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Estimation and inference in the linear-quadratic inventory model*

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We summarize some recent work of ours on estimation and hypothesis testing on the parameters of the linear-quadratic inventory model. For some data-generating processes calibrated to estimates from some existing studies, this work uses (1) asymptotic theory to compare alternative estimators on the basis of the asymptotic efficiency of parameter estimates, (2) asymptotic theory and simulations to consider how likely one will be to get sharp estimates of the parameters of the model, and (3) simulations to see how accurately sized are hypothesis tests about the parameters of the model.

Key words: Linear-quadratic inventory model; Asymptotic theory; Simulation
JEL classification: C10

1. Introduction

In this paper, we summarize some work we have done recently analyzing estimation and hypothesis testing in the linear-quadratic inventory model [West (1993), West and Wilcox (1993a, b)]. Our objective in this work is to use conventional asymptotic theory and simulations to assess how well one might expect commonly-used econometric techniques to work in practice.

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Table 1
Some estimates of the linear quadratic model. [a]

Sources	(1) a_0/a_2	(2) a_1/a_2	(3) a_3
Eichenbaum (1989)	0.0	(0.2, 0.8)	(0.6, 1.6)
Ramey (1991)	(0.1, 0.1)	(− 0.7, − 0.4)	(0.3, 0.4)
West (1986a)	(− 0.9, 7.7)	(13.8, 28.5)	(1.1, 4.9)

[a] a_0, a_1, a_2, and a_3 are cost function parameters defined in eq. (1).

The table is based on the estimates for monthly, U.S. postwar data, for six two-digit industries, and, for Eichenbaum and West, aggregate nondurables manufacturing as well. For the seven sets of estimates (six for Ramey), the published estimates of a_0, a_1, and a_2 (relative to one or another linear combination of these three variables) were first recomputed relative to a_2. The second lowest and second highest estimates are given in parentheses.

The sample period, left-hand-side variable, instrument list, techniques to correct for serial correlation of cost shocks (if any), and list of observable cost variables (if any) varies across the three papers.

Sources: Eichenbaum: table 3 (p. 862), top panel; Ramey: table 1 (p. 323); West: table 6 (p. 393), top panel. Eichenbaum reports parameters that he denotes λ and α. We set $a_0 = 0$; $a_1/a_2 = [\lambda + (1/\lambda) - 1 - (1/b)]^{-1}$, with $b = 0.995$; $a_3 = 2(1-\alpha)(a_1/a_2)$.

To motivate such research, consider some recent estimates of the parameters of this model. Table 1 summarizes the range of estimates produced by Eichenbaum (1989), Ramey (1991), and West (1986a), all of which applied similar versions of the model to roughly the same data on inventories and sales at the two-digit level in the manufacturing sector in the United States. The a_i's capture various production and inventory holding costs, and will be defined precisely in the next section of the paper.

As indicated in the notes to the table, the ranges were constructed by throwing out each paper's high and low estimates of each parameter. Once this is done, there is little overlap in estimates across the three papers. In addition, plots of one author's estimates of a given parameter against another author's (not presented) suggest little or no correlation – that is, if one author found a relatively high estimate of, say, a_1/a_2 for a given industry, there was little tendency for another author to find that as well. Some of the differences in estimates are economically important: Ramey's estimates imply that marginal production cost slopes down, which is suggestive of an imperfectly competitive market, while Eichenbaum and West's indicate that it slopes up, which is not.

One possible explanation for this disparity of results is that one or more of the papers uses an inappropriate specification or estimation technique; the papers differ in their treatment of, for example, unobserved serial correlation, observable measures of factor costs, and choice of instruments. More generally, it may be that all three papers are fundamentally misspecified. While we recognize the need to consider such possibilities, we have focused thus far on exploring the possibility that sampling error alone accounts for such differences in results across authors.

In section 2, we present the model and describe the data-generating processes we use in our analysis. In section 3, we outline the instrumental variables and maximum likelihood estimators that we evaluate. In section 4, we use conventional asymptotic theory to see whether these estimators are so disperse that sampling error might plausibly account for disparities such as those shown in table 1. For some but not all of our data-generating processes (DGP's), the answer appears to be yes. In section 5, we describe simulations that evaluate, for an instrumental variables estimator, the reliability of the predictions of conventional asymptotic theory. While such theory usually gives a good guide to the behavior of our estimators, sometimes it does not, and, insofar as our DGP's accurately capture the salient aspects of the real data used in the studies summarized in table 1, some of the dispersion in table 1 may be due to certain finite sample biases. Section 6 concludes.

