBS 1949–1987, the BLS 1988–2016.
- Return on the market portfolio: S&P 500, with nominal December prices and January to December nominal dividends from Robert Shiller's website (www.econ.yale.edu/~shiller/data.htm); real annual returns computed by deflating nominal prices and dividends.
- Conditional second moments in (2). Constructed from the variance-covariance matrix of the residuals of a bivariate VAR(1) in a measure of inflation and a measure of either (a) aggregate growth or (2) the market return. The VAR was estimated using 20-year rolling regressions.
- Growth in labor hours and in capital per hour. We used hours in private business: Kendrick (1961) 1890–1947, BLS series PRS84006033 (average of quarterly figures) 1948–2016. Capital growth is from Gordon (2016) 1890–1948, and from the BLS (capital services in private business) 1949–2016.
- Demographic measures. (a) US: 1890 and 1900–1959 from Carter et al. (2006); 1891–1899 obtained by linear interpolation of the 1890 and 1900 data. Life expectancy: 1960–2016 from FRED series SPDYNLE00INUSA. Other US demographic variables: 1960–2000 from Carter et al. (2006); 2001–2016 from Haver Analytics. (b) World, 1950–2015: The UN's *World Population Prospects*, using data for the several dozen countries defined as high income in 2015 (United Nations 2017a, 156).
- Income inequality. Share of income that goes to the top 10 percent of the income distribution. From the World Wealth and Income Database, 1913–2015.
- Relative price of investment. Relative to total consumption. Kuznets (1961) 1889–1929, BEA (lines 2 and 7 of NIPA table 1.1.4) 1930–2016.
- Government dissaving. (a) Federal government primary deficit relative to GDP. Carter et al. (2006) for 1869–1939; FRED series FYFSD for 1940–2016 (multiplied by -1 so that a positive value means deficit). (b) Federal debt/GDP. Carter et al. (2006) for 1869–1938; FRED series FYGFD for 1939–2016.
- Current account, expressed relative to GDP. Jordà, Schularick, and Taylor (2017) for 1870–1928; BEA (line 33 of NIPA table 4.1) for 1929–2016.
- Spread between public and private borrowing rates: BAA minus ten-year Treasuries. FRED series BAA 1920–2016, with Treasury rates from Homer and Sylla (2005) 1920–1953 and FRED series GS10 1954–2016.
- Money growth. M1 1916–1947 and M2 1869–1947 from Carter et al. (2006); both series 1948–1958 from Rasche (1987); both series 1959–2016 from FRED (series M1NS and M2NS), average of monthly figures.

To keep tables of manageable length, we put in the online Appendix results with correlates that in our view serve mainly to establish the robustness of the results presented here. Those correlates are: labor productivity growth; growth in per capita capital; conditional moments with the price level measured by alternative deflators; the change of the US and world dependency ratios and the level and difference of US and world values of Geanakoplos, Magill, and Quinzii's (2004) MY ratio; federal deficit/GDP and world debt/GDP.

The Appendix also contains the following plots for each of our correlates: scatterplots of ten-year moving averages of *r* versus the correlate, one with all observations 1890–2016 and one with every tenth observation (the latter to give an uncluttered look at the progression of the relationship over time); bivariate time series plots of Müller and Watson (2018) filtered *r* and filtered correlate, 1890–2016 and 1950–2016 sample. We present some time series plots of ten-year moving average data in our discussion below.

IV. Empirical Results

A. *Correlations: Overview*

Estimated correlations are in Table 3 (1890–2016) and Table 4 (1950–2016), with 68 percent confidence intervals. We present 68 percent confidence intervals because as discussed above we only have a small sample of observations on ten-year intervals. With a small sample, power is low. So a less stringent standard for rejecting a correlation of zero seems warranted. (See the discussion in Müller and Watson 2018). Our online Appendix presents 90 percent confidence intervals.

As stated in notes to Table 3, due to data availability, a few series end in 2015 or (in Table 3) start later than 1890. Estimates whose confidence interval excludes zero will be referred to as significant. The "(+)" and "(−)" in column 1 repeats, for convenience, entries in Table 1. In interpreting the estimated sign of a correlation, we refer to "expected" and "unexpected" signs, though, as noted above, the models described above generally make predictions about signs holding all other correlates constant rather than an unconditional prediction. We defer discussion of economic significance to a brief illustration, using conditional forecasts, at the end of this section; we do note that analysis suggests that correlations whose absolute values are 0.20 or larger can be economically significant. We use ρ^{MA}, $\rho^{LP} - I(0)$, and $\rho^{LP} - I(d)$ to denote the population values of the estimates in columns 2, 3a, and 4a.

Some general comments, before discussing specific entries. First, for a given correlate, the estimates of ρ^{MA}, $\rho^{LP} - I(0)$ and $\rho^{LP} - I(d)$ are similar. The signs of all three point estimates are the same in 19 of the 25 rows in Table 3 and 23 of the 28 rows in Table 4. (There are three more entries in Table 4 than in Table 3 because of world demographic variables.) When signs conflict across measures, it is generally the case that all three point estimates are insignificant.

