THE SOURCES OF FLUCTUATIONS IN AGGREGATE INVENTORIES AND GNP*

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A simple real linear-quadratic inventory model is used to determine how cost and demand shocks interacted to cause fluctuations in aggregate inventories and GNP in the United States, 1947–1986. Cost shocks appear to be the predominant source of fluctuations in inventories and are largely, though not exclusively, responsible for the fact that GNP is more variable than final sales. Cost and demand shocks are of roughly equal importance for GNP. These estimates, however, are imprecise. With different, but plausible, values for a certain target inventory-sales ratio, cost shocks are less important than demand shocks for GNP fluctuations.

I. INTRODUCTION

This paper is concerned with how real cost and demand shocks interact to determine aggregate real inventories and GNP in the postwar United States. Its aim is to answer such questions as the following. Do inventories respond mainly to demand shocks [Holt et al., 1960]? Are demand shocks of secondary importance in explaining fluctuations in GNP [Prescott, 1986a, 1986b]? What is the dynamic pattern of the response of inventories to cost and demand shocks? What is the pattern of response of GNP [Blanchard and Quah, 1988]?

A long tradition attributes the bulk of movements in inventories to demand shocks. Accelerator models, pioneered by Metzler [1941] and Lovell [1961], posit that inventories are proportional to expected sales. Production smoothing models, pioneered by Holt et al. [1960], suggest that because of increasing marginal costs of production, the desire to smooth production relative to demand will also cause adjustment of inventories in response to demand.

Some recent evidence, however, has suggested that inventories may also (or instead) be responding to cost shocks. One simple stylized fact that suggests this is that for virtually any U. S. industry or aggregate, production is more variable than demand [Blinder, 1981b, 1986a; Blanchard, 1983].[1] This is logically inconsistent with

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1. An exception is production to order manufacturing, when demand is measured by orders rather than sales [West, 1988a].

a simple production smoothing model with increasing production and inventory costs, because such a model argues that the sole reason to hold inventories is to smooth production relative to demand [West, 1986]. It is also empirically inconsistent with more complex production smoothing models that allow for accelerator effects and for quadratic costs of changing production, since these additional complexities do not appear to explain the excess variability [West].

Cost shocks, however, rationalize the excess production variability quite naturally. This is most easily seen in an extreme case when demand is constant (there are no demand shocks). Production will still vary as costs vary, since production will be high (low) when costs are low (high), with procyclical adjustment of inventories covering the gap between production and sales. Production will therefore be more variable than sales.

Partly because cost shocks provide a simple explanation of the excess variability of production, recent inventory research has emphasized the potential role of cost shocks (e.g., Blinder [1986b], Maccini and Rossana [1984], and Miron and Zeldes [1987]). It appears, however, that there is as yet no direct evidence, still less a consensus, on how important cost shocks are relative to demand shocks. Christiano and Eichenbaum [1987], for example, find the excess variability of production suggestive of a predominant role for cost shocks, while Blinder [1986a] constructs an example in which the excess variability is consistent with a very small role. As stated above, one aim of this paper is to quantify the relative importance of cost and demand shocks as determinants of aggregate inventories.

Simultaneously, the paper studies how these shocks interact to determine GNP. Recent work in real business cycles has argued that most of the movements in GNP can be explained by fluctuations in costs. Prescott [1986a], for example, suggests that 75 percent of these movements are cost related. Consistent with this, two very recent vector autoregressive (VAR) studies [Blanchard and Quah, 1988; King et al., 1987] have found that well over half of the variance of GNP forecasts more than twelve quarters ahead is due to permanent rather than transitory shocks; both studies interpret permanent shocks as cost rather than demand related, while acknowledging that other interpretations are possible. By contrast, in an earlier VAR study Blanchard and Watson [1986] found that demand shocks are the primary source of GNP fluctuations, as did Fair [1988] in a recent study.

The present paper uses comovements of inventories and GNP to help determine the sources of fluctuations in GNP. Given the importance of movements of inventory stocks at cyclical turning points [Blinder, 1981b; Blinder and Holtz-Eakin, 1986], this seems likely to contain significant information about the sources of U. S. business cycles. The basic intuition is suggested by a simple production smoothing model, where the only cost terms are ones quadratic in the level of production and inventories: demand shocks will tend to cause inventories to move countercyclically; cost shocks will tend to cause them to move procyclically.

Since it is well-known that inventory movements are procyclical [Summers, 1981], this simple model would, of course, attribute much of the movement in GNP and inventories to cost shocks. The model used, however, allows for a target inventory-sales ratio (as do, e.g., Blanchard [1983], Ramey [1988], and West [1986]). This can induce procyclical movements in inventories in response to demand shocks, and no simple mapping between shocks and comovements is expected to obtain. But estimation of the parameter that determines the target inventory-sales ratio, together with the other parameters of the model, allows one to disentangle movements due to cost from those due to demand shocks. These parameters may be computed from the estimates of a bivariate VAR in inventories and GNP. The VAR is estimated on quarterly data, 1947–1986, for both stationary and unit root specifications.

The point estimates suggest that cost shocks are the predominant source of fluctuations in inventories. They are largely, though not exclusively, the reason that GNP is more variable than final sales; some excess variability appears to be due as well to increasing returns in production. Cost and demand shocks are of roughly equal importance in GNP fluctuations. Cost shocks are especially important for inventories at relatively long horizons and for GNP at short horizons. Over 90 percent of the variance of inventory forecasts 20 quarters ahead is due to cost shocks. The comparable figure for GNP is about 40 to 60 percent.

GNP and inventories both display hump-shaped responses to demand and cost shocks, with the peak effect occurring about four quarters out. When the shocks are assumed to have unit roots, new steady states are essentially achieved in about ten to twelve quarters; when the shocks are assumed stationary, the variables are markedly different from the steady state even forty quarters out.

For the usual reasons, however, these results should be interpreted with caution: tests of overidentifying restrictions strongly reject the model (as in Christiano and Eichenbaum [1987], for example), and confidence intervals are rather large (as in Blanchard and Quah [1988], for example). In connection with the latter point, it should be noted that the estimates are quite sensitive to the parameter that determines the target inventory-sales ratio. The point estimate of the relevant parameter is lower than that obtained in some previous studies (e.g., Blanchard [1983], Ramey [1988], and West [1986]). When this parameter is constrained to some higher values consistent with these previous studies, much less—as little as 10 percent—of the movement in GNP over 20 quarters ahead is attributed to cost shocks.

Section II describes the model. Section III presents empirical results. Section IV concludes. An Appendix has some technical details, with an additional appendix available on request from the author containing additional results and details not of central importance.

II. Model

The basic model is a generalization of the linear-quadratic inventory models in, for example, Blinder [1982], Blanchard [1983], Belsley [1969], and West [1986], and was suggested by Sargent [1979, Ch. XVI]. A similar model was developed independently by Christiano and Eichenbaum [1987]. To focus on interactions between inventories and output fluctuations, it is assumed that storage in inventories is the only means of smoothing production or demand in response to shocks. Demand is linear (the area under the demand curve is quadratic). Production and storage costs also are quadratic.

Let S_t be real demand (sales), Q_t real production, and H_t real inventories. The variables are linked by the identity $Q_t = S_t + \Delta H_t$. Let L_t be labor supply, P_t the real price of output, and R_t real profits, with the wage rate the numeraire.

Utility is separable over time. The per period utility function of the representative consumer depends on labor and current consumption S_t:

$$-fL_t - fg_{0S}S_t^2 + 2fU_{dt}S_t, \tag{1}$$

where f and g_{0S} are positive and U_{dt} is a demand shock. Constant and linear terms in (1) and throughout are suppressed, for notational simplicity. The first term in (1) reflects disutility from work, and the second diminishing marginal benefit of additional demand. The demand shock U_{dt} captures shocks to preferences, policy, and the like. A positive value raises demand.

For the representative firm, production and storage costs $L_t \equiv C_t$ are

$$(2) \quad C_t = g_{0Q}Q_t^2 + g_{1Q}\Delta Q_t^2 + g_{0H}(H_{t-1} - g_{HS}S_t)^2 + 2U_{ct}(hH_t + Q_t),$$

where h and g_{HS} are positive, the other g parameters are such that the maximization problem stated below is well defined (see footnote 3 below), and U_{ct} is a cost shock.

The first term in (2) reflects increasing costs to production if $g_{0Q} > 0$, and decreasing costs if $g_{0Q} < 0$. The second term reflects costs of adjusting production (e.g., hiring and firing costs). Simple forms of costs of adjustment are often assumed present in inventory models (e.g., Eichenbaum [1984] and Maccini and Rossana [1981, 1984]). The quadratic specification can be considered an approximation to an arbitrary cost function that is convex in production. The accelerator term, $g_{0H}(H_{t-1} - g_{HS}S_t)^2$ appears in many studies of manufacturing and retail inventories (e.g., Blanchard [1983] and Irvine [1981]). It reflects a balancing of inventory holding and stockout costs [Holt et al., 1960], capturing a tendency of inventories to track a target level $g_{HS}S_t$, and g_{HS} is the target inventory-sales ratio that was mentioned in the introduction. See Blanchard [1983] or West [1986] for additional discussion of this and the other terms in the cost function.