2. The model and data-generating processes

The model follows Holt et al. (1960). A representative firm maximizes the expected present discounted value of future cash flows, with a cost function that includes linear and quadratic costs of production and of changing production and of holding inventories. Let p_t be real price, S_t real sales, Q_t real production, H_t real end-of-period inventories, C_t real costs, b a discount factor ($0 \leq b < 1$), E_t mathematical expectations conditional on information known at time t, assumed equivalent to linear projections, and u_t a cost shock that is observable to the firm but unobservable to the econometrician. The objective function is

$$\max \mathrm{E}_t \sum_{j=0}^{\infty} b^j (p_{t+j} S_{t+j} - C_{t+j}),$$

subject to

$$Q_{t+j} = S_{t+j} + H_{t+j} - H_{t+j-1},$$

$$C_{t+j} = 0.5 a_0 \Delta Q_{t+j}^2 + 0.5 a_1 Q_{t+j}^2 + 0.5 a_2 (H_{t+j-1} - a_3 S_{t+j})^2 + H_{t+j} u_{t+j}. \tag{1}$$

The a_i's are unknown parameters. In (1) and throughout, constant and linear terms are omitted for notational economy.

An optimizing firm will not be able to cut costs by increasing production by one unit this period, storing the unit in inventory, and producing one less unit next period, holding revenue unchanged throughout. Formally, differentiating

(1) with respect to H_t gives

$$\mathrm{E}_t\{a_0(\Delta Q_t - 2b\Delta Q_{t+1} + b^2\Delta Q_{t+2}) + a_1(Q_t - bQ_{t+1}) + ba_2(H_t - a_3S_{t+1}) + u_t\} = 0\,. \tag{2}$$

To solve the model and obtain a data-generating process, it is necessary to specify the structure of demand and of the cost shock. We make the simplest possible assumption, that sales are exogenous – specifically, an AR(2) – and that u_t is iid. It may then be shown [West (1993)] that the reduced form of the model is

$$H_t = (\lambda_1 + \lambda_2)H_{t-1} - \lambda_1\lambda_2 H_{t-2} + \pi_1 S_{t-1} + \pi_2 S_{t-2} + \varepsilon_{Ht}\,, \tag{3a}$$

$$S_t = \phi_1 S_{t-1} + \phi_2 S_{t-2} + \varepsilon_{St}\,, \tag{3b}$$

where ϕ_1 and ϕ_2 are given parameters in the assumed process for sales, λ_1 and λ_2 are the two smallest (in modulus) roots of

$$\lambda^4 - b^{-2}a_0^{-1}[ba_1 + 2a_0 b(1+b)]\lambda^3 + b^{-2}a_0^{-1}[a_0(1+4b+b^2) + a_1(1+b) + ba_2]\lambda^2 - b^{-2}a_0^{-1}[a_1 + 2a_0(1+b)]\lambda + b^{-2} = 0\,, \tag{4}$$

and $\varepsilon_{Ht} = -(\lambda_1\lambda_2/a_0)u_t + (\pi_2/\phi_2)\varepsilon_{St}$, for π_1 and π_2 defined in a footnote.[1] Since S_t and u_t are stationary, so, too, are H_t and Q_t.

We aim to evaluate estimators of the a_i's. Note that a_0, a_1, and a_2 are identified only up to scale: in (4) and footnote 1 it may be seen that only the ratios of these parameters affect λ_1, λ_2, π_1, and π_2; alternatively, it may be seen from (2) that if $\{a_0, a_1, a_2\}$ set u_t orthogonal to a set of instruments, then so does $\{\alpha a_0, \alpha a_1, \alpha a_2\}$ for any nonzero α. Thus ratios but not levels of the parameters are identified; the ratios that we analyze happen to be a_0/a_2 and a_1/a_2, but that choice is arbitrary. We also analyze a_3, which is identified.