Second, the three measures tend to agree not only in sign but in relative magnitude. We use rank order correlation to summarize concordance of relative magnitude:

(8)

Rank order correlation of estimates	ρ^{MA} versus $\rho^{LP}-I(0)$	$\rho^{LP}-I(0)$ versus $\rho^{LP}-I(d)$	ρ^{MA} versus $\rho^{LP}-I(d)$
1890–2016	0.94	0.94	0.88
1950–2016	0.88	0.94	0.77

TABLE 3—LONG-RUN CORRELATIONS, 1890–2016

	Correlate (expected sign) (1)	10Y moving average $\hat{\rho}^{MA}$ (2)	Low pass filter–$I(0)$ $\hat{\rho}^{LP}$ (3a)	R^2 (3b)	Low pass filter–$I(d)$ Mean $\hat{\rho}^{LP}$ (4a)	R^2 (4b)
(1a)	GDP growth (+)	−0.15 (−0.31, 0.01)	0.01 (−0.19, 0.20)	0.00	−0.02 (−0.19, 0.13)	0.04
(1b)	Consumption growth (+)	0.06 (−0.11, 0.23)	0.04 (−0.16, 0.23)	0.00	0.03 (−0.13, 0.14)	0.03
(1c)	World GDP growth (+)	0.09 (−0.06, 0.24)	0.18 (−0.02, 0.36)	0.03	0.12 (−0.06, 0.34)	0.06
(1d)	TFP growth (+)	−0.55 (−0.64, −0.45)	−0.36 (−0.52, −0.17)	0.13	−0.31 (−0.50, −0.08)	0.14
(2a)	var_t(GDP growth) (−)	−0.62 (−0.78, −0.46)	−0.49 (−0.62, −0.31)	0.24	−0.30 (−0.65, −0.10)	0.12
(2b)	var_t(π_GDP) (−)	−0.17 (−0.29, −0.03)	−0.04 (−0.24, 0.16)	0.00	0.11 (−0.40, 0.39)	0.08
(2c)	var_t(mkt return) (−)	−0.59 (−0.73, −0.46)	−0.51 (−0.64, −0.34)	0.26	−0.35 (−0.75, −0.11)	0.17
(3a)	cov_t(π_GDP, GDP growth) (−)	−0.42 (−0.57, −0.26)	−0.31 (−0.47, −0.11)	0.10	−0.18 (−0.60, 0.08)	0.11
(3b)	cov_t(π_GDP, mkt return) (−)	0.02 (−0.14, 0.18)	−0.08 (−0.28, 0.13)	0.01	−0.18 (−0.42, 0.40)	0.09
(4a)	Labor hours growth (+)	0.31 (0.09, 0.53)	0.27 (0.06, 0.44)	0.07	0.24 (0.05, 0.43)	0.09
(4b)	Growth in capital per hour (−)	0.23 (0.07, 0.39)	0.13 (−0.07, 0.32)	0.02	0.12 (−0.05, 0.34)	0.05
(5a)	Dependency ratio (+)	0.36 (0.17, 0.56)	0.38 (0.19, 0.53)	0.14	0.25 (0.10, 0.65)	0.09
(5b)	Percent aged 40–64 (−)	−0.43 (−0.61, −0.25)	−0.43 (−0.58, −0.25)	0.19	−0.26 (−0.55, −0.23)	0.08
(5c)	ΔPercent aged 40–64 (+)	0.14 (0.00, 0.27)	0.18 (−0.02, 0.36)	0.03	0.19 (−0.03, 0.70)	0.10
(5d)	ΔLife expectancy (−)	−0.14 (−0.27, 0.00)	0.17 (−0.04, 0.35)	0.03	0.08 (−0.13, 0.21)	0.04
(6a)	Fed deficits/GDP (+)	−0.56 (−0.68, −0.43)	−0.48 (−0.61, −0.30)	0.23	−0.43 (−0.59, −0.26)	0.22
(6b)	Fed debt/GDP (+)	−0.59 (−0.79, −0.39)	−0.57 (−0.68, −0.40)	0.32	−0.44 (−0.75, −0.27)	0.22

(*continued*)

The high rank order correlations of the estimates of course means that a correlation that is relatively large by one measure also tends to be relatively large by the other two measures.

Thus, our discussion for the most part will not have need to distinguish between measures of correlation, for either sign or relative magnitude.