A positive cost shock U_{ct} raises the cost of both production and inventory storage. The parameter h measures the shock's impact on inventory storage costs relative to its impact on production costs. The shock captures random fluctuations in technology.

Let E_t denote mathematical expectations (linear projections) conditional on period t information. The representative consumer maximizes the expected present discounted value of utility, and the representative firm the expected present discounted value of profits, using a common discount rate b, $0 < b < 1$:

$$(3) \qquad \max \lim_{T\to\infty} E_0 \sum_{t=0}^{T} b^t \left(-fL_t - fg_{0S}S_t^2 + 2fU_{dt}S_t\right)$$

subject to

$$P_tS_t = L_t + R_t,$$

$$\max \lim_{T\to\infty} E_0 \sum_{t=0}^{T} b^t R_t \tag{4}$$

subject to

$$R_t = P_tS_t - L_t,$$
$$L_t = C_t,\ S_t = Q_t - \Delta H_t.$$

The constraints in (3) and (4) assume that all profits are remitted to consumers as profits are earned.

The model is solved as follows. For algebraic convenience assume that all markets are competitive, and set the number of firms and consumers equal to one. (If, instead, there is a single monopolistic firm, as in Blinder [1982], the same first-order conditions result; the monopolistic and perfectly competitive versions of the model are observationally equivalent. The analysis below therefore is robust to possible imperfection in the product market and allows sticky prices in the sense of Blinder.)[2] Use $P_tS_t = L_t + R_t$ to eliminate L_t from (3):

$$\max \lim_{T\to\infty} E_0 \sum_{t=0}^{T} b^t \left[-f(P_tS_t - R_t) - fg_{0S}S_t^2 + 2fU_{dt}S_t\right]. \tag{5}$$

Differentiate with respect to S_t. The resulting first-order condition may be written as an aggregate demand curve,

$$P_t = -2g_{0S}S_t + 2U_{dt}. \tag{6}$$

For the firm use $S_t = Q_t - \Delta H_t$ to write the sum in (4) in terms of H_t and Q_t. Let $c_t \equiv E_t\Sigma_{j=0}^{\infty} b^j C_{t+j}$. Differentiate with respect to H_t and Q_t. The resulting first-order conditions may be written as

$$-P_t + bE_tP_{t+1} = \frac{\partial c_t}{\partial H_t},$$
$$P_t = \frac{\partial c_t}{\partial Q_t}, \tag{7}$$

2. Readers who prefer the monopolist interpretation should note that under that interpretation the parameter estimate called g_{0S} is instead an estimate of $0.5g_{0S}$. In a related context Eichenbaum [1984] states that an oligopolistic structure results in an observationally equivalent equilibrium, provided that individual firms follow symmetric open loop Nash strategies.

where $\partial c_t/\partial H_t$ and $\partial c_t/\partial Q_t$ may be computed in straightforward fashion and are written out explicitly in the Appendix. The first equation in (7) says that the firm is indifferent between adding a unit to inventory this period to be sold next period (excess of discounted expected revenue over cost is $bE_tP_{t+1} - \partial c_t/\partial H_t$) and selling the unit this period (revenue is P_t). The second equation in (7) says that the firm produces until marginal production cost equals price. See Blanchard and Melino [1986] for additional interpretation.

Equilibrium P_t, Q_t, S_t, and H_t are determined by the three equations in (6) and (7) and the identity $Q_t = S_t - \Delta H_t$. The equilibrium is perturbed as demand shocks shift the aggregate demand curve (6), cost shocks shift the aggregate inventory and output supply curves (7).

To solve for how the shocks interact to determine Q_t, H_t, and S_t, it is convenient to eliminate P_t and E_tP_{t+1} from (7) by substituting (6) and (6) led one time period into (7), and then eliminate S_t using the identity $S_t = Q_t + \Delta H_t$. This leaves two first-order conditions in the two variables H_t and Q_t. See equation (A1) in the Appendix for the exact equations.

To estimate the model, it is necessary to specify the stochastic processes for the shocks U_{ct} and U_{dt}. The empirical work assumes that the cost and demand shocks follow uncorrelated AR(1) (possibly random walk) processes with parameters ϕ_c and ϕ_d, with $|\phi_c|$, $|\phi_d| \leq 1$: $E_{t-1}U_{dt} = \phi_d U_{dt-1}$, $E_{t-1}U_{ct} = \phi_c U_{ct-1}$. Let Y_t be the (2×1) vector $(H_t\ Q_t)$, Φ a 2×2 diagonal matrix, and $\Phi \equiv \text{diag}\,(\phi_c, \phi_d)$. The Appendix shows that the solution to the model is

$$Y_t = \Pi Y_{t-1} + FU_t, \tag{8}$$

where Π and F are 2×2 matrices that depend on b, Φ, h, and g_{ij}.[3] Since U_t follows a vector AR(1) with coefficient matrix Φ, FU_t follows a vector AR(1) with coefficient matrix $F\Phi F^{-1}$: $FU_t = (F\Phi F^{-1})FU_{t-1} + F(U_t - \Phi U_{t-1})$. To obtain an equation with a

3. Necessary conditions for equations (8) and (A1) to be the optimal solution to the model include that (1) A_0 is positive definite (the Legendre-Clebsch condition for optimality [Stengel, 1986, p. 213] and (2) the two smaller of the four roots to $|bA_1'z^{-1} + A_0 + A_1z| = 0$ are strictly less than $b^{-1/2}$ in modulus. These conditions are guaranteed to hold if $g_{0Q}, g_{0H}, g_{0S}, g_{1Q} > 0$. See Hansen and Sargent [1981]. I thank Tryphon Kollintzas for clarifying this point.

serially uncorrelated disturbance, quasi-difference (8) to obtain

$$(9)\quad Y_t - F\Phi F^{-1}Y_{t-1} = \Pi Y_{t-1} - F\Phi F^{-1}\Pi Y_{t-2} + V_t,$$
$$V_t \equiv F(U_t - \Phi U_{t-1}).$$

The aim of this paper is to use (9) to determine how cost and demand shocks interact to determine inventories, production, and sales. This requires estimates of Π, $\Omega_v \equiv EV_tV_t'$, Φ, and F. Given $F\Phi F^{-1}$, the first two are easily obtained from (9) by linear regressions; calculating $F\Phi F^{-1}$ entails some work (see below and the Appendix). Given F, Ω_v may be diagonalized by multiplying it by F^{-1}. One may then apply standard VAR techniques to compute impulse response functions and variance decompositions.

A basic check on the plausibility of the results is the pattern of impulse responses. While complicated and perhaps counterintuitive dynamics are possible [Blinder, 1986a], intuition suggests that the initial impact of a cost shock will be to cause inventories, production, and sales to fall, with a negative long-run impact as well when there are unit roots. One expects the initial impact of a demand shock to cause production and sales to rise, with the effect on inventories indeterminate: production smoothing will tend to make the effect negative, $g_{HS} > 0$ in equation (2) will tend to make the effect positive (see also Blinder [1986a]). When there are unit roots ($\phi_d = \phi_c = 1$), the long-run impact of a demand shock on production and sales is positive, on inventories indeterminate (again because of conflicting forces from production smoothing and $g_{HS} > 0$).

This section closes with an overview of the procedure used to identify the shocks, and may be skipped without loss of continuity. The first step is to obtain an estimate of $F\Phi F^{-1}$, which is used to construct the right-hand- and left-hand-side variables in (9). Estimation of $F\Phi F^{-1}$ when ϕ_c and ϕ_d are unknown is discussed in the Appendix. Consider instead when ϕ_c and ϕ_d are imposed a priori. This was true, with $\phi_d = \phi_c = 1$, for one of the specifications estimated below. Then $\Phi = F\Phi F^{-1} = I$; the shocks follow uncorrelated random walks; and equation (9) is just $\Delta Y_t = \Pi\Delta Y_{t-1} + V_t$. Upon defining $(v_{1t}v_{2t})' = V_t$, this may be written out in scalars as

$$(10)\quad \begin{aligned} \Delta H_t &= \pi_{11}\Delta H_{t-1} + \pi_{12}\Delta Q_{t-1} + v_{1t}, \\ \Delta Q_t &= \pi_{21}\Delta H_{t-1} + \pi_{22}\Delta Q_{t-1} + v_{2t}. \end{aligned}$$

Now, the g_{ij}s are identified only up to a normalization, as are h, σ_c^2, σ_d^2, and F. (This is apparent in equation (A1); doubling all the g_{ij}

terms except g_{HS} leaves the first-order conditions unchanged, apart from a rescaling of the disturbances. Variance decompositions, and impulse responses to a one standard deviation shock, however, are invariant to the normalization.[4] The normalization chosen was $g_{0S} + g_{0Q} + g_{0H}g_{HS}^2 + (1 + b)\, g_{1Q} = 1$. Given this (or any) normalization, as well as a value of the discount rate b, the g_{ij}s can be computed from the OLS estimates of the four π_{ij}s. One can then compute h, σ_c^2, σ_d^2, and F. See the Appendix for details.

