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Hansen and Sargent's *Recursive Models of Dynamic Linear Economies*: A Review Essay†

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Lars Peter Hansen and Thomas J. Sargent's book, Recursive Models of Dynamic Linear Economies, *exposits, extends, and applies methods for solution and analysis of dynamic stochastic linear quadratic models. The book, which can be used as a monograph or in a graduate course, integrates theory, econometrics, and computation. This essay provides a summary and offers some mild complaints about material not included in what is already a remarkably comprehensive book.* (*JEL* C32, C61, D40, D50, E10)

1. *Introduction*

This terrific book exposits, extends, and applies methods for solution and analysis of dynamic stochastic linear quadratic models. A well-known example is the permanent income model as studied in Hall (1978). The book considers both partial- and general-equilibrium versions of that and other models. It establishes a basic methodological framework and develops and applies that framework in a number of examples and contexts. The framework integrates theory, econometrics, and computation, with the authors even supplying MATLAB code to implement their examples. In those examples, the environments are real, perfectly competitive and complete markets, though as noted below, the methodological framework is applicable more generally.

In this review, I summarize the book (sections 2 and 3) and give some thoughts on the book's uses and limitations (sections 4 and 5).

2. *Overview*

After an introductory chapter 1, the book is divided into three sections: "Tools"; "Components of Economies"; and "Representations and Properties." "Tools" and "Components of Economies" lay out methodological foundations. The "Representations and Properties" section applies and extends those foundations.

The "Tools" section includes two chapters. Chapter 2 presents some basics on linear time series (prediction from a vector autoregression (VAR), for example). Chapter 3 presents basics on linear quadratic optimal control (matrix Ricatti equation, for example), along with some nonstandard material such as risk-sensitive preferences.

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The section “Components of Economies” includes four chapters. Chapter 4 presents the basic framework: a social planner maximizes utility of a representative agent, subject to a set of linear constraints. Per period utility depends on (i) a service flow s_t relative to a (possibly) stochastic bliss point b_t, i.e., $(-1/2)(s_t - b_t)^2$ in the scalar case; and (ii) labor ℓ_t. The flow s_t is linear in consumption purchases c_t and possibly a habit stock. Habit stock, if present, is a distributed lag on current and past consumption.

To illustrate, here is the setup in the simple case in which all variables are scalars. With the exception of labor ℓ_t, each of the variables about to be defined can be vectors.

(1) Utility:

$$\max -(1/2)E_0 \sum_{t=0}^{\infty} \beta^t[(s_t - b_t)^2 + \ell_t^2]$$

subject to

$$s_t = \Lambda h_{t-1} + \Pi c_t,$$

$$h_t = \Delta_h h_{t-1} + \Theta_h c_t,$$

resource constraints given below,

where

(2)

s_t: service flow from consumption;

b_t: bliss point (possibly nonstochastic, with $b_t = \bar{b}$ for all t);

ℓ_t: labor;

h_t: household capital or habit stock;

c_t: consumption;

β: discount factor, $0 < \beta < 1$;

$\Lambda, \Pi, \Delta_h, \Theta_h$: parameters, which obey certain conditions (e.g., the modulus of each of the eigenvalues of Δ_h is less than 1), some of which are system-wide and involve other parameters introduced below.

It is possible that some or all of Λ, Π, Δ_h, and Θ_h are zero, and it is possible that ℓ_t^2 does not appear in the objective function.

The conventional per period linear-quadratic utility function is $(-1/2)(c_t - \bar{b})^2$ (e.g., Hall 1978, p. 974), with $\bar{b}$ a constant. In this conventional formulation, a consumption purchase this period yields utility this period, but not in subsequent periods. This suggests that consumption is exclusively nondurables, and there is no habit formation. Clearly, this is a special case of (1), with $\Pi = 1, \Lambda = 0$, and $b_t = \bar{b}$. In conjunction with (1)’s specification of service flows and habit stock, the book’s use of $(-1/2)(s_t - b_t)^2$ allows durable consumption goods, with consumption purchases in previous periods possibly yielding utility flows this period. (Algebra: the constraints in (1) imply $s_t = \Pi c_t + \Lambda(\Delta_h h_{t-2} + \Theta_h c_{t-1})$, so if $\Pi \neq 0$, $\Lambda \neq 0$, and $\Theta_h \neq 0$ a consumption purchase c_{t-1} yesterday yields utility s_t today. If, further, $\Delta_h \neq 0$, then, so, too, do consumption purchases in previous periods.) Indeed, the setup even allows a seasonal component in habit persistence (see p. 74 of the book for details). As well, allowing b_t to vary over time permits preference shocks that shift the marginal utility of consumption. Thus (1) is a very general specification.

