K

k. *See* QUANTITY THEORY OF MONEY.

Kalman filter. The Kalman filter is a technique to use noisy measurements of an unobservable vector of random variables to make an optimal estimate of the value of that vector, when the vector evolves according to a dynamic linear equation. I first describe some applications, then sketch some algebra.

There are two broad classes of applications. One is in algorithms for maximum likelihood estimation and forecast of time-series models. The Kalman filter is often computationally convenient here; see Harvey (1989) for many illustrations.

The second class of applications is to economic models in which variables are unobservable, and only measured with noise. One possibility is that all the relevant variables are observable to economic agents but not to the research economist. Hamilton (1985), for example, assumes essentially that market expectations of US inflation and the real rate on government bonds follow a vector autoregressive process (the 'transition equation', in the terminology defined below). Economists such as Hamilton observe only the realizations of these variables, which, under rational expectations, differ from the unobserved market expectations by a serially uncorrelated error (the 'measurement equation'). Hamilton uses the Kalman filter to estimate the market expectations.

Another possibility is that some relevant variables are unobservable to economic agents, who use the Kalman filter to estimate the values of these variables. Lin, Engle and Ito (1991), for example, assume that stock returns in Japan and the US respond to both local and global factors (the measurement equation); these factors are unobserved. Investors apply the Kalman filter to extract estimates of the local and global factors, which are used to forecast stock returns and volatility.

Formally, it is assumed that the system in question may be written in *state-space form*, evolving according to a pair of equations,

$$x_{t+1} = Ax_t + Bu_t + \varepsilon_{1t},\ \varepsilon_t \sim \text{iid}\, N(0,V_1), \tag{1}$$

$$y_t = Cx_t + \varepsilon_{2t},\ \varepsilon_{2t} \sim \text{iid}\, N(0,V_2). \tag{2}$$

Equation (1) is the *transition equation*, equation (2) is the *measurement equation*, x_t is an $(s \times 1)$ unobserved *state vector* that one wishes to estimate, and y_t is the $(m \times 1)$ noisy, observed measurement of x_t. The unobserved disturbances ε_{1t} $(s \times 1)$ and ε_{2t} $(m \times 1)$ are uncorrelated both with each other and with x_0, the presample value of x_t. The $(c \times 1)$ vector u_t follows a known sequence. In most applications in which $B \neq 0$, u_t is a *control vector* chosen to influence the path of x_t optimally. The analyst starts with a prior on the mean and variance of the presample values of x_0, $x_0 \sim N(x_0|_0, \Sigma_0|_0)$. A, B and C, which obey some technical conditions, are dimensioned commensurately with x_t, y_t and u_t. The algebra below can be generalized to allow A, B and C to vary with time, and to allow correlation between ε_{1t} and ε_{2t}; see Anderson and Moore (1979) and Harvey (1989).

The aim is to use observations on $\{y_j\}$, $j = 1, \ldots, t$, to compute estimates of x_t, $t = 1, \ldots, T$. Let $x_t|_n$ denote $E(x_t|y_n, y_{n-1}, \ldots, y_1, u_n, u_{n-1}, \ldots, u_1)$, $\Sigma_t|_{t-1}$ the estimate of $E(x_t - x_t|_{t-1})(x_t - x_t|_{t-1})'$ available at time $t-1$, with an analogous definition for $\Sigma_t|_t$. Given that all variables are normally distributed, one could in principle use standard formulas for conditional expectations of normally distributed random variables (equivalently, formulas for signal extraction) to solve for $x_t|_t$. (For example, if $A = B = 0$ and $s = m = 1$, so that all variables are scalars, it is easy to see that $x_t|_t = [CV_1/(C^2V_1 + V_2)]y_t$.) But such computations (illustrated in Bertsekas 1976) quickly get very complicated in the general case. Kalman's contribution (1960, cited in Harvey 1989) was to note that the computational burden is greatly lessened by doing the computations recursively, first using $x_0|_0$ and $\Sigma_{0|0}$ to get $x_{1|0}$ and $\Sigma_{1|0}$, then using these to get $x_1|_1$ and $\Sigma_{1|1}$, and so on.

We have

$$\begin{aligned} x_t|_t &= x_t|_{t-1} + E[(x_t - x_t|_{t-1})|(y_t - y_t|_{t-1})] \\ &= x_t|_{t-1} + D_t(y_t - y_t|_{t-1}) \\ D_t &\equiv \{[E(y_t - y_t|_{t-1})(y_t - y_t|_{t-1})']^{-1} \\ &\quad [E(y_t - y_t|_{t-1})(x_t - x_t|_{t-1})']\}'. \end{aligned} \tag{3}$$

It follows from (1) and (2) that $x_t|_{t-1} = Ax_{t-1}|_{t-1} + Bu_{t-1}$, $y_t|_{t-1} = Cx_t|_{t-1}$. D_t can be derived with routine but tedious calculations, yielding

$$\begin{aligned} x_t|_t = Ax_{t-1}|_{t-1} + Bu_{t-1} + \Sigma_t|_{t-1}C'(C\Sigma_t|_{t-1}C' + V_2)^{-1} \\ [y_t - C(Ax_{t-1}|_{t-1} + Bu_{t-1})], \end{aligned} \tag{4a}$$

$$\Sigma_t|_{t-1} = A\Sigma_{t-1}|_{t-1}A' + V_1, \tag{4b}$$

$$\Sigma_t|_t = \Sigma_t|_{t-1} - \Sigma_t|_{t-1}C'(C\Sigma_t|_{t-1}C' + V_2)^{-1}C\Sigma_t|_{t-1}. \tag{4c}$$

Equations (4a)–(4c) can be used to solve recursively forward for $x_t|_t$ from $t = 0$. Under suitable conditions, as $t \to \infty$, $\Sigma_t|_{t-1}$ approaches a constant matrix, say, Σ, which is the unique positive semidefinite solution to the 'Riccati equation' $\Sigma = A[\Sigma - \Sigma C'\ (C\Sigma C' + V_2)^{-1}C\Sigma]A' + V_1$; $\Sigma_t|_t$ and D_t approach the constant values implied by (4a) and (4c).