It should be noted that whether or not ϕ_c and ϕ_d are known a priori, this is not the usual procedure for orthogonalizing vector autoregressive residuals (e.g., Haltiwanger and Maccini [1987]), and issues such as sensitivity of results to orderings of variables are not relevant. The basic algebraic reason for this is that the three unknowns h, σ_c^2, and σ_d^2 are determined uniquely by the three unknowns in Ω_v.

III. Empirical Results

A. Data and Estimation Technique

The data were real (1982 dollars), quarterly, seasonally adjusted, and expressed at annual rates, 1947:1 to 1986:4. Figures for GNP, final sales (demand) and inventory investment were obtained from CITIBASE files GNP82, GNS82, and GV82. The implied series for inventories was obtained by setting the 1982:1 figure to match the corresponding entry in the CITIBASE file for real inventories, GL82, and then using the series for inventory investment (GV82) to compute the level in other quarters.

The first step in the empirical work was to model deterministic and stochastic trends. Regressions of log levels of the data on a constant and time trend yielded estimated growth rates of 0.786 percent per quarter for inventories and 0.828 percent for production; when inventories and GNP were constrained to have a common deterministic growth rate, the figure was 0.807 percent per quarter. This suggested that it is reasonable to model the two variables as having a common deterministic trend, and indeed, neither asymptotic nor Monte Carlo tests could reject the null of a common deterministic trend. Details on these tests, as well as on the consistency of geometric growth with the model, are in the additional appendix available on request. The data used in all the

4. Actually, impulse response functions are invariant only up to a sign change.

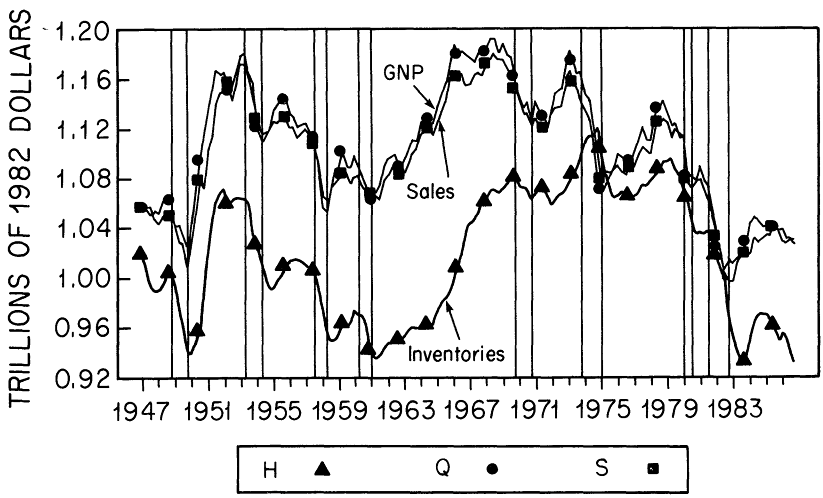

FIGURE I
Basic Data

estimation below therefore are data "scaled" by this common deterministic trend, i.e., the 1982 data just described divided by $(1.00807)^t$. The mean values for scaled inventories, GNP, and sales were 1,017, 1,106, and 1,099 billion 1982 dollars. The scaled data are plotted in Figure I, where NBER business cycle peaks and troughs are noted with vertical lines.

All the inference reported below is conditional on the estimated growth rate of 0.807 percent per quarter and, except for the results in Tables I and II below, on an imposed discount rate $b = 0.98$ as well. Related work [West, 1986] and some tests described in a footnote suggest that the empirical results are not likely to be sensitive even to large errors in the estimate of this deterministic trend, or to the exact choice of discount rate.[5]

The Said and Dickey [1984] test for a unit root in the scaled data does not reject the null of a unit root in either GNP or inventories at even the 10 percent level, for either four lags

5. For the $\phi_c = \phi_d = 1$ specification described below, I calculated the infinite horizon variance decompositions described in the next paragraph, for data scaled by $(1.012)^t = [1 + (1.5) \times (.008)]^t$ and $(1.004)^t = [1 + (0.5) \times (0.008)]^t$. None of these estimates were more than four percentage points different than the figures reported in Table V below. See West [1986] on the insensitivity of results to exact choice of b.

TABLE I

ESTIMATES OF SERIAL CORRELATION PARAMETERS

	(1)	(2)	(3)		(4)	(5)
					Left-hand-side variable in regressions	
	ϕ_c	ϕ_d	$F\Phi F^{-1}$		$H_t^{\sim}$	$Q_t^{\sim}$
(1)	1.0	1.0	1.0	0.0	$H_t - H_{t-1}$	$Q_t - Q_{t-1}$
			0.0	1.0		
(2)	0.949	0.949	0.949	0.0	$H_t - 0.949H_{t-1}$	$Q_t - 0.949Q_{t-1}$
			0.0	0.949		
(3)	0.969	0.997	0.976	−0.005	$H_{t-1} - 0.976H_{t-1} + 0.005Q_{t-1}$	$Q_t + 0.027H_{t-1} - 0.990Q_{t-1}$
			−0.027	0.990		

TABLE II
SUMMARY STATISTICS ON REDUCED FORM

(1) ϕ_c, ϕ_d	(2) π_{11}	(3) π_{12}	(4) π_{21}	(5) π_{22}	(6) $Q(36)$ $H_t^{\sim}$	(7) (p-value) $Q_t^{\sim}$	(8) χ^2 (4) (p-value) lags = 1 vs. lags = 2
(1) 1.00,1.00	0.500	0.231	−0.299	0.489	49.56	29.45	21.68
	(0.338,0.612)	(0.126,0.340)	(−0.539,−0.106)	(0.299,0.650)	(0.07)	(0.77)	(0.00)
(2) 0.949,0.949	0.537	0.227	−0.256	0.508	49.72	29.16	21.79
	(0.370,0.644)	(0.127,0.333)	(−0.500,−0.088)	(0.333,0.676)	(0.06)	(0.78)	(0.00)
(3) 0.969,0.997	0.515	0.231	−0.278	0.490	49.11	29.69	21.90
	(0.357,0.610)	(0.133,0.339)	(−0.518,−0.067)	(0.297,0.652)	(0.07)	(0.76)	(0.00)

In columns (2) to (5), 95 percent confidence intervals are in parentheses, from bootstrap. Columns (6) and (7) report the Box-Ljung Q-statistic, where the left-hand-side variables $H_t^{\sim}$ and $Q_t^{\sim}$ are defined in Table I.

(t-statistic for H_0: coefficient on lagged dependent variable = 1 is −1.96 for GNP, −1.20 for inventories) or twelve lags (t-statistic = −1.62 for GNP, −1.16 for inventories), using either the asymptotic or Monte Carlo levels in Schwert [1987]. This suggested the importance of a differenced ($\phi_c = \phi_d = 1$) specification. On the other hand, extreme serial correlation of GNP and inventories is consistent with a stationary model as well, with the persistence coming from ϕ_c and ϕ_d less than but near unity. This suggests the plausibility of an undifferenced specification as well. A cointegrated specification seemed of secondary interest because the null of no cointegration of GNP and inventories was not rejected at even the 10 percent level using the Engle and Granger [1987] CRDW test, when either GNP was regressed on inventories (Durbin-Watson = 0.070) or inventories on GNP (Durbin-Watson = 0.042).

This suggests the importance of two of the specifications estimated: a differenced one, with $\phi_c = \phi_d = 1$ imposed, and a quasi-differenced one, where $\phi_c = 0.969$ and $\phi_d = 0.997$ was estimated as described in the Appendix. In this stationary specification demand disturbances were overwhelmingly dominant at distant horizons, for GNP (see the discussion of Table V below). To check whether this result followed simply because ϕ_d was very near one, and slightly larger than ϕ_c, a third and final specification estimated ϕ_d and ϕ_c subject to the constraint that $\phi_c = \phi_d$. The maximum likelihood estimate was $\phi_c = \phi_d = 0.949$. See Table I for a summary of the serial correlation parameters for the three specifications, as well as the left-hand-side variables used in the regression estimates of (9).

In all three specifications, confidence intervals for various estimates were bootstrapped [Efron, 1982; Freedman, 1982; Runkle, 1987], using 1,000 repetitions.[6] For each of the 1,000 repetitions: (a) a time series of Y_t was generated recursively using the estimated Π and $F\Phi F^{-1}$, and sampling the estimated residuals with replacement; (b) equation (9) was reestimated (holding $F\Phi F^{-1}$ fixed), to get another Π. Inference was thus conditional on the estimated or imposed serial correlation matrix $F\Phi F^{-1}$ (and, as noted above, on the value of the discount rate b and the estimated growth rate).

Impulse responses and variance decompositions over various finite horizons were calculated in the standard way, using the RATS

6. For the differenced specification asymptotic standard errors were also calculated for some of the parameter estimates, in a fashion similar to that described in West [1988b]. The results were about the same.

computer program. The results also report variance decompositions at an infinite horizon, computed simply as the limit of the finite horizon variance decompositions.