I presented the maximization problem (1) without a budget constraint. Via the welfare theorems for competitive economies with complete markets, the first part of the book works from the point of view of a social planner who maximizes (1) subject to economy-wide resource constraints. These constraints are as follows, again in a simple case in which all variables except (possibly) d_t are scalars:

(3)

$$\Phi_c c_t + \Phi_g g_t + \Phi_i i_t = \Gamma k_{t-1} + d_t,$$

$$g_t = \ell_t,$$

$$k_t = \Delta_k k_{t-1} + \Theta_k i_t,$$

where

(4)

g_t: intermediate goods;

i_t: investment goods;

k_t: capital stock;

d_t: endowment or technology shock (possibly a vector, even when other variables are scalars);

$\Phi_c, \Phi_g, \Phi_i, \Gamma, \Delta_k, \Theta_k$: parameters, or vectors of parameters, that obey some system-wide conditions to insure stability and uniqueness.[1]

When $\Delta_k = 1 - \delta$ and $\Theta_k = 1$, the final equation in (3) is the usual equation for accumulation of capital with a depreciation rate of δ. The first equation is a linear production function in which capital and labor combine to produce consumption and investment. To see this, set $g_t = \ell_t$ and rearrange the equation as

$$(5) \quad \Phi_c c_t + \Phi_i i_t = \Gamma k_{t-1} - \Phi_g \ell_t + d_t.$$

[1] Actually, even with all variables scalars, Φ_c and Φ_g will not be scalars but will be vectors padded out in a certain way. See p. 64 of the book.

With $\Phi_g \leq 0$, the right-hand side is a linear production function, with d_t an additive productivity shock. The left-hand side says that output is split across consumption and investment.

The constraint (5) is more flexible than it may at first appear. Continue to suppose for simplicity that not only ℓ_t but c_t, i_t, and k_{t-1} are scalars. Here are some parameterizations of (5).

- A pure endowment economy is a special case of (5). One may set parameters in (5) so that $c_t + i_t = d_{1t}$, where d_{1t} is the first element of d_t.
- A Leontief production function is also a special case. Here, the parameters Φ_c, Φ_i, Γ, and Φ_g are, in the simplest case, (2×1) vectors. One may set the first row of (5) to state $c_t + i_t = \gamma k_{t-1} + d_{1t}$ and the second row to state that $\ell_t = \gamma_2 k_{t-1}$ or perhaps $\ell_t = \gamma_2 k_{t-1} + d_{2t}$ for a shock d_{2t}.
- The reader may wonder whether one can also interpret the variables as logs, rather than levels, with the left-hand side of (5) reflecting the usual first-order log-linearization of an additive budget identity (e.g., Walsh 2003, p. 72) and the right-hand side capturing, for example, the log of a Cobb–Douglas production function. This is not referenced in the book, but is discussed in section 5 below where I give the answer "yes."

The book introduces g_t in (3) and ℓ_t in (1) not to model intermediate goods or to focus on the role of labor in the utility function. The latter could as well be accomplished by making s_t a vector with one of the components labor. Rather, g_t and ℓ_t serve to introduce costs of adjustment. In particular, the examples in the book generally involve a restriction like $g_t = \varphi i_t$, implying, after substitution, that $\ell_t^2 = \varphi^2(k_t - \Delta_k k_{t-1})^2$ = costs of adjusting capital. Costs of adjustment are the means by which the model endogenously generates rich dynamic patterns.

Finally, the vector of bliss point and technology shocks $(b_t d_t')$ follows a Markov process such as a VAR.