B. *Intertemporal IS*

In rows 1a–1c of Table 3, we see that GDP and consumption growth have correlations with safe real rates that are small in magnitude and, in one case, negative over the 1890–2016 sample. However, the same rows in Table 4 show that GDP and

Table 3—Long-Run Correlations, 1890–2016 (*continued*)

	Correlate (expected sign) (1)	10Y moving average $\hat{\rho}^{MA}$ (2)	Low pass filter–$I(0)$ $\hat{\rho}^{LP}$ (3a)	R^2 (3b)	Low pass filter–$I(d)$ Mean $\hat{\rho}^{LP}$ (4a)	R^2 (4b)
(7)	Current account/GDP (+)	−0.15 (−0.35, 0.05)	−0.18 (−0.36, 0.03)	0.03	−0.22 (−0.49, 0.15)	0.11
(8)	Relative price inv. goods (+)	−0.01 (−0.17,0.15)	0.07 (−0.13, 0.27)	0.01	0.12 (−0.11, 0.42)	0.08
(9)	Top 10% income share (−)	−0.03 (−0.22,0.15)	−0.05 (−0.27, 0.17)	0.00	0.11 (−0.40, 0.38)	0.07
(10)	BAA-10-yr Treasury spread (−)	0.30 (0.11,0.48)	0.21 (−0.02, 0.41)	0.04	0.17 (−0.20, 0.47)	0.10
(11a)	π_GDP (−)	−0.30 (−0.45,−0.16)	−0.43 (−0.57, −0.25)	0.19	−0.42 (−0.67, −0.10)	0.23
(11b)	π_PCE (−)	−0.38 (−0.55,−0.22)	−0.48 (−0.62, −0.31)	0.23	−0.46 (−0.71, −0.21)	0.27
(12a)	M1 growth (−)	−0.34 (−0.56,−0.12)	−0.44 (−0.60, −0.24)	0.20	−0.39 (−0.59, −0.16)	0.21
(12b)	M2 growth (−)	−0.33 (−0.50,−0.16)	−0.33 (−0.49, −0.14)	0.11	−0.30 (−0.53, −0.05)	0.14

Notes: In rows 1–3: GDP and consumption are real and per capita; consumption is total consumption; π_GDP is GDP inflation. World GDP is constructed from 23 countries given in footnote 9. "mkt return" is the real return on the S&P 500. The second moments in lines 2 and 3 are constructed from rolling samples of 20 years as described in the text. The dependency ratio in row 5a is defined in Table 1. Rows 6 and 7 are ratios of nominal variables. In row 8, the GNP/GDP deflator for business fixed investment is expressed as a ratio to the deflator for total consumption. In row 11b, π_PCE is PCE inflation. All data are annual. Asymptotic 68 percent confidence intervals are in parentheses. "10Y moving avg." reports correlations of ten-year moving averages of r_t and the indicated variable. "Low pass filter" uses Müller and Watson's (2018) low pass filter to cut off frequencies higher than ten years. The point estimate $\hat{\rho}^{LP}$ is the correlation between the filtered series, while the R^2 is described in Section II in the text. Column 3 applies Müller and Watson (2018) assuming the data are $I(0)$. Column 4 reports Müller and Watson's (2018) confidence interval that controls coverage allowing r and the correlate to be $I(d)$ for any d between −0.4 and 1.0; the point estimate in this column is the posterior mean from a Bayesian procedure whose prior allows r and the correlate to be $I(d)$ in the range just given. See Section II for further discussion. Data end in 2015 for the correlates in rows 1c and 9. The sample start for the low pass filter is 1890 except for the correlates in the following rows: 2a, 2b, 3a–1891; 2c, 3b–1893; 5c and 5d–1891; 9–1913; 10–1920; 12a–1916. The sample start is nine years later in the "10Y" column.

consumption growth fare better over the 1950–2016 sample. These findings that positive correlations between economic growth and real rates are episodic corroborate earlier research (Leduc and Rudebusch 2014, Hamilton et al. 2016). Further, in our view, the mixed picture of signs and significance is consistent with the large literature that finds the intertemporal IS equation wanting, absent second-order terms considered below (e.g., Canzoneri, Cumby, and Diba 2007).

Figure 2A presents a time series plot of ten-year moving averages of r (identical to the r-MA plot in Figure 1C) and of GDP growth. The positive correlation in the 1950–2016 sample is evident. That the two series generally did not move together prior to 1950 is also evident, with the two series moving in opposite directions almost every year from throughout the period from 1914 to 1945.

Figure 2B replaces GDP growth with TFP growth (again, ten-year moving averages), with the plot of trend r repeated.[10] It is patently obvious that the correlation is

[10] The plot for TFP growth begins in 1899 because the underlying data start in 1890. Thus, the first possible observation on ten-year moving averages is 1899.