A specialization of the model that involves a simple form of costs has simple implications for the relative variabilities of GNP and final sales. Suppose in particular that $h = g_{1Q} = g_{HS} = 0$, so that $C_t = g_{0Q}Q_t^2 + g_{0H}H_t^2 + 2Q_tU_{ct}$. Assume tentatively that all variables have a zero unconditional mean. Then in a stationary environment (a) in the presence of demand shocks only ($U_{ct} \equiv 0$), $0 \leq E(S_t^2 - Q_t^2) = ES_t^2 - EQ_t^2 = \text{var}(S) - \text{var}(Q)$, or $\text{var}(Q)/\text{var}(S) \leq 1$ [West, 1986]; and (b) in the presence of cost shocks only ($U_{dt} \equiv 0$), $\text{var}(Q)/\text{var}(S) \geq 1$ (see the additional appendix available on request).

In the presence of unit roots, variances do not exist, but analogous inequalities nonetheless hold [West, 1988a]. Since $S_t = Q_t - \Delta H_t$, $S_t^2 - Q_t^2 = -2Q_t\Delta H_t + \Delta H_t^2$. Under fairly general conditions—including in particular when $(\Delta H_t, \Delta Q_t)$ follows a vector autoregression, as in the present paper—$EQ_t\Delta H_t \equiv E[(\Delta Q_t + \Delta Q_{t-1} + \ldots)\Delta H_t] = E[\Sigma_{j=0}^{\infty} \Delta Q_{t-j})\Delta H_t]$ exists (is finite). The simplified model defined in the previous paragraph then implies that (a) in the presence of demand shocks only ($U_{ct} \equiv 0$), $0 \leq E(S_t^2 - Q_t^2)$, and (b) in the presence of cost shocks only ($U_{dt} \equiv 0$), $0 \geq E(S_t^2 - Q_t^2)$.

That the data do not have zero means, and are first scaled by g^t, is irrelevant for the stationary specification but introduces some minor complications for the unit root specification. As explained in the additional appendix available on request, it is necessary to examine not $-2\text{cov}(Q_t, \Delta H_t) + \text{var}(\Delta H_t)$ but $-2\text{cov}(Q_t, \Delta H_t) + g^{-1}$ $\text{var}(\Delta H_t)$. This was calculated in a straightforward fashion from the $(\Delta H_t, \Delta Q_t)$ autoregression, and is reported in the Table IV entries for $E(S_t^2 - Q_t^2)$.

B. Empirical Results

Estimates of the reduced form, of cost and demand parameters, of impulse response functions, of production and sales variability, and of variance decompositions will be discussed in turn. Table II has estimates of the reduced-form (8), where $\Pi = [\pi_{ij}]$. (Constant terms were included in all the regressions, but are not reported to conserve space.) Given how close the values of ϕ_c and ϕ_d, are, the reduced-form estimates are of course quite similar (columns (2) to (5)).

The results of three diagnostic tests are reported in columns (6) to (8). The Q-statistics in columns (6) and (7) cannot reject the null of no serial correlation in the residuals at the 5 percent level, though they do reject at the 10 percent level for the inventory equation all three specifications. Column (8) reports maximum likelihood tests of the null of a lag length of one versus a lag length of two (after differencing or quasi-differencing by $F\Phi F^{-1}$). These reject the null of a lag length of one quite strongly. For the stationary specifications, tests of an unrestricted lag length of two, in levels, versus the restricted second-order VAR implied by (9) also reject the null at the 0.05 level (not reported in the table).

The rejection of the overidentifying restrictions reported in column (8) suggests that this model is too simple to fully characterize the data. Qualitatively similar results obtain, however, when a more complicated model that implies a longer length VAR is used (see subsection C below). Since the present model is simpler to interpret and since the parameter estimates and impulse response functions are for the most part quite plausible (see below), I shall focus on this simple model.

Cost and demand parameters are reported in Table III, with the normalization as stated in the table. Most parameters are correctly signed. The demand curve slope g_{0S}, the inventory cost g_{0H}, and the cost of adjustment g_{1Q} are all fairly precisely estimated, and are consistent with those for the automobile industry [Blanchard, 1983] and for two-digit nondurables manufacturers [West, 1986]. The target level parameter g_{HS}, however, is incorrectly signed, although the 95 percent confidence interval is so large that it includes values such as 0.4 and 0.7 that are consistent with Blanchard, Ramey [1988], and West. I therefore interpret this as a noisy and imprecise sample estimate of a population parameter that is positive (though perhaps small).[7] Particularly interesting are the estimates of the quadratic production cost g_{0Q}. As in Blanchard, this cost is insignificantly different from zero and constant returns to scale cannot be rejected. As in Ramey, however, the point estimates are negative, implying a tendency to bunch production.[8]

7. While $g_{HS} < 0$ is not sensible, this model still generates a positive level of inventories (see Schutte [1983]). In the underlying model that allows for deterministic growth in inventories and production, which is described in detail in the additional appendix available on request, the quadratic costs in (2) are interpreted as costs around a minimum point that grows over time. This growth can lead to positive inventory levels even if g_{HS} is negative (or, more plausibly, zero).

8. The estimated value of g_{0Q} is small enough relative to the other parameters that the conditions noted in footnote 3 are met. It should be emphasized that the conditions in that footnote are necessary but not sufficient. James Hamilton has pointed out to me that these conditions therefore do not establish that the point estimates are consistent with equations (8) and (A1) characterizing the optimal policy.

TABLE III
ESTIMATES OF COST AND DEMAND PARAMETERS

(1) ϕ_c, ϕ_d	(2) g_{0Q}	(3) g_{1Q}	(4) g_{0S}	(5) g_{0H}	(6) g_{HS}
(1) 1.00,1.00	−0.072	0.344	0.392	0.145	−0.040
	(−0.320,0.232)	(0.214,0.407)	(0.111,0.657)	(0.050,0.320)	(−0.663,0.680)
(2) 0.949,0.949	−0.044	0.366	0.317	0.111	−0.127
	(−0.303,0.260)	(0.251,0.417)	(0.074,0.589)	(0.041,0.287)	(−0.947,0.551)
(3) 0.969,0.997	−0.055	0.347	0.367	0.129	−0.057
	(−0.315,0.218)	(0.203,0.407)	(0.108,0.625)	(0.037,0.299)	(−0.636,0.874)

Ninety-five percent confidence intervals are in parentheses, from bootstrap. The five parameters are related by the normalization $1 = g_{0S} + g_{0Q} + g_{0H} g_{HS}^2 + (1 + b) g_{1Q}$.

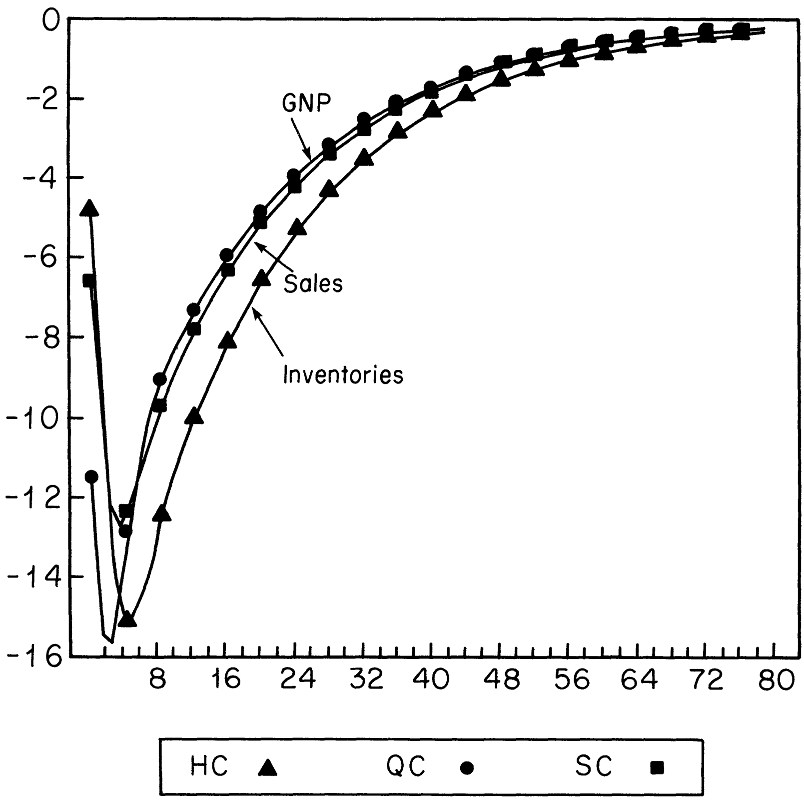

FIGURE II
Responses to Cost Shock

Figures II and III plot the response to one standard deviation cost and demand shocks for the $\phi_c = \phi_d = 0.949$ specification. Figures IV and V do the same for $\phi_c = \phi_d = 1.0$. (To conserve space, plots for $\phi_c = 0.969$, $\phi_d = 0.997$ are not presented, but any differences from $\phi_c = \phi_d = 0.949$ are noted below.) The signs of the shocks are as in equations (1) and (2): a positive cost shock raises costs, a positive demand shock raises demand. The units on the vertical axis are billions of 1982 dollars. Note that the vertical scale in Figure III is slightly more compact than in the other three figures, and that the horizontal scales are different for the $\phi_c = \phi_d = 0.949$ and $\phi_c = \phi_d = 1.0$ specifications.