To generate the data, we need to specify values for the cost parameters, the ϕ_i's, and the variance–covariance matrix of the disturbances u_t and ε_{St}. Here, we limit ourselves to four sets of parameters that are the main focus of West and Wilcox (1993b), and refer the reader to that paper for a discussion of a broader set of parameters.

We label the four sets of cost parameters A, B, C and D; the assumed values for each are given in table 2. (We will explain the table 2 figures in brackets in the

[1] Define the scalars ρ_1, ρ_2, w_1, w_2, w_3, and w_4 as follows: $\rho_1 = \lambda_1 + \lambda_2$, $\rho_2 = -\lambda_1\lambda_2$, $w_1 = b^2\rho_2$, $w_2 = -\rho_2[b^2 + 2b + b(a_1/a_0) + (ba_2a_3/a_0)]$, $w_3 = \rho_2[2b + 1 + (a_1/a_0)]$, $w_4 = -\rho_2$. Define the (1×2) vector $e' \equiv (1\ 0)$, the (2×2) matrices Φ and D as $\Phi(1,1) = \phi_1$, $\Phi(1,2) = \phi_2$, $\Phi(2,1) = 1$, $\Phi(2,2) = 0$, $D = [I - b\rho_1\Phi - b^2\rho_2\Phi^2]^{-1}$. Then $(\pi_1, \pi_2)' = e'D(w_1\Phi^3 + w_2\Phi^2 + w_3\Phi + w_4 I)$.

Table 2
Probability of a parameter estimate with an incorrect sign, $T = 300$, according to asymptotic approximation.[a]

DGP	Estimator	Parameter		
		a_0/a_2	a_1/a_2	a_3
A		10.00	1.00	0.10
	IV4	[0.02]	[0.43]	[0.40]
	FIML	[0.00]	[0.32]	[0.35]
B		0.17	− 0.33	0.50
	IV4	[0.00]	[0.07]	[0.00]
	FIML	[0.00]	[0.03]	[0.00]
C		10.00	20.00	1.00
	IV4	[0.12]	[0.26]	[0.25]
	FIML	[0.01]	[0.03]	[0.04]
D		10.00	− 5.00	0.50
	IV4	[0.00]	[0.00]	[0.10]
	FIML	[0.00]	[0.00]	[0.07]

[a]The cost function [eq. (1) in the text] includes $0.5a_0\Delta Q_t^2 + 0.5a_1Q_t^2 + 0.5a_2(H_{t-1} - a_3S_t)^2$.
The first row of each section gives the parameter values.

IV4 is an instrumental variables estimator that uses two lags each of H_t and S_t as instruments (four instruments altogether), and is applied to a first-order condition for cost minimization; FIML is full information maximum likelihood, and is applied to the bivariate system consisting of the decision rule for H_t and the process for S_t. The estimators are described in section 3 of the text.

In brackets are the probability of an incorrectly signed estimate for a sample of size 300, computed by (a) solving analytically for $V \equiv (3 \times 3)$ asymptotic variance–covariance matrix of a given estimator, which varies across estimators and DGP's, (b) assuming that the parameter estimate is $N(\beta, V/300)$, where $\beta \equiv (a_0/a_2, a_1/a_2, a_3)$, and (c) using the diagonal elements of $V/300$ to compute the probability that such normal variables will have an incorrect sign. See section 4 of the text for details. By construction, the probability is less than 0.50, and the entries for the maximum likelihood estimator FIML will be less than those for the instrumental variables estimator IV4.

next section of the paper.) All four are based on studies using U.S. data of one sort or another. Parameter set A is roughly consistent with the estimates for post-war aggregate data in West (1990) and those for automobile data in Blanchard and Melino (1986), parameter sets B and C with those for post-war two-digit manufacturing industries in Ramey (1991) and West (1986a) respectively (see table 1), parameter set D with those for auto data from the 1920's and 1930's in Kashyap and Wilcox (1993). See Ramey (1991) for an argument for the reasonableness of the negative values for a_1 in parameter sets B and D. Once again, only the relative values of a_0, a_1, and a_2 are relevant. Throughout, we also set

$$b = 0.995, \quad \phi_1 = 0.75, \quad \phi_2 = 0.20,$$
$$\text{var}(\varepsilon_S) = 0.120833, \quad \text{var}(u) = 3.5, \tag{5}$$
$$\text{corr}\,(\varepsilon_S, u) = -0.5\,.$$