Table 4—Long-Run Correlations, 1950–2016

	Correlate (expected sign)	10Y moving average $\hat{\rho}^{MA}$	Low pass filter–$I(0)$ $\hat{\rho}^{LP}$	R^2	Low pass filter–$I(d)$ $\hat{\rho}^{LP}$	R^2
	(1)	(2)	(3a)	(3b)	(4a)	(4b)
(1a)	GDP growth (+)	0.61 (0.48, 0.74)	0.23 (−0.06, 0.46)	0.05	0.16 (−0.07, 0.43)	0.09
(1b)	Consumption growth (+)	0.56 (0.40, 0.72)	0.46 (0.19, 0.64)	0.21	0.31 (0.03, 0.60)	0.16
(1c)	World GDP growth (+)	0.05 (−0.23, 0.33)	0.00 (−0.27, 0.28)	0.00	−0.02 (−0.50, 0.27)	0.08
(1d)	TFP growth (+)	−0.23 (−0.46, −0.01)	−0.25 (−0.49, 0.04)	0.06	−0.18 (−0.46, 0.06)	0.11
(2a)	var_t(C growth) (−)	−0.22 (−0.48, 0.03)	−0.40 (−0.60, −0.13)	0.16	−0.37 (−0.65, 0.00)	0.20
(2b)	$\text{var}_t(\pi$_NDS) (−)	−0.16 (−0.43, 0.12)	−0.31 (−0.53, −0.03)	0.10	−0.23 (−0.60, 0.00)	0.11
(2c)	var_t(mkt return) (−)	−0.34 (−0.58, −0.11)	−0.59 (−0.73, −0.35)	0.34	−0.48 (−0.80, −0.27)	0.28
(3a)	$\text{cov}_t(\pi$_NDS, C growth) (−)	−0.76 (−0.83, −0.69)	−0.64 (−0.77, −0.42)	0.41	−0.45 (−0.80, −0.23)	0.25
(3b)	$\text{cov}_t(\pi$_NDS, mkt return) (−)	−0.27 (−0.54, 0.00)	0.02 (−0.25, 0.30)	0.00	0.15 (−0.30, 0.46)	0.10
(4a)	Labor hours growth (+)	0.81 (0.76, 0.87)	0.38 (0.10, 0.58)	0.15	0.36 (0.06, 0.65)	0.20
(4b)	Growth in capital per hour (−)	0.10 (−0.14, 0.33)	0.34 (0.05, 0.55)	0.11	0.24 (−0.01, 0.50)	0.12
(5a)	Dependency ratio (+)	−0.04 (−0.30, 0.23)	0.10 (−0.18, 0.36)	0.01	0.13 (−0.11, 0.50)	0.09
(5b)	Dependency ratio, world (+)	0.00 (−0.27, 0.26)	0.08 (−0.21, 0.34)	0.01	0.08 (−0.23, 0.32)	0.06
(5c)	Percent aged 40–64 (−)	−0.62 (−0.79, −0.45)	−0.54 (−0.70, −0.29)	0.29	−0.28 (−0.65, −0.10)	0.12
(5d)	Percent aged 40–64, world (−)	−0.41 (−0.64, −0.18)	−0.40 (−0.60, −0.12)	0.16	−0.21 (−0.55, 0.00)	0.09
(5e)	ΔPercent aged 40–64 (+)	0.18 (−0.01, 0.36)	0.33 (0.04, 0.55)	0.11	0.28 (0.01, 0.70)	0.14
(5f)	ΔPercent aged 40–64, world (+)	0.20 (−0.01, 0.42)	0.24 (−0.04, 0.48)	0.06	0.18 (−0.07, 0.55)	0.11
(5g)	ΔLife expectancy (−)	−0.23 (−0.37, −0.08)	−0.33 (−0.54, −0.04)	0.11	−0.32 (−0.60, 0.15)	0.17
(6a)	Fed deficits/GDP (+)	−0.26 (−0.58, 0.05)	−0.26 (−0.49, 0.03)	0.07	−0.17 (−0.48, 0.08)	0.11
(6b)	Fed debt/GDP (+)	−0.58 (−0.76, −0.40)	−0.62 (−0.76, −0.40)	0.39	−0.37 (−0.80, −0.13)	0.18
(7)	Current account/GDP (+)	0.18 (−0.09, 0.44)	0.12 (−0.17, 0.38)	0.01	0.06 (−0.23, 0.39)	0.09
(8)	Relative price inv. goods (+)	0.22 (−0.06, 0.50)	0.20 (−0.09, 0.44)	0.04	0.10 (−0.13, 0.32)	0.07
(9)	Top 10% income share (−)	−0.25 (−0.50, 0.00)	−0.29 (−0.51, −0.01)	0.08	−0.18 (−0.40, 0.01)	0.08

(*continued*)

TABLE 4—LONG-RUN CORRELATIONS, 1950–2016 (*continued*)

	Correlate (expected sign)	10Y moving average $\hat{\rho}^{MA}$	Low pass filter–I(0) $\hat{\rho}^{LP}$	R^2	Low pass filter–I(d) $\hat{\rho}^{LP}$	R^2
	(1)	(2)	(3a)	(3b)	(4a)	(4b)
(10)	BAA-10-yr Treasury spread (−)	−0.04 (−0.37, 0.29)	−0.07 (−0.33, 0.22)	0.00	−0.09 (−0.43, 0.35)	0.09
(11a)	π_GDP (−)	0.33 (0.15, 0.51)	0.26 (−0.02, 0.50)	0.07	0.06 (−0.25, 0.60)	0.09
(11b)	π_NDS (−)	0.39 (0.21, 0.56)	0.31 (0.02, 0.53)	0.09	0.08 (−0.21, 0.60)	0.09
(12a)	M1 growth (−)	0.22 (−0.08, 0.51)	0.00 (−0.28, 0.27)	0.00	−0.04 (−0.36, 0.27)	0.08
(12b)	M2 growth (−)	0.11 (−0.12, 0.34)	0.15 (−0.14, 0.40)	0.02	0.08 (−0.21, 0.38)	0.09