In response to a positive stationary cost shock (Figure II), GNP, inventories, and final sales all fall initially, then rise back to

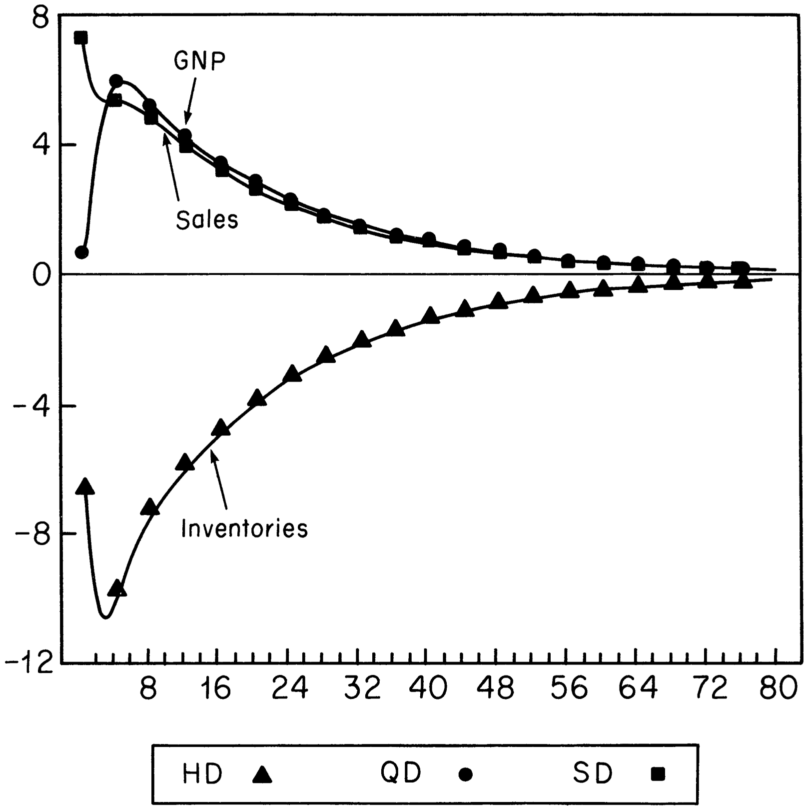

FIGURE III
Responses to Demand Shock

the initial steady state. The smoothing role of inventories is illustrated by the sharper initial fall of GNP than demand, in response to the increase in costs; without inventories, this sharper fall would not be possible. The smoothing pattern appears to make GNP more variable than sales, as is expected in simplified versions of the model in the presence of cost shocks alone.

In Figure III the GNP and sales responses to a stationary demand shock are familiar hump-shaped ones. As in Blanchard and Quah [1988], the peak response occurs at about four quarters. Inventories are initially drawn down, thereby buffering GNP from the shock. They are then built up, accumulating above the steady state level before falling back down. The pattern is similar to

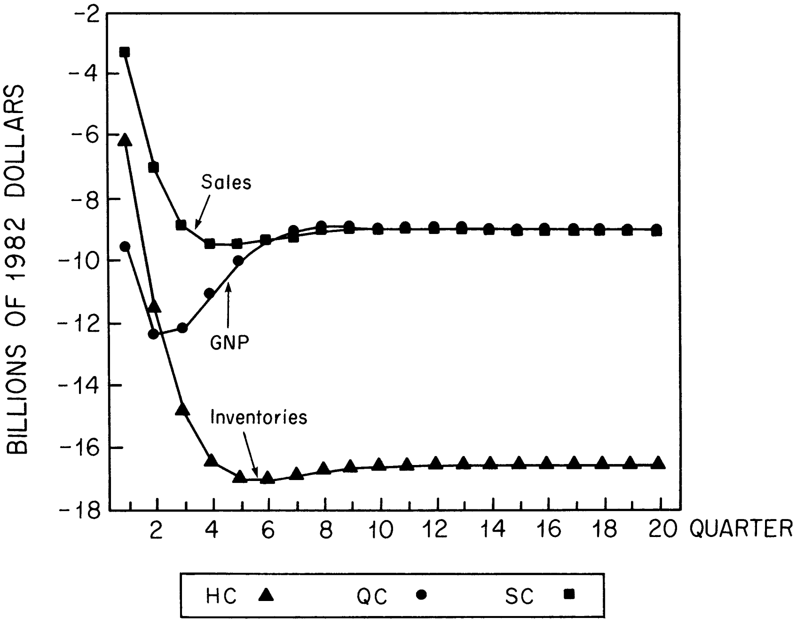

FIGURE IV
Responses to Cost Shock

Haltiwanger and Maccini's [1987] estimates of the response of finished goods inventories to new orders shocks. The smoothing by inventories appears to make GNP more variable than demand. This is inconsistent with the standard production smoothing model with g_{0Q} positive, but is unsurprising given that the estimated g_{0Q} is negative.[9]

In Figure IV inventories, demand, and GNP all fall in response to a positive random walk cost shock. Once again, inventories perform their smoothing role, allowing demand to fall less than GNP. The decline in both inventories and final sales is almost monotonic; GNP displays a hump shape (as in Blanchard and Quah [1988] and King et al. [1987]). The new steady state is essentially

9. For $\phi_c = 0.969$, $\phi_d = 0.997$, the response to a cost shock is quite similar to that in Figure II, but the response to a demand shock is somewhat different from that in Figure III, in that (a) even after 80 quarters no return to the steady state is obvious (this of course results since ϕ_d is so near unity), and (b) after initially falling, inventories rise above the steady state before finally falling back toward the steady state.

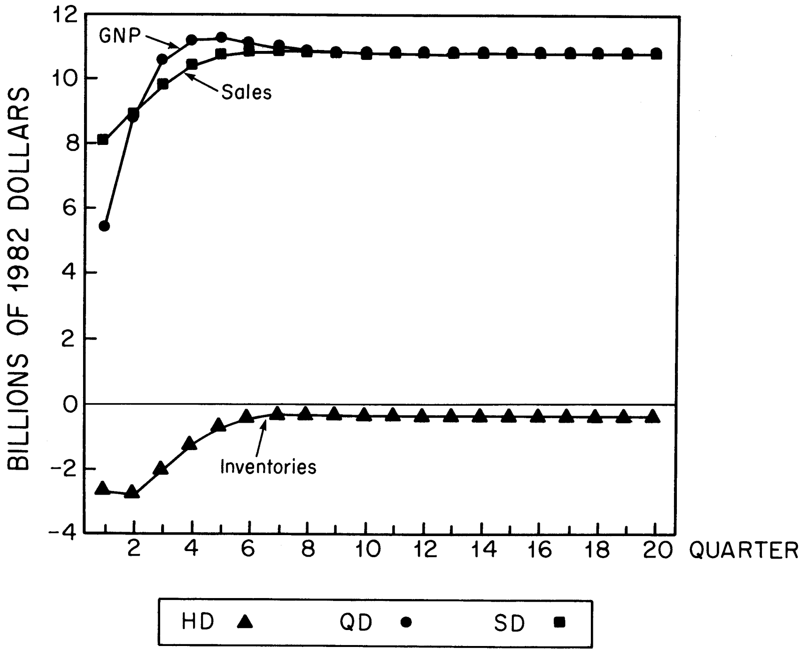

FIGURE V
Responses to Demand Shock

obtained in about two years, again as in Blanchard and Quah and King et al. (The similarity of the steady state changes in final sales and GNP results because inventory investment (the first difference of inventories) is assumed stationary.)

In response to a positive random walk demand shock (Figure V), inventories are drawn down, and demand and GNP rise. Some smoothing is apparent initially, although GNP quickly rises past demand. Inventories show a hump shape; the return toward the initial level again suggests a target level. The steady state is again reached in about two years.

Table IV reports the relative variability of GNP and final sales, using the $\phi_c = \phi_d = 0.949$ and $\phi_c = \phi_d = 1$ specifications. As is well-known [Blinder, 1981b], the variance of GNP exceeds that of final sales (line 1, column 2). The appropriate inequality holds as well when unit roots are assumed present (line 1, column 1). The impression from the figures that GNP is more variable than sales, in response to either cost or demand shocks, is borne out by the

TABLE IV
VARIABILITY OF GNP VERSUS FINAL SALES

	Specification	
	(1) $\phi_c = \phi_d = 1.00$ $E(S_t^2 - Q_t^2)$	(2) $\phi_c = \phi_d = 0.949$ var(Q)/var(S)
(1) Raw data	−303.2	1.09
	(−491.3,−161.7)	(1.06,1.14)
(2) Just demand shocks	−13.4	1.002
	(−241.8,70.0)	(0.90,1.05)
(3) Just cost shocks	−289.7	1.17
	(−472.3,−58.1)	(1.09,1.40)

Ninety-five percent confidence intervals are in parentheses, from bootstrap. For column 1, units are billions of 1982 dollars squared. To interpret these entries, it may help to note that the values of var(ΔQ) corresponding to the three lines in column 1 are 146.8, 44.5, and 101.2.

relevant point estimates (lines 2 and 3), though the excess variability is statistically insignificant at the 95 percent level when there are demand shocks only (line 2).