This chapter, and indeed the entire book, includes a wealth of examples. Here is one of my own, a simplified version of a model in my own inventory work (West 1986). This example, in contrast with most of the book, is partial equilibrium, and is chosen in part to counteract a possible perception that the setup requires specification of a general-equilibrium environment. In my example, a representative firm minimizes the present value of per period costs, taking sales as given:

$$(6)\quad \max -E_0 \sum_{t=0}^{\infty} \beta^t \Big[(1/2)Q_t^2 + Q_t V_t + (1/2)aN_t^2\Big]$$

subject to

$$Q_t = N_t - N_{t-1} + M_t$$

where

Q_t: production;

V_t: exogenous cost shock;

N_t: inventories;

M_t: exogenous sales;

a: strictly positive parameter.

This can be mapped into the framework above with:

$$(7)\quad Q_t = c_t = s_t;\ \Lambda = 0, \Pi = 1;$$
$$b_t = -V_t;\ N_t = k_t = i_t;$$
$$\Delta_k = 0, \Theta_k = 1;\ g_t = \ell_t = \sqrt{a}N_t;$$

Φ_c, Φ_g, Φ_i, Γ, and d_t are defined as the 2×1 vectors in the following equation:

$$\begin{pmatrix}1\\0\end{pmatrix} c_t + \begin{pmatrix}0\\1\end{pmatrix} g_t + \begin{pmatrix}-1\\-\sqrt{a}\end{pmatrix} i_t = \begin{pmatrix}-1\\0\end{pmatrix} k_{t-1} + \begin{pmatrix}M_t\\0\end{pmatrix}.$$

Then using $\ell_t^2 = aN_t^2$, we have $(1/2)[(s_t - b_t)^2 + \ell_t^2] = (1/2)Q_t^2 + Q_t V_t + (1/2)aN_t^2 + V_t^2$. The exogenous term V_t^2 is irrelevant to the maximization problem, so the above mapping succeeds.

To return to my summary: a central goal of the analysis is to solve the model numerically, for various parameterizations of the setup given above, as well as for some extensions and generalizations of that setup. Chapter 5 in the "Components of Economies" part of the book describes how to solve the maximization problem using optimal control methods (matrix Ricatti equation and all that). Let

$$(8)\quad x_t \equiv (h_{t-1} k_{t-1} b_t d_t')' = \text{the state vector, of dimension } n_x \times 1,$$

$$(9)\quad w_t = \text{vector of martingale difference shocks driving } b_t \text{ and } d_t.$$

Then the solution for the state vector will be of the form

$$(10)\qquad x_{t+1} = A^o x_t + Cw_{t+1}.$$

The book gives the equations that are used to solve for A^o and C in terms of the parameters of the model (1)–(4). Endogenous variables that are not elements of the state are linear in the state. For example, for $1 \times n_x$ vectors S_s and S_c,

$$(11)\qquad s_t = S_s x_t,\ c_t = S_c x_t,$$

and similarly for other endogenous variables $(i_t, \ldots)$ for suitable $1 \times n_x$ vectors. Once again, the book gives explicit formulas.

Chapter 5 also writes out first-order conditions using Lagrange multipliers, presenting formulas that allow one to solve for those multipliers in terms of the state.

In the scalar example (6) that I presented, one could instead solve "by hand" (p. 114), and the authors do so in the appendix to chapter 5. But in more complicated problems, numerical solutions are the natural and practical method. To that end, the authors have supplied MATLAB code that implements the solutions described in chapter 5. This code is posted at http://www.tomsargent.com/books.html (note that the URL given in the book is defunct). Thus, while the book is explicit and detailed about that the logic and algebra to that solution, the authors aim to stimulate the reader to not only understand the solution method, but also to use the supplied code to solve the wide range of models that fit into the framework of the book.

Chapters 6 and 7 in the "Components of Economies" part of the book present material relevant for analyzing decentralized versions of the model. Chapter 6 covers prices, chapter 7 defines a competitive equilibrium in a complete-markets environment. Prices in the decentralized system are explicitly linked to the Lagrange multipliers used in chapter 5. Thus, the equivalence between decentralized and planned allocations is established constructively and values for prices can be computed given chapter 5 formulas for the Lagrange multipliers.