Notes: See notes to Table 3. Consumption is nondurables and services, with π_NDS the corresponding inflation rate. In 5b, 5d, and 5f, world demographic variables are constructed from the several dozen countries that are labeled high income by the United Nations in 2016 (United Nations 2017a, 156). For the low pass filter correlation, the sample period is 1950–2016. For 10Y moving averages, the sample period is 1959–2016. Data end in 2015 for the correlates in rows 1c, 5b, 5d, 5f, and 9. Data start in 1951 for the correlates in rows 2b, 3b, and 5f.

(unexpectedly) negative. The two series move in opposite directions not only during the Great Depression and World War II—when real rates were low and TFP growth was high—but more generally. For example, during the period from the mid-1980s to the mid-2000s, trend TFP growth rose and trend r fell. This striking result is quantified in Tables 3 and 4 in row 1d: all six estimated correlations are unexpectedly negative ($6 = 2$ sample periods $\times$ 3 estimates of long-run correlations). Moreover, these estimates are usually significant.

We are not aware of prior use of TFP data to examine the low-frequency relationship between the safe real rate and TFP growth. Our finding of a negative correlation is inconsistent with earlier work that has emphasized trend growth as a positive correlate of real rates (Laubach and Williams 2003, 2016; Yi and Zhang 2017). It is also inconsistent with standard economic theory (Baker, Delong, and Krugman 2005). To explain persistently negative real rates following the 2008–2009 recession, some economists have given pride of place to persistently low productivity growth (e.g., Fischer 2016). But the recent combination of low real rates and low TFP growth is not reflective of the overall historical pattern of low-frequency movement between the two variables. We leave explanation of our finding to future research.

The results for GDP and TFP growth may seem to conflict with the well-known work of Laubach and Williams (2003, 2016), which uses a stripped down macro model and gives pride of place to trend GDP growth as a determinant of the natural rate of interest. Laubach and Williams do not use TFP data, however, so our negative TFP results are not in conflict with theirs in the narrow sense of yielding different results for the same correlate. As well, GDP growth works as expected in our postwar sample, and Laubach and Williams rely on postwar data. Thus, in a narrow sense, our results are consistent with theirs. But in a broader sense, our results for TFP growth, and for GDP growth over the 1890–2016 sample, suggest that Laubach and Williams should not be interpreted as finding that trend growth explains real

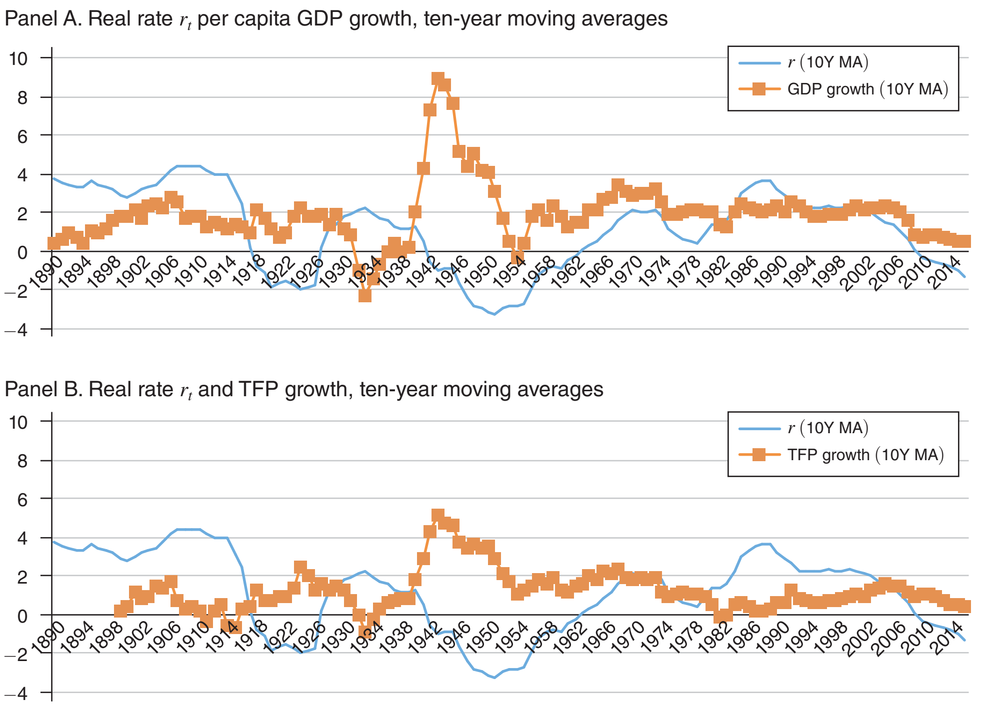

Figure 2

rates. Indeed, we shall shortly offer evidence that some other correlates, which do not even appear in Laubach and Williams's model, have stronger low-frequency ties to the real rate than does GDP growth.