Table IV suggests an explanation of the seeming contradiction between the Blinder [1982] version of the production smoothing model and the fact that GNP is more variable than final sales. The bulk of the explanation is that cost shocks are important. But even in the absence of cost shocks, GNP would possibly continue to be more variable. This excess variability appears to be attributable at least in part to a small tendency to bunch production (i.e., to the small negative value of g_{0Q}). If the Table IV figures are recalculated for the $\phi_c = \phi_d = 1.0$ specification under the counterfactual assumption that $g_{0Q} = 0$ (constant rather than increasing returns to scale), with all other parameters held constant, the entry in line (2) for just demand shocks falls to 0.994.

Table V has variance decompositions. In all three specifications the variance of inventories is largely attributable to cost disturbances. This is especially true at relatively long horizons. The point estimates suggest that over 90 percent of the variance is attributable to cost shocks at horizons of four quarters or more, the confidence intervals suggest that it is unlikely that less than half the variance is due to cost shocks. This dominance of cost shocks is consistent with the marked procyclicality of inventory stocks (see Figure I) and is perhaps unsurprising given that the estimates of the accelerator parameter g_{HS} were negative.

All three specifications attribute to cost shocks about 40 to 60

TABLE V
PERCENTAGE OF VARIANCE DUE TO COST SHOCKS

Variable horizon		Specification (1) $\phi_c = \phi_d = 1.00$	(2) $\phi_c = \phi_d = 0.949$	(3) $\phi_c = 0.969, \phi_d = 0.996$
H	1	89	77	83
		(64.7,99.9)	(60.3,98.2)	(60.6,99.9)
	4	97	94	96
		(74.0,99.7)	(75.5,98.9)	(60.5,99.3)
	8	98	97	99
		(68.1,99.5)	(78.2,99.2)	(51.6,99.4)
	12	99	98	99
		(66.7,99.6)	(78.3,99.4)	(48.7,99.5)
	20	100	98	99
		(65.6,99.7)	(78.2,99.4)	(46.1,99.6)
	∞	100	98	93
		(64.5,100.0)	(78.0,99.5)	(36.7,98.5)
Q	1	75	83	76
		(17.7,91.4)	(47.0,94.8)	(12.1,90.7)
	4	60	71	59
		(9.9,81.5)	(31.4,90.4)	(7.2,78.4)
	8	51	63	48
		(6.7,74.9)	(24.6,87.3)	(5.8,69.4)
	12	48	60	43
		(5.3,71.9)	(22.9,85.9)	(4.4,66.0)
	20	45	58	37
		(4.8,70.4)	(21.7,85.0)	(3.7,60.4)
	∞	41	57	13
		(3.2,68.2)	(21.3,84.4)	(2.9,34.9)
S	1	14	22	15
		(0.1,38.6)	(1.0,48.1)	(0.1,41.1)
	4	40	51	39
		(4.2,61.3)	(18.2,74.5)	(4.2,59.7)
	8	41	53	38
		(3.7,64.9)	(19.1,78.8)	(2.9,61.4)
	12	41	53	36
		(3.6,66.2)	(19.0,80.0)	(2.7,60.2)
	20	41	53	33
		(3.4,66.7)	(18.7,81.3)	(2.4,57.1)
	∞	41	53	12
		(3.2,68.2)	(18.8,81.7)	(2.3,34.3)

Ninety-five percent confidence interval are in parentheses, from bootstrap. For the column (1) specification, $h = 0.81$ $(-0.38,4.70)$, $(\sigma_c/\sigma_d) = 0.85$ (0.07,2.03). For the column (2) specification, $h = 0.65$ $(-0.14,2.87)$, $(\sigma_c/\sigma_d) = 1.16$ (0.07,9.86). For the column (3) specification, $h = 0.71$ $(-0.41,3.97)$, $(\sigma_c/\sigma_d) = 0.92$ (0.09,1.94).

percent of the variability of GNP at horizons of about 20 quarters. At longer horizons, however, there are marked differences between the two specifications that impose $\phi_c = \phi_d$ (columns (1) and (2)) and the one that does not (column (3)). When $\phi_d = \phi_c$, the infinite horizon figure is still about 40 to 60 percent, but for $\phi_c = 0.967$, $\phi_d = 0.996$, the figure is about only a little above 10 percent. A comparison of columns (2) and (3) indicates that this is an artifact of the slightly higher point estimate of ϕ_d: if $\phi_d = 1$, $\phi_c < 1$, the contribution of cost shocks at an infinite horizon would of course be zero. Here, instead, ϕ_d is slightly less than one, so the contribution of cost shocks at that horizon is not exactly zero.[10] I am therefore inclined to downplay the infinite horizon decompositions in column (3). In this connection the reader should recall that the confidence intervals are conditional on the estimates of ϕ_c and ϕ_d, so the upper bound of 34.9 in the infinite horizon confidence interval in column (3) probably is consistent with a point estimate in the 40 to 60 range.

This 40 to 60 percent range is bracketed by the somewhat higher estimates in Blanchard and Quah [1988] and King et al. [1987], and the somewhat lower estimates in Blanchard and Watson [1986] and Fair [1988].[11] A possible reconciliation with the two papers that find higher estimates is that permanent shocks, tentatively linked in those papers to cost rather than demand, are in fact partly demand related: in the present context, at least, nothing in the model or results argues for allowing for cost but not demand shocks to be permanent.

A possible reconciliation with the three papers that find a smaller role for costs is suggested by the only one of the papers that has an inventory equation [Fair, 1988]. Fair uses a standard flexible accelerator–production smoothing model. Desired inventories are proportional to sales; actual inventories adjust only partially toward the desired level [Fair, 1984, pp. 131–32]. In Fair [1988] the shock to the inventory equation is interpreted as one of the components of the aggregate demand shock. In the present paper, however, the shock to the inventory equation in both (8) depends on cost as well as on demand. Inventory investment therefore responds to cost shocks. The same plausibly applies to other types of business investment. Insofar as the shocks to the aggregate demand curve in

10. The "∞" entry for H_t in column 3 indicates that this argument does not yet apply to inventories with $\phi_d = 0.996$; it would of course eventually apply for some ϕ_d arbitrarily near unity.

11. Fair [1988] only calculates decompositions up to eight quarters out; these, too, attribute a much lower figure to costs than does Table V.

Blanchard and Watson [1986] are due to business investment, some of the GNP variability that those papers attribute to aggregate demand shocks might more properly be attributed to aggregate cost shocks. In any case, whether or not I am correct in arguing that shocks to investment equations plausibly reflect cost as well as demand, my argument does suggest why I find a more important role for cost shocks than do Blanchard and Watson and Fair [1988]. Whether this argument is persuasive of course will require further research.

In Table V cost shocks are less important for GNP as the forecast horizon increases. This pattern holds quite rigidly. Although not reported in Table V, the fraction of GNP variability attributable to cost shocks declined monotonically as the horizon increased. Evidently, demand shocks are estimated to have increasing real effects for GNP, with inventories serving as a buffer. This is illustrated in the impulse responses. In both specifications GNP responses to cost shocks show an earlier peak and a quicker approach to the steady state.

The decreasing importance of demand shocks is consistent with Maccini and Haltiwanger [1987], who report an analogous tendency for shocks to new orders to account for an increasing fraction of the variance of manufacturing inventories as the forecast horizon increases. The contradictory Blanchard and Quah [1988] and King et al. [1987] result that cost shocks are increasingly important as the horizon increases again can be potentially reconciled with Table V if permanent disturbances are demand as well as cost related.

Finally, fluctuations in final sales appear to be attributable in roughly equal shares to cost and demand shocks. (I again discount the results in column (3), for the reasons given above.) There does not appear to be a marked tendency for cost shocks to be particularly important at any particular horizon. (Once again, for the differenced specification the similarity of the infinite horizon decompositions for GNP and sales results because inventory investment is stationary.)

C. Additional Empirical Results

To check and extend the preceding results, three additional sets of estimates were obtained. For simplicity, I imposed $\phi_c = \phi_d = 1$ in all three and did not compute any confidence intervals. The first set of estimates, already mentioned in the discussion of diagnostic tests, used a more complicated model that implied a longer length VAR. The cost function in equation (2) was expanded to

$$(11)\quad C_t = g_{0Q}Q_t^2 + g_{1Q}\Delta Q_t^2 + g_{0H}(H_{t-1} - g_{HS}S_t)^2 + 2U_{ct}(hH_t + Q_t) + g_{1H}\Delta H_t^2 + g_{1HQ}\Delta H_t \Delta Q_t + g_{2H}\Delta^2 H_t^2 + g_{2Q}\Delta Q_t^2.$$

The four additional terms are suggested by Eichenbaum [1984]. With $\phi_c = \phi_d = 1$ this can be shown to lead to an exactly identified second-order VAR in (ΔQ_t, ΔH_t). Diagnostic tests on the OLS estimates of this second-order VAR are as follows: for the ΔH_t equation, $Q(36)$ for the residual was 32.81 (p-value = 0.62); for the ΔQ_t equation, $Q(36)$ = 28.37 (p-value = 0.82); $\chi^2(4)$ for lags = 2 against lags = 3 (second-order against third-order VAR) yields 4.33 (p-value = 0.36).

