

line: shocks have transitory effects, with GNP reverting rapidly to a stable trend.

A contrary view, first proposed by Nelson and Plosser (1982), is that GNP is *nonstationary*, or has a *unit root*. To oversimplify somewhat, Nelson and Plosser argue that the current value of GNP is essentially the base for the future path of GNP. In this view, if we unexpectedly enter a recession, GNP even ten or twenty years from now is likely to be lower than had there been no recession.

Which of these contrasting views is correct has important implications for economic theory and policy. Nelson and Plosser argue that if the unit root model is appropriate, monetary theories of the business cycle are not very attractive, since monetary shocks are typically thought to have only transitory effects. Campbell and Mankiw (1987) argue that the concept of a stationary natural rate of unemployment is not very compelling if shocks are not routinely offset with a return to trend.

This article briefly outlines the Dickey-Fuller test, the most commonly used statistical technique for testing the null of a unit root in a time series against the alternative that the series is stationary around trend line. Additional discussion, as well as other tests, may be found in the references. Unfortunately, these references are for the most part fairly technical; a good textbook discussion may be found in Hamilton (1994).

The simplest example of a Dickey-Fuller test in a time series x_t involves the model:

$$x_t = m_0 + m_1 t + \phi x_{t-1} + \varepsilon_t \quad (1)$$

where $-1 < \phi \leq 1$ and $m_1 = 0$ if $\phi = 1$ (these conditions rule out behavior that is rarely seen in economic data). If $\phi = 1$, the process has a unit root, this phrase deriving from the fact $z = 1$ is a root to the equation $1 - \phi z = 0$ if $\phi = 1$, but not otherwise. If $|\phi| < 1$, the process does not have a unit root but instead is stationary around the trend $m_0 + m_1 t$.

A Dickey-Fuller test for whether there is a unit root in the process (1) begins by rewriting the equation as:

$$\begin{aligned} x_t - x_{t-1} &\equiv \Delta x_t = m_0 + m_1 t + (\phi - 1)x_{t-1} + \varepsilon_t \\ &\equiv m_0 + m_1 t + \delta_0 x_{t-1} + \varepsilon_t \end{aligned} \quad (2)$$

Note that $\delta_0 = 0$ if and only if $\phi = 1$. So one estimates (2) by ordinary least squares, and calculates the usual t -statistic for $H_0: \delta_0 = 0$. The

Unit Root Tests

When the economy goes into a recession, how long is it before GNP returns to its prerecession trend line? Textbooks such as Robert J. Gordon's *Macroeconomics* (4th ed., 17) suggest that it does not take long, often requiring no more than a couple of years. In this view, GNP is *stationary*, or *mean-reverting*, around a trend

usual critical values for this t -statistic, however, should not be used, because under the null hypothesis of a unit root this statistic is *not* normally distributed, even if ε_t is normally distributed, and even in large samples. Instead, critical values tabulated in Fuller (1976, 373) may be used. At the 1-, 5- and 10-percent levels, the asymptotic critical values are -3.96 , -3.66 and -3.41 . Use of the usual asymptotic normal distribution, which would, for example, suggest rejecting at the 5-percent level in a one-tailed test if the t -statistic were less than -1.65 , will, under the unit root null, lead to too many rejections; at this significance level one should instead reject the null only if the t -statistic is less than -3.66 .

The reader familiar with standard macroeconomic data will recognize that most such data, including GNP, display behavior that is too complex to be captured by equation (1). One can test for unit roots in such series by adding additional lags of Δx_t , as additional regressors:

$$x_t - x_{t-1} \equiv \Delta x_t = m_0 + m_1 t + \delta_0 x_{t-1} + \delta_1 \Delta x_{t-1} + \dots + \delta_p \Delta x_{t-p} + \varepsilon_t \quad (3)$$

Once again, x_t has a unit root if and only if $\delta_0 = 0$.

One can use equation (3) to test the unit root null exactly as before, by estimating equation (3) by OLS, computing the t -statistic for $H_0: \delta_0 = 0$, and using the same non-standard critical values that were used for equation (2). This is true even if in the population the disturbance to equation (2) is serially correlated, provided many lags of Δx_t are included. Somewhat more precisely, equation (3) can be used even if Δx_t has what time-series analysts call a moving-average component. In a Monte Carlo study, Schwert (1988) found that setting the number of lags equal to the integer part of $12 \times (\text{sample size}/100)^{1/4}$ provides an adequate number of lags. Thus, if the sample size is 100, set $p = 12$ in equation (3); if the sample size is 200, set $p = 14$.

Some technical points: First, it is important to include the time trend (the $m_1 t$ term) in equations (2) and (3), if, as usual, the alternative is that the data are stationary around a time trend. Otherwise one is very likely to accept the unit root null even when the data are in fact stationary (West 1987). Second, Campbell and Mankiw (1987), Cochrane (1988), and Phillips and Perron (1988) among others have proposed alter-

natives or extensions to the Dickey-Fuller test. The Dickey-Fuller test is, however, the simplest, and is more reliable than some others in samples of the size usually used by economists (Schwert 1988). Finally, there are multivariate tests for the number of unit roots in vector time-series processes. The pioneering work here is by Engle and Granger (1987). Alternative, more general tests have been proposed by a number of authors. Some of these are presented in the June/September 1988 issue of the *Journal of Economic Dynamics and Control*. A consensus has yet to develop on which test is preferred.

For most macroeconomic data, tests for unit roots do not reject the null. See, for example, Nelson and Plosser (1982) for tests using annual United States data (including GNP). Taken at face value, these tests indicate that when GNP falls unexpectedly, one should expect a lower value for GNP indefinitely far into the future. Campbell and Mankiw (1987, 867), for example, suggest that if GNP falls unexpectedly by 1 percent, the forecast of GNP twenty years later falls by about 1.5 percent.

A much debated question is whether these results in fact *should* be taken at face value. Dickey and Fuller (1981) show that their tests are likely not to reject the null of a unit root if there is no unit root, but there still is considerable serial correlation in the series. (In the model (1), this is equivalent to ϕ being near but not quite equal to unity.) This has led West (1988) and Christiano and Eichenbaum (1989) to argue that the tests do not establish the presence of a unit root, or even that mean reversion to a trend line is unlikely in, say, five years.

The debate over the presence of unit roots in economic time series is unsettled. Ongoing work on tests for unit roots may help a consensus emerge.

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See also COINTEGRATION; COMPOSITE TRENDS; TRENDS AND RANDOM WALKS

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