While the mixed results for economic growth and the negative results for TFP growth suggest that the intertemporal IS equation is wanting, the signs of the second moment variables in rows 2 and 3 of Tables 3 and 4 are generally as expected in both samples. Further, by one or more measures of long-run correlation, the estimates are significant. This is consistent with the view that time-varying second moments are an essential part of the intertemporal IS equation (Campbell and Cochrane 1999 or Bansal and Yaron 2004).

For the intertemporal IS equation, then, the picture is mixed. For further insight, see the online Appendix where we execute some low-frequency regressions of r_t on the linear combination of correlates given on the right-hand side of the intertemporal IS equation (4). The results are consistent with those in Tables 3 and 4.

C. *Aggregate Savings and Investment*

We now move to correlates suggested by models of aggregate saving and investment. Across the two samples in Tables 3 and 4, the results in rows 4–10 can be divided into three categories: the variable produces correlations of expected sign, sometimes significant, in both sample periods (labor force growth, demographic variables); signs are sometimes as expected, sometimes not (current account, relative price of investment goods, top 10 percent income share, BAA-10-year

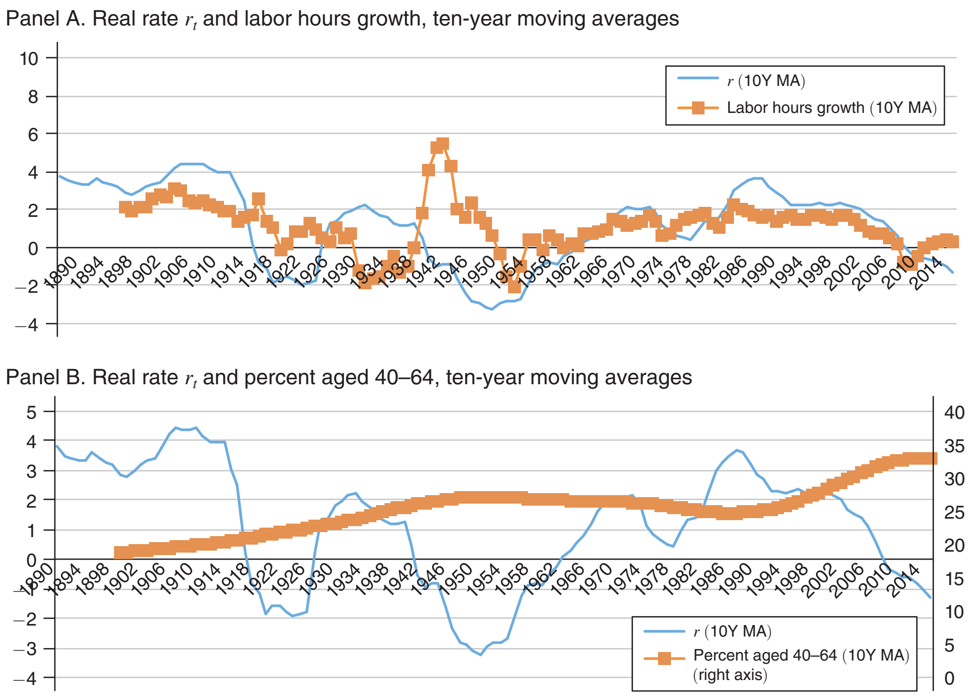

Figure 3

spread, inflation/money growth); signs are not as expected (growth in capital per hour, measures of government dissavings).

The first of several correlates to produce correlations of expected sign is labor hours growth in row 4 of Tables 3 and 4. It is strongly positively correlated with r in both sample periods.[11] The strong positive correlation is evident in Figure 3, panel A, which plots ten-year moving averages of r and of labor hours growth. In contrast to GDP growth or TFP growth, labor hours growth trends down with r in the last decades of the sample. Indeed, with the exception of the 1930s, trend labor hours growth tends to move in the same direction as trend r through the entire sample.

We are not aware of previous reduced-form research that has quantified a link between labor hours growth and r. However, changes in trend employment growth associated with demographic change are important for understanding real interest rate trends in the structural models of Gagnon, Johannsen, and Lopez-Salido (2016) and Kara and von Thadden (2016). As well, the informal calibration in Bullard (2017) uses trend labor force growth as one of the determinants of trend r.

Demographic variables comprise a second set of aggregate saving and investment correlates to produce estimated correlations with the expected sign. There are

[11] The point estimates for 1950–2016 are, however, quite different for $\hat{\rho}^{MA}$ and $\hat{\rho}^{LP}-I(0)$—two measures that ordinarily yield very similar estimates. This appears to result from $\hat{\rho}^{LP}-I(0)$ surprisingly (to us) producing a sharp rise in trend labor hours growth at the end of the sample, while $\hat{\rho}^{MA}$ produces a more plausible low estimate.