Point estimates for the $g_{...}$ are given in part A of Table VI. Of the five parameters present in the model used above, four fall within the 95 percent confidence intervals in line 1 of Table III (the exception is g_{1Q}, which is a little larger than one would expect from the Table III confidence interval). Most of the four additional parameters are small relative to the original parameters, with three of the four (g_{2Q}, g_{1H}, g_{2H}) having negative signs. The interpretation of these negative signs is unclear. Perhaps this suggests a tendency to bunch inventory holdings as well as production. The entries corresponding to the "∞" line in Table V are reported in part B of Table VI. As may be seen, they are consistent with the Table V entries.

I conclude that even though the model in part B was, as usual, rejected by tests of overidentifying restrictions, substantively different results are unlikely to be produced by extensions to models

TABLE VI

Estimates for expanded model, $\phi_c = \phi_d = 1$
A. Estimates of cost and demand parameters

g_{0Q}	g_{1Q}	g_{0S}	g_{0H}	g_{HS}	g_{2Q}	g_{1H}	g_{2H}	g_{1HQ}
−0.222	0.453	0.323	0.099	0.118	−0.088	−0.003	−0.045	0.115

B. Percentage of variance due to cost shocks

Horizon	H	Q	S
∞	99	45	45

No confidence intervals are available; $h = 1.83$; $(\sigma_c/\sigma_d) = 0.72$.

that are complicated and more difficult to interpret, but are unrejected.

A second set of additional estimates considered the implications of the imprecise estimates of the accelerator parameter g_{HS}. In the three specifications in this set, g_{HS} was fixed at values of 0.17, 0.34, and 0.68 instead of its estimated value of −0.04; 0.68 is the upper bound of the 95 percent confidence interval in line (1) of Table III, and the other two values are one fourth and one half of this upper bound. These higher values for g_{HS} are consistent with the estimates of some earlier studies [Blanchard, 1983; West, 1986]. For each value of g_{HS} I held the other $g_{..}$ parameters fixed at the values reported in Table III, solved for the reduced form, and then used this in all subsequent calculations.

For each g_{HS} the impulse response functions (not shown) looked similar to those in Figures IV and V. The only notable difference was that demand shocks caused a rise in the steady state level of inventories, and for $g_{HS} = 0.68$, demand shocks caused inventories to rise immediately as well as in the steady state. As noted in Section II, this is exactly what one would expect with relatively large values for g_{HS}.

A higher value of g_{HS} might also lead to a less important role for cost shocks, since more of the procyclical movement of inventories might be attributed to movements in demand; such an outcome is not guaranteed since, in equilibrium, S_t is moved by shocks to both cost and demand. But, in fact, the implied infinite horizon fraction of the variance of inventories, GNP, and sales due to cost shocks, falls for all three specifications, with the fall the more dramatic the larger is g_{HS}. See Table VII.

It will be noted that even in the $g_{HS} = 0.68$ specification, much of the variability of inventories is attributed to cost shocks. The basic technical characteristic of the data that accounts for this is

TABLE VII
PERCENTAGE OF VARIANCE DUE TO COST SHOCKS, IMPOSED VALUES OF g_{HS}

(1) g_{HS}	(2) Horizon	(4) Q	(5) S	(3) H	(6) p-value for H_0: g_{HS} = column (1) value
(1) 0.17	∞	35	35	98	> 0.50
(2) 0.34	∞	27	27	93	> 0.20
(3) 0.68	∞	9	9	66	0.05

In all three rows $\phi_c = \phi_d = 1$. No confidence intervals are available. In rows (1) to (3) the values of h are 0.88, 1.03, and 4.98; the values of (σ_c/σ_d) are 0.62, 0.44, and 0.07.

probably that inventories and GNP are not cointegrated or, if cointegrated, are barely so in the sense that any stationary linear combination is barely stationary. (See the tests for unit roots and cointegration above.) What this probably means is that the model is unhappy assigning all, or even almost all, of the movement in both inventories and output to one common factor. Even when most of the movement in output can be attributed to demand, as in Table VII, explanation of much of the movement in inventories requires turning to cost.

Note that according to the bootstrap confidence intervals used to compute column (6) in Table VII, the fit of the model does not deteriorate appreciably as g_{HS} is varied. The estimates in Table V, then, are sufficiently imprecise that fixing g_{HS} at a plausible value that is rather different from its estimated value results in a variance decomposition that is rather different, and more consistent with some earlier studies (e.g., Fair [1988, p. 232], who finds that supply shocks account for 7 percent of the variance of the eight-quarter-ahead forecast error in GNP).[12]

The third and final set of additional empirical results followed Blanchard and Watson [1986] by looking quarter by quarter at the role of cost and demand shocks in eight-step-ahead forecast errors in GNP. For the $\phi_c = \phi_d = 1$ specification, with g_{HS} at its freely estimated value of -0.040, this is plotted in Figure VI; for the $g_{HS} = 0.34$ specification used in Table VII, this is plotted in Figure VII. In both figures, as in Blanchard and Watson, peaks and troughs in the forecast errors line up nicely with NBER business cycle peaks and troughs, with the latter noted by the vertical lines in the figures.

An unsatisfactory aspect of the results is that in neither specification is the 73:4 to 75:1 recession singled out as one in which cost shocks are unusually important—on the contrary, the figures make it evident that *demand* shocks are claimed to be unusually important. Figure I indicates why: this recession is unique in that inventories moved procyclically through most of the recession, with the inventory liquidation not taking place until the *end* of the recession. The model of course attributed this countercyclical movement to demand shocks. And at least one author [Blinder, 1981a, pp. 46–52] seems to suggest that the inventory accumulation in 1974 is ultimately explained by demand shocks (though his analysis, which essentially concludes that the inventory buildup

12. In defense of the present paper, it should be noted that these estimates do not seem to be any less precise than those in Blanchard and Quah [1988] or King et al. [1987].

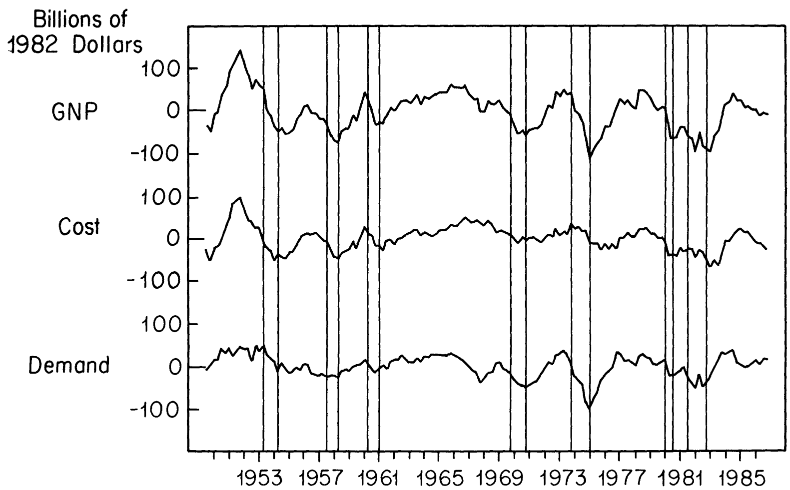

FIGURE VI
Components of GNP Forecast Error, $g_{HS} = -0.04$

was a sluggish accelerator-driven response to earlier (1973) positive demand shocks, does not help explain why such a sluggish response did not occur in other recessions).

But even if the inventory accumulation is consistent with patterns of demand shocks, the broader inference that demand

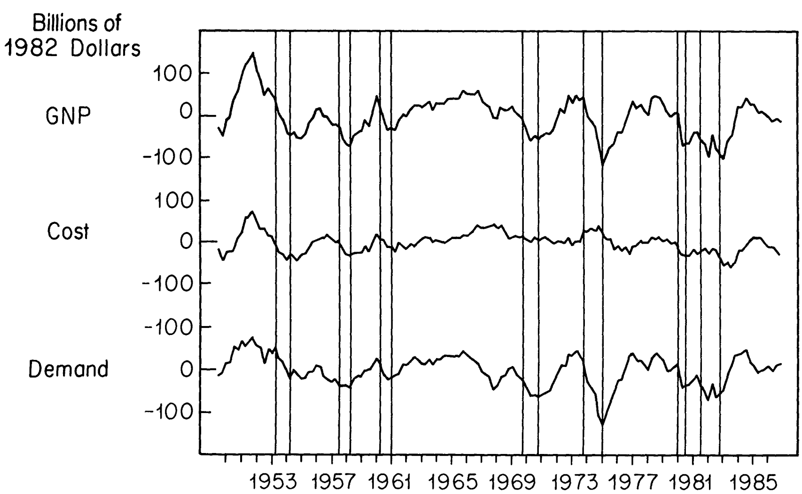

FIGURE VII
Components of GNP Forecast Error, $g_{HS} = 0.34$

shocks are largely responsible for the fall in output is unlikely to be correct. In this recession, then, use of the comovements of output and inventories leads to unlikely inferences about the sources of fluctuations. As is suggested by Figures VI and VII, however, in other recessions the model's results are more in accord with what we think we know about the sources of these recessions. The 1981:3 and 1982:4 recession, for example, is one in which demand played a prominent role; in the $g_{HS} = -0.04$ and $g_{HS} = 0.34$ specifications demand shocks account for, respectively, 44 and 61 percent of the forecast error. More generally, both cost and demand shocks play a role in most recessions, a result consistent with, for example, Blanchard and Quah [1988].