The final section of the book is "Representations and Properties." This section covers a miscellany of topics. Some, but not all, of these topics are connected to one another, but all rely on the general setup that has been established in the first three parts of the book. Chapter 8 allows for the state x_t to be measured with error. It describes using the Kalman filter to construct the log likelihood of the model under normality. Frequency domain maximum likelihood and simulation estimation methods are outlined, allowing for unit roots in shocks. The chapter also applies results related to the Kalman filter to time aggregation (say, decisions are made monthly, data are quarterly) and to when private agents have more information than does the econometrician.

Chapter 9 describes and illustrates a certain transformation that is necessary to analyze some versions of the model. Chapter 10 considers partial-equilibrium versions of the model, with solutions derived when one takes as given the parameters of demand curves facing firms. (Previous chapters also assume competitive firms. Those earlier chapters differ from chapter 10 because they take parameters of utility as primitive, while chapter 10 takes parameters of a demand curve as primitive.) The chapter illustrates firm decisions in a number of examples.

Chapter 11 applies the results to permanent income models. These are models in which there is a single capital good with gross return $1 + \gamma$, with parameters satisfying $\beta(1 + \gamma) = 1$, which is Hall's (1978) random-walk condition that the discount rate equals the interest rate. This chapter derives a set of results and conditions without being specific about the presence or absence of durability or habit persistence (i.e., without being specific about the form of the constraints written below equation (1) above).

Chapters 12 and 13 develop results for economies with heterogeneous households, described by the authors as "among the most interesting in the book" (p. xiv). Chapter 12 gives conditions under which possibly heterogenous households will have parallel linear Engel curves, which is Gorman's (1953) condition for aggregation. These conditions allow heterogeneous preference and endowment shocks (i.e., heterogeneous b_t in (1) and d_t in (3)) and heterogeneous initial wealth (heterogenous initial k), but require common parameters in (1) and (3). Chapter 13 allows parameters of the household technology (1) to vary across two types of households and shows how to construct a "mongrel" representative household.

Finally, chapter 14 presents results for some models that involve seasonality.

3. *Examples*

The book contains a wealth of examples. Some of these do not obviously fit into the framework outlined above. So to preempt a concern that the framework is too specialized to handle all but a narrow range of questions, here is a partial list of linear quadratic examples worked out in the text:

- Permanent income/stochastic growth models of consumption, as in Hall (1978), Brock and Mirman (1972), and Jones and Manuelli (1990).
- Real asset prices, as in Lucas (1978).
- A rational addiction model, as in Becker and Murphy (1988).
- Investment under uncertainty, as in Lucas and Prescott (1971).
- A housing model, as in Topel and Rosen (1988).
- Equilibrium in a market for a given occupation, as in Ryoo and Rosen (2004).

The last four examples are considered in a partial-equilibrium context. The fact that (1)–(4) can accommodate these examples illustrates the generality of the setup. In particular, despite the book's emphasis on general equilibrium, one should not be discouraged from considering using the book's tools simply because one is working with a partial-equilibrium model.

4. *Possible Uses*

The book serves a number of purposes. At a narrow level, anyone solving dynamic linear models will find code, solutions, and tips: solutions and often code relating to doubling algorithms, time aggregation, seasonality, estimation of models written in state space form, and a host of other topics that are directly relevant even to models that do not map exactly into the framework used in the book. In conjunction with perusal of notation in earlier chapters, perhaps assisted by the index, one could easily extract information on one or a few of the many aspects of solving dynamic models that the book covers.

Next, the book is a monograph that supplies a superb guide to linear quadratic optimization and indeed to dynamic optimization more generally.