33 entries for demographic variables in rows 5a–5d in Table 3 and 5a–5g in Table 4; 29 of these have expected sign, about half of which are significant. In terms of support for demographic variables as correlates of real rates, these results fall roughly midway between the regression results of Poterba (2001) on the one hand, who finds modest support for demographic variables, and Geanakoplos, Magill, and Quinzii (2004) and Favero, Gozluklu, and Yang (2016) on the other, who find exceptionally strong support.

Figure 3, panel B, plots ten-year moving averages of r and the percent aged 40–64. The expected negative correlations presented in row 5b of Table 3 and row 5c of Table 4 reflect long periods when the two moved in opposite directions: the mid-1900s–mid-1920s, 1930–1970, and the mid-1980s–2016. Note that the trend value of this correlate moves quite slowly. This perhaps suggests that one put more weight on the $I(d)$ estimates, or on the first-difference estimates in rows 5c of Table 3 and 5e and 5f of Table 4. The dependency ratio is the only other correlate whose trend value appears equally slow moving.

The remaining aggregate saving and investment correlates do not consistently work as expected. We begin with measures of government dissaving—federal deficits and debt, in rows 6a and 6b of Tables 3 and 4. These correlates consistently yield estimates that have unexpected signs, and often are significant, especially in the 1890–2016 full sample. This seems to reflect in part the two world wars, periods in which r was quite low (indeed, negative) even though debt and deficits were high. As noted above, inflation and expected inflation rose during those wars. But nominal rates did not rise commensurately. This likely reflects financial repression as defined in Reinhart and Sbrancia (2015): nominal rates were kept low and government debt was sold in large part to a captive market.[12] In addition, our recent bout of negative real rates came with large increases in federal deficits and debt following the 2008–2009 recession. Thus, our results suggest that government dissaving may be of second-order importance for trends in real rates and that other factors, such as demographics, may dominate.

Growth in capital per hour is a second correlate that consistently yields estimates that have unexpected signs, though in this case, the estimates are generally insignificant. Inspection of a plot of the series of ten-year moving averages (not shown) indicates two extended periods when this correlate and r moved down together: from the mid-1930s to the mid-1940s and the last ten years. If we drop the last ten years of the sample, the estimated correlation in the postwar sample changes from 0.10 (Table 4) to -0.16, for example. The anemic growth of fixed investment in recent low interest rate years has of course been often noted (e.g., Gutiérrez and Philippon 2017); apparently, such growth was also weak for ten-year averages from the Depression through World War II, at least relative to labor growth.

The final set of aggregate saving and investment correlates, in rows 7–10, display mixed results. Estimates for the current account, relative price of investment goods, top 10 percent income share, and BAA-ten-year spread generally have unexpected signs for the 1890–2016 sample in Table 3, but expected signs for the 1950–2016

[12] Unfortunately, there is no obvious quantitative measure of financial repression, so we have not included it as a correlate.

sample in Table 4. However, even when the signs are as expected, the point estimates are generally small and insignificant. Since our sample is unusually long, these results need not contradict the evidence in research that focuses on relatively recent years (e.g., Bernanke 2005, Sajedi and Thwaites 2016, or Del Negro et al. 2017).

D. *Mundell-Tobin Effect*

The variables in rows 11 and 12 of Tables 3 and 4 work as expected, and strongly so, in the 1890–2016 period. They do not work as expected in the 1950–2016 period. Over the longer sample, it seems the correlations are dominated by trend rises in the nominal variables in the two world wars and the 1970s, periods in which the trend safe real rate was low. During the 1950–2016 period, behavior over the last four decades gets more weight: trend real rates and trend inflation have drifted down, while trend money growth has stayed more or less flat.

E. *Summary*

As noted in the introduction, much current conventional wisdom views trend GDP growth as the primary driver of the secular trend in safe real rates. The results reported here and in earlier research (e.g., Hamilton et al. 2016) suggest that in the data the low-frequency link with GDP growth is episodic, and the link with TFP growth is negative. A reduced-form result that, so far as we know, is new to this paper is that labor hours is a strongly positive low-frequency correlate of the safe real rate. Hence, if forced to rely on a growth variable, labor hours growth seems preferable to GDP growth as a low-frequency correlate of real rates.

In the standard overlapping generations model, labor hours growth and TFP have a symmetric effect on the real rate in steady state. Hence, our finding of oppositely signed correlations for labor hours growth and TFP is unexpected. We suspect that labor hours works as expected because it partly reflects the age variables that are conventionally captured by our other demographic variables. Specifically, hours growth is the same as population growth in standard overlapping generations models. Thus, increases in aggregate investment that are needed to match capital with labor are met with equal increases in aggregate savings due to a larger population of savers. However, empirically, hours growth also captures low-frequency trends in labor force participation and in the length of the work week. This will cause low-frequency fluctuations in the demand for capital without corresponding changes in population of savers, generating an extra source of fluctuation in real rates from labor hours growth.