The model thus produces an unsatisfactory interpretation of one of the most interesting time periods in the sample, that of the first OPEC recession: to paraphrase Fiorella LaGuardia, when this model made a mistake, it made a doozy. But the model also yields sensible results for the remainder of the sample. My overall conclusion from this exercise is that the present paper's analysis of periods of recession supports the judgment made in some earlier work that there typically is no single shock that drives the economy into a recession.

IV. Conclusions

Fluctuations in aggregate inventories in the postwar United States appear to be due more to fluctuations in cost than to fluctuations in demand. Despite some long-standing difficulties in linking movements in inventories to those in costs (a recent example is Miron and Zeldes [1987]), the implication is that future inventory research should emphasize the role of costs. Fluctuations in GNP appear to be due in roughly equal proportions to fluctuations in cost and demand. The point estimates are, however, noisy. With different, and plausible, values for the parameter that determines a target inventory-sales ratio, cost shocks are less important than demand shocks for GNP fluctuations.

Appendix

This Appendix discusses (1) how to solve the model, and how to calculate F (defined in equation (8)), given estimates of $F\Phi F^{-1}$ and Π; and (2) how to estimate $F\Phi F^{-1}$ and Π.

1. Tentatively ignore the scaling for growth discussed in the

text. The right-hand side of (7) is

$$\frac{\partial c_t}{\partial H_t} = 2g_{0H}g_{HS}(H_{t-1} - g_{HS}S_t) + 2bg_{0H}(1 - g_{HS})(H_t - g_{HS}E_tS_{t+1}) + 2hU_{ct}$$

$$\frac{\partial c_t}{\partial Q_t} = -2bg_{1Q}E_tQ_{t+1} + 2[g_{0Q} + (1 + b)g_{1Q}]Q_t - 2g_{1Q}Q_{t-1} - g_{0H}g_{HS}(H_{t-1} - g_{HS}S_t) + 2U_{ct}.$$

Upon substituting (6) into (7), using the above for the right-hand side of (7), and then substituting out for S_t, we get

(A1) $E_t[bA_1'Y_{t+1} + A_0Y_t + A_1Y_{t-1} + \beta_0^{-1}(D_0U_t + bD_1U_{t+1})] = 0.$

In (A1), $\beta_0 \equiv g_{0S} + g_{0Q} + g_{0H}g_{HS}^2 + (1 + b)g_{1Q}$; A_0 and A_1 are 2×2 matrices that depend on the discount rate b and the parameters in (1) and (2), with A_0 symmetric and positive definite, with nonzero off-diagonal elements,

$$A_0 \equiv \begin{vmatrix} \beta_2 & \beta_1 \\ \beta_1 & 1 \end{vmatrix}, \qquad A_1 \equiv \begin{vmatrix} \beta_4 & 0 \\ -\beta_4 & \beta_3 \end{vmatrix},$$

where $\beta_1 \equiv -\beta_0^{-1}[g_{0S} + g_{0H}g_{HS}^2]$, $\beta_2 \equiv \beta_0^{-1}[(1 + b)g_{0S} + g_{0H}g_{HS}^2 + bg_{0H}(1 - g_{HS})^2]$, $\beta_3 \equiv -\beta_0^{-1}g_{1Q}$, $\beta_4 \equiv -\beta_0^{-1}[g_{0S} - g_{0H}g_{HS}(1 - g_{HS})]$; U_t is the 2×1 vector $(U_{ct}, U_{dt})'$; the D_i are 2×2 matrices,

$$D_0 = \begin{vmatrix} h & 1 \\ 1 & -1 \end{vmatrix}, \qquad D_1 = \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix}.$$

Let

$$D = D_0 + b\begin{vmatrix} 0 & -\phi_d \\ 0 & 0 \end{vmatrix},$$

$$\Omega_u = E(U_t - \Phi U_{t-1})(U_t - \Phi U_{t-1})' = \mathrm{diag}(\sigma_c^2, \sigma_d^2).$$

Inserting (8) led once into (A1) yields

$$bA_1'(\Pi Y_t + F\Phi U_t) + A_0Y_t + A_1Y_{t-1} - DU_t = 0 \rightarrow$$

$$bA_1'[\Pi(\Pi Y_{t-1} + FU_t) + F\Phi U_t] + A_0(\Pi Y_{t-1} + FU_t) + A_1Y_{t-1} - DU_t = 0 \rightarrow$$

(A2) $$bA_1'\Pi^2 + A_0\Pi + A_1 = 0$$

(A3) $$[bA_1'(F\Phi F^{-1} + \Pi) + A_0]F = D.$$

After estimating the reduced-form (9), one uses the four equations in (A2) to linearly recover the four elements of A_0 and A_1.

Given estimates of A_0 and A_1, one can calculate the three unknowns h, σ_c^2, and σ_d^2 from the three equations in $[bA_1'(F\Phi F^{-1} + \Pi) + A_0]\Omega_v[bA_1'(F\Phi F^{-1} + \Pi) + A_0]' = D\Omega_u D'$. (An estimate of Ω_v is available from the covariance matrix of the reduced-form residuals.) One then calculates $F = [bA_1'(F\Phi F^{-1} + \Pi) + A_0]^{-1}D$.

As stated in the text, the data were scaled by a growth rate of $(1.00807)^t \equiv g^t$ prior to estimation of (9). The model that allows such growth (described in detail in the additional appendix available on request) implies that the first-order condition (A1) should be written as

$$E_t\{bgA_1'Y_{t+1} + A_0Y_t + g^{-1}A_1Y_{t-1} + DU_t\} = 0,$$

where $D = D_0 + bgD_1$, and D_0 and D_1 are defined below equation (A1). The calculations just described are then modified in a straightforward fashion.

2. When $\phi_c = \phi_d = \phi$ for some scalar ϕ, $F\Phi F^{-1} = \Phi I$, and it is straightforward to estimate ϕ and Π subject to the restriction that $Y_t = (\phi I + \Pi Y_{t-1}) - \phi\Pi Y_{t-2} + V_t$ for some scalar ϕ. Then $F\Phi F^{-1} = \phi I$, and one proceeds as above. When $\phi_c \neq \phi_d$, maximum likelihood is very cumbersome. (The constraint is not only nonlinear but involves both the regression parameters and the variance–covariance matrix.) The following procedure, which yields consistent though not efficient estimates, was therefore used instead. (a) OLS was used to estimate the second-order VAR $Y_t = \Pi_1Y_{t-1} + \Pi_2Y_{t-2} + V_t$. (b) The matrix lag polynomial $I - \Pi_1L - \Pi_2L^2$ must be factored as $(I - F\Phi F^{-1}L)$ $(I - \Pi L)$. For given Π_1 and Π_2, there may be zero, two, or four real factorizations. (Analogously, if $F\Phi F^{-1}$, Π, Π_1, and Π_2 were all scalars, there would be zero or two factorizations: zero if both roots to $1 - \Pi_1L - \Pi_2L^2$ are complex or two if the roots are real. In the latter case one obtains two factorizations by assigning first one and then the other root to the serial correlation parameter $F\Phi F^{-1}$.) For the Π_1 and Π_2 actually estimated, there happened to be two real factorizations. (c) Let $P = F\Phi F^{-1}$. For each factorization (each P and Π); (i) compute A_0 and A_1 as described above. (ii) With some manipulation (A3) implies that $D\Phi = [bA_1'(P + \Pi) + A_0]P[bA_1'(P + \Pi) + A_0]^{-1}D$. Imposing that Φ is diagonal allows one to solve for ϕ_d. Given h (computed as described above), one can also use this to compute ϕ_c. This yields Φ and D. (iii) Compute F as above—$F = [bA_1'(P + \Pi) + A_0]^{-1}D$. (iv) Compute $F\Phi F^{-1}$ using the diagonal Φ produced in step (ii). Call this matrix P^*. (v) The implied restricted VAR is $Y_t = (P^* + \Pi)Y_{t-1} - P^*\Pi Y_{t-2} + V_t$. Compute the likelihood (the log determinant of the variance–covariance matrix of V_t). (d) Select the factorization that

yields the highest likelihood. This P^* is what is reported as $F\Phi F^{-1}$ in Table I.

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