Finally, the book could be used as the basis for a second-year graduate course. The student should probably be familiar with topics in time series such as autoregressions and topics in dynamic optimization such as the Bellman equation and use of Lagrange multipliers in dynamic models. ("Probably" because the right student, under the guidance of the right instructor, could follow the book even without prior exposure to one or more of such topics.) The book is self-contained on linear quadratic dynamic optimization: no prior knowledge of the Ricatti equation or related material is presumed. The book does not contain exercises or problems, so that would have to come from the instructor. As well, the book does not attempt big-picture motivation of the choice of topics to cover in the "Representations and Properties" part of the book, nor of the linear quadratic setup versus the loglinear (see below) and nonlinear setups that dominate macro today. Connection to the larger literature will have to come from the instructor.

5. *Complaints*

The book is comprehensive. Its arguments are rigorous and elegant. Every chapter has insights for even the experienced macroeconomist.

But if you are like me, you expected nothing less from these two authors. So rather than expand on the obvious, I have assigned myself the task of finding things to complain about. Some of my complaints are substantive, some expositional. My central complaint is that the authors do not compare or link their linear quadratic setup to the loglinearized or linearized setups that dominate current work in linear models in macro. In the remainder of this piece, I will use the term "linearized" to encompass loglinearizations as well as linearizations in levels.

1. To introduce the topic of linear quadratic versus linearized models, let me begin with one of the book's examples. Chapter 4 of the book uses as one of its applications a model that it attributes to Jones and Manuelli (1990). When Jones and Manuelli (1990) parameterize utility, it follows much of macro literature and employs the familiar constant elasticity form. In self-evident notation, constant elasticity utility is

$$\text{(12)} \qquad \mathrm{U}(C_t) = (1-\sigma)^{-1} C_t^{1-\sigma}.$$

The reader—or at least this reader—might therefore expect the book to comment on mapping Jones and Manuelli's constant elasticity utility (12) into the book's quadratic utility (1). Clearly a second-order Taylor series expansion may be employed. Perhaps that Taylor series expansion can be applied to the log of consumption rather than consumption (though perhaps not, since isoelastic utility is strictly concave in log consumption only when $\sigma > 1$). The book is very user-friendly in all sorts of calculations, but does not assist us here.

That is, the book assumes a linear quadratic setup from the get-go. Jones and Manuelli (1990) preferred to parameterize utility as constant elasticity; the authors choose instead quadratic utility. If we want to think of a quadratic objective function as an approximation to a constant elasticity of other non-quadratic objective functions, there is of course nothing to prevent us from so doing. But should we do our Taylor series expansion around steady state C_t, or steady state $\ln(C_t)$, and how should our expansion accommodate growth or unit roots? Such questions are not discussed.

Jones and Manuelli (1990) is an example of a general issue. In the circles I travel in, the leading area for application of dynamic linear models is via models whose first-order conditions and possibly constraints are linearized with an underlying objective function that is not quadratic. New Keynesian models are a prominent example.[2] Other examples can be found across the spectrum of macroeconomics. For the reader unfamiliar with this sort of linearization, let me pull a simple example from recent inventory work. As in equation (6), let Q_t denote production and M_t sales. Morley and Singh (2016) construct a variable, call it Δn_t, that they label "inventory investment as a percentage of sales," via: $\Delta n_t \equiv \ln(Q_t) - \ln(M_t)$. The label is rationalized by taking the equation (6) identity $Q_t = N_t - N_{t-1} + M_t$, dividing both sides by M_t, and taking logs, yielding $\Delta n_t \equiv \log\left(\frac{N_t - N_{t-1}}{M_t} + 1\right)$.

Morley and Singh (2016) is a trivial illustration of a technique widely used in modeling macroeconomic data. Given that this literature uses models that are dynamic and linear, the subject of the book is directly relevant. But application of the book's techniques to this leading area for dynamic linear models will not always be immediate. Presumably such application would require a second-order expansion of the objective function and perhaps an expansion or

[2] Yes, I know, New Keynesian models are not always solved via linearization. I discuss the linearized solutions since they potentially map directly into the framework of the book.

loglinearization of a constraint. That will often result in a system that maps immediately into (1), although if that second-order expansion results in cross-product terms between choice variables—say, because utility is not separable in two types of consumption, or between consumption and leisure—some additional work would be required before (1) is applicable. Alternatively, in some models one could perhaps reverse engineer a quadratic objective function by integrating an existing set of linearized or log-linearized first-order conditions.