Overall, labor hours growth and demographic variables seem to evidence the most reliable long-run correlation with the safe rate, with estimated long-run correlations that come with expected signs and generally are significant. Most other variables deliver a mixed picture in terms of such correlations. Because of this, we view trends in labor hours growth and in demographic variables as being most appealing if one is looking to use a small number of correlates to explain the trend in the decline in safe real interest rates over the past four decades.

F. *Economic Significance*

We have focused on statistical significance. We close with concise evidence on economic significance, illustrated by the numerical magnitude of the change in a forecast of the trend real rate when one makes a plausible change in the forecast of the trend value of the change in the percentage of the population aged 40–64. Our illustration uses this correlate because of its good performance in Tables 3 and 4.

Let r^{MA}_{2026} and x^{MA}_{2026} denote the 2026 trend values of the real rate and the percentage aged 40–64, measured via the ten-year moving average. Define

(9a) $\hat{r}^{MA}_{2026}$ and $\hat{x}^{MA}_{2026}$ are "direct" forecasts of r^{MA}_{2026} and x^{MA}_{2026};

(9b) $\tilde{r}^{MA}_{2026}$ and $\tilde{x}^{MA}_{2026}$ is an external forecast for the change in the percentage of the population aged 40–64, with $\tilde{r}^{MA}_{2026}$ a forecast of the trend real rate conditional on $\tilde{x}^{MA}_{2026}$.

The direct forecasts in (9a) rely on coefficients from a regression r^{MA}_{2026} and x^{MA}_{2026} on the current and first lagged values of r_t and x_t; the regression sample runs from $t + 10 = 1960$ to $t + 10 = 2016$. In (9b), the external forecast of the trend value of the percentage of the population aged 40–64, $\tilde{x}^{MA}_{2026}$, comes from the Social Security Administration (2017) and the United Nations (2017b). To construct $\tilde{r}^{MA}_{2026}$, we follow literature such as Clark and McCracken (2015), relying in part on an estimate of the correlation between the residuals of the regression used to obtain the direct forecasts $\hat{r}^{MA}_{2026}$ and $\hat{x}^{MA}_{2026}$. Details on the construction of $\hat{r}^{MA}_{2026}$, $\hat{x}^{MA}_{2026}$, $\tilde{r}^{MA}_{2026}$, and $\tilde{x}^{MA}_{2026}$ are in the working paper version of this paper (Lunsford and West 2017).

We consider both $\hat{x}^{MA}_{2026}$ and $\tilde{x}^{MA}_{2026}$ to be reasonable forecasts of the trend value of the change in the percentage of the population aged 40–64. We ask whether moving from $\hat{x}^{MA}_{2026}$ to $\tilde{x}^{MA}_{2026}$ results in an economically significant change in trend forecasts of the real rate. The answer is yes. Equations (10a) and (10b) illustrate for the United States and world Δpercentage aged 40–64:

		$\hat{r}^{MA}_{2026}$	$\hat{x}^{MA}_{2026}$	$\tilde{r}^{MA}_{2026}$	$\tilde{x}^{MA}_{2026}$
(10a)	Δpercentage aged 40–64	1.74	−0.19	1.47	−0.24
		(0.69, 2.79)		(0.73, 2.20)	
(10b)	Δpercentage aged 40–64, world	1.25	0.03	0.30	−0.08
		(0.29, 2.22)		(−0.26, 0.85)	

The implied elasticities are large. From (10a), we see that a 0.05 fall from -0.19 to -0.24 in the the forecast for trend Δpercentage aged 40–64 in the United States lowers the forecast of the trend real rate by 0.27 percent ($= 1.74 - 1.57$). From (10b), the comparable figures for the world demographics are a 0.11 fall in the correlate and a 0.95 percentage point fall in the forecast of the trend real rate. The 68 percent confidence intervals indicate that there is a lot of uncertainty about these forecasts. Nonetheless, the point estimates suggest considerable economic significance for these two correlates. Lunsford and West (2017) has additional results showing economic significance for other correlates.[13]

V. Conclusions

Motivated by the decline in the safe real interest rate over the last several decades, we study long-run correlations between the safe real rate and over 20 variables that have been posited to move with safe rates. We find that the safe real rate in the United States has statistically and economically important long-run correlations with aggregate labor hours and demographic variables. For most other variables, we found substantive long-run correlations in some samples and measures but not in others. Based on these results, we view demographic change as a reasonable starting point for understanding the recent secular decline in real rates. Further, we prefer labor hours growth to GDP growth or TFP growth for modeling trends in safe real rates.

Our reduced-form analysis did not attempt to provide a structural explanation for the results we found. One priority for future research is better understanding why some correlates work as expected while others do not.

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[13] That working paper also constructs direct forecasts of r_{2026}^{MA} using bivariate regressions in (r_t, x_t) for each correlate x_t. This produces 25 forecasts for the 1890–2016 sample and 28 forecasts for the 1950–2016 sample. The median across the 25 and 28 forecasts is 0.45 percent and 1.13 percent, respectively. Since our end of sample value is $r_{2026}^{MA} = -1.33$, this suggests a considerable rise in the trend real rate, with a partial or perhaps full return to the 0.97 percent mean given in Table 2.

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