If the normality assumption is dropped in favour of an alternative finite variance distribution, equations (4) describe an estimator of $x_t|_t$ that is optimal from the point of view of a certain mean-squared error criterion (Anderson and Moore 1979).

Equations (4) above give the *filtered* estimates of x_t – those computed using observations on y_s, $s \leq t$. There are analogous formulas for the *smoothed* estimates of x_t, computed from a sample of size $T > t$ using y_s, $1 \leq s \leq T$, or *predicted* values of x_t, computed using y_s, $s < t$ (Anderson and Moore 1979; Harvey 1989).

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See also BAYESIAN INFERENCE IN TIME SERIES; PREDICTION; SIGNAL EXTRACTION; STATISTICAL INFERENCE IN TIME SERIES; VECTOR AUTOREGRESSION METHODS; VOLATILITY.

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key currency. The term used to denote a national currency that is widely used and held for international transactions, including transactions by nationals of third countries. A key currency typically takes on quasi-official functions; governments use it to define the foreign exchange values of their national currencies, buy and sell it in the foreign exchange market to stabilize the values of their currencies, and hold it as a reserve asset. The pound sterling was the key currency before World War I; the US dollar played that role after World War II. There are several key currencies today, including the dollar, the Deutschemark and the Japanese yen.

P.B.K.

killer bees. *See* TAKEOVER LANGUAGE.

kiting. The use of 'float' (funds or cheques already sent out by the payer but not yet collected) by consumers, banks, businesses, and other entities. Kiting provides the payer with temporary and usually free credits prior to the time the appropriate value of funds can be collected from his deposit account. At time of collection the float collapses and the possibility of kiting ends abruptly, but in the meantime the holder enjoys a temporary subsidy. In order that cheques not 'bounce' due to insufficient funds, the beneficiary of kiting must of course be sure that bank balances at time of collection are adequate to cover all incoming debits against his account.

E.H.S.

Knights Templar. The leading facts of the history of this military order are well known: at the time of the first crusade they were founded to defend pilgrims to the Holy Land against the infidel; they enjoyed exemptions, granted by special papal bulls, from ordinary ecclesiastical jurisdictions; they acquired immense wealth, became unpopular both in England and in France, and, in the latter country, were suppressed by an unscrupulous stroke of authority of King Philip the Fair, who condemned the grand master Molay and other dignitaries to death, and confiscated, in 1307, a large part of the wealth of the order. Though in England such extreme proceedings were not taken, Edward I, in 1295, carried away by force from the Temple a sum of £10,000, and Edward II, shortly after his accession, seized £50,000 in silver, besides gold and jewels, which had been deposited in their treasury (Cunningham, *Growth of English Industry*, p. 254).

During almost the whole of the 13th century the house of the Templars in Paris acted as bankers to the kings of France, the royal princes, noblemen, rich burghers and merchants. Its dealings in this capacity were for the first time submitted to a searching and exhaustive analysis by M. Léopold Delisle in his *Mémoire sur les Opérations financières des Templiers (Mémoires de l'Académie des Inscriptions et Belles Lettres*, vol. 33, 1889), of which the following is a summary.

Owing to the sanctity ascribed to their precincts, monasteries were, during the middle ages, favourite places for deposits of the precious metals, jewels, chattels, etc., but the *Commanderies* of the Temple distinctly acted as bankers by (1) being chosen as deposits for disputed funds, (2) granting loans and acting as securities for the fulfilment of contracts, (3) transmitting monies and paying them at a distance and (4) accepting and effecting payments for customers who had a running account with them. All these operations have been identified by M. Delisle and are authenticated by original documents printed in his appendix. Deposits in cash were sometimes locked up in special *hutches* marked with the names of the owners, in which case they could not be touched without the express consent of the depositors, but generally the Order was allowed to make use of the deposits at its discretion, but of course under its responsibility.

Fragments of one of the books kept in the Temple at Paris for the daily receipts of money, and printed in the appendix (pp. 162–223), afford an insight into their daily transactions, and show how the payments effected were either put to the credit of the owner of an account (*super talem*) or carried over to another account such as *in parvo libro novo, in magno libro*, etc. For each day the name of the brother in charge heads the entry; and at night the monies received are as a rule transferred to the central office (*solvit in turre*). About 800 different names are entered in the relatively short space of fifteen months (12 March 1295 to 4 July 1296); the reference to about ten distinct other registers, such as *in magno libro ad debemus*, etc., show that the Templars understood the advantages of systematic book-keeping.

From 1202, the Temple became the central treasury of the kings of France, and under Louis IX the royal auditors even held their meetings in the Temple; it also paid the pensions granted by the king, the amounts of which were transferable. From the balance sheets, which have been