So, yes, the book's algorithms, techniques, and analysis are applicable to not only linear quadratic models but dynamic linearized models. But connecting to the prominent line of work that uses dynamic linearized models will, I think, generally require some work. I may be wrong, but I suspect that many researchers will find this off-putting.

2. A corollary of assuming a linear quadratic setup from the get-go is that we do not get a comparison of behavior for (i) linear quadratic (this book) versus (ii) non-quadratic with first-order conditions that are linearized around a steady state (lots of macro). Are we spending too much time specifying isoelastic utility and Cobb–Douglas production functions, and then working with linearized first-order conditions? Should we instead be specifying quadratic utility and linear production functions from the outset?

If we specify linear quadratic from the outset, one issue is calibration. The book's examples generally do not calibrate parameters. By calibrate I mean, choose a value to, say, match a moment computed from the steady state, or to fall within a range that is conventionally considered plausible. In principle one could do so in linear quadratic models. To use isoelastic utility once again as an example, in a static world one can map isoelastic utility into quadratic utility by choosing quadratic utility parameters to fix the (wealth-dependent) coefficient of relative risk aversion at a desired level, given initial wealth (e.g., West, Edison, and Cho 1993 for an illustration). And there is a (large!) literature on what values of risk aversion are plausible. Is this a productive route to calibrating a linear quadratic model? The general point is that in thinking about using a linear quadratic model, it would have been helpful if the book illustrated how to choose plausible values for parameters.

The book makes a powerful case that one can get very far with quadratic from the outset. But because the book is not concerned with the questions that I am raising, it does not try to directly make a case that we should start with quadratic from the outset.

3. So as to not overstate: Objective functions in linearized models are sometimes specified in a quadratic form. This quite often applies to solutions for optimal policy in linearized New Keynesian models (e.g., Walsh 2003, pp. 524–25, for a simple example). And here the book's code and analytical results are immediately applicable. Also this New Keynesian example illustrates the important fact that even though the book assumes a real and perfectly competitive environment throughout, its presentation of linear quadratic optimization is applicable quite generally, whether the world is monetary or real or competitive or monopolistically competitive.

4. On a different topic, related to possible use as a text: as noted above, the book focuses on solving for a decision rule. First-order conditions—the trade-offs that are at the heart of economic analysis—play a subsidiary role. First-order conditions of course are essential in the book. But discussion of first-order conditions sometimes occurs at a high level, with the goal of moving us toward the decision rule or a numerical value. For example, the asset-pricing discussion

in chapter 7 does not explicitly state (in self-evident notation) $1 = \beta E_t[(1 + r_{t+1}) \times U'(C_{t+1})/U'(C_t)]$, though that relation is implicit in the solution. If the book is used as a text, some instructors will likely want to supplement the discussion in places with first-order conditions, either to remind or enlighten the student about certain economic aspects of the solution.

On the other hand, because the book is focused on moving us toward solutions, the asset-pricing discussion presents neat and revealing solutions for asset prices that will benefit student and instructor alike. So, too, does the neat chapter on the permanent income model, where, using the generality of (1), the authors derive general analytical conclusions about that model when the number of consumption goods is arbitrary and there may or may not be durability or habit persistence.

5. If I try hard to think of something methodological that might be in the book but is not, I nearly draw a blank. But maybe the following would qualify: Markov switching in shocks or parameters (Hamilton 1989). The chapter on seasonality (chapter 14) assumes a particular Markov switching form in which there are switches of parameters with probability one between seasons. It would have been great to have the same skillful and complete solution for a general Markov switching process.

The reason I had to try hard to come up with something methodological that might have been but was not included in the book is that the book is remarkably comprehensive. Risk-sensitive preferences, time aggregation, unit roots, geometric growth, serially correlated measurement error, techniques for rapid computation, even techniques for rapid calculation in seasonal models—you name it, the book covers it. And, needless to say, does so in elegant and rigorous fashion.

6. *Conclusion*

This book is chock full of results that will be useful to all interested in dynamic linear models, including material that will be novel to even the experienced macroeconomist. Buy it, read it, use it.

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