The eq. (5) values were chosen so that (a) the ϕ_i's roughly match trend-stationary estimates for sales in U.S. aggregate nondurables manufacturing, monthly, seasonally adjusted, 1967–1990, and (b) in conjunction with parameter set A, the implied variance–covariance matrix of (H_t,S_t) is approximately proportional to that of nondurables manufacturing. In addition, constant and trend terms were included in the DGP's. See West and Wilcox (1993b) for details.

3. The estimators

We consider instrumental variables (IV) estimators of eq. (2) and the full information, maximum likelihood (FIML) estimator of the bivariate system (3a)–(3b). We use asymptotic theory to analyze both estimators (section 4) and simulation methods to analyze an IV estimator (section 5).

The IV estimator is used by (among others) West (1986a), Eichenbaum (1989), and Kashyap and Wilcox (1993). It begins by choosing a left-hand-side variable, and replacing expectations of variables with their realized values. Assume for concreteness that the left-hand-side variable is chosen by moving ba_2H_t to the left-hand side of (2), and then dividing both sides of the equation by ba_2 [e.g., Ramey (1991)]. The result is

$$\begin{aligned}
H_t &= X_t'\beta + v_{t+2},\\
X_t &\equiv [-b^{-1}(\Delta Q_t - 2b\Delta Q_{t+1} + b^2\Delta Q_{t+2}), -b^{-1}(Q_t - bQ_{t+1}), S_{t+1}]'\\
&\equiv [X_{0t+2}, X_{1t+1}, S_{t+1}]',\\
\beta &\equiv (\beta_1, \beta_2, \beta_3)' \equiv (a_0/a_2, a_1/a_2, a_3)',\\
v_{t+2} &\equiv -(ba_2)^{-1}\{u_t + ba_0(X_{0t+2} - \mathrm{E}_tX_{0t+2}) + ba_1(X_{1t+1} - \mathrm{E}_tX_{1t+1})\}\\
&\quad - a_3(S_{t+1} - \mathrm{E}_tS_{t+1}).
\end{aligned} \tag{6}$$

As is typical in empirical work, we impose a value of b, which allows us to construct X_{0t} and X_{1t}.

We then estimate β as follows: Let Z_t be a $(q\times 1)$ vector of instruments consisting of lags of H_t and S_t, where $q \geq 3$. Let T be the sample size, and stack Z_t, X_t, and H_t into $T\times q$, $T\times 3$, and $T\times 1$ matrices $Z \equiv [Z_t']$, $X \equiv [X_t']$, and $H \equiv [H_t]$. The estimator uses the combination of instruments that has the smallest possible asymptotic variance–covariance matrix given the set of instruments used,

$$\hat{\beta} = (X'Z\hat{W}Z'X)^{-1}X'Z\hat{W}Z'H, \tag{7}$$

where $\hat{W}$ is a $q \times q$ matrix such that $\hat{W} \xrightarrow{p} W$, $W^{-1} \equiv \Sigma_{j=-2}^{2} \mathrm{E} Z_t Z_{t-j}' v_{t+2} v_{t+2-j}$ [the bounds on the summation follow because $Z_t v_{t+2} \sim$ MA(2), which in turn results because u_t is iid]. In our experiments, we calculated $\hat{W}$ following the method in Newey and West (1993).

In all our simulations and most of our asymptotic calculations, we set $q = 4$, $Z_t = (H_{t-1}, S_{t-1}, H_{t-2}, S_{t-2})'$, so that Z_t is the 4×1 vector of instruments in the reduced form. Since Z_t has four elements, we refer to this estimator as 'IV4'. In some of our asymptotic calculations, we also describe results for IV estimators that use $q > 4$ instruments, with q an even number (for convenience) and with Z_t consisting of the first $q/2$ lags of H_t and of S_t. It may be shown that because of the serial correlation in the disturbance v_{t+2}, smaller asymptotic variance–covariance matrices result from use of additional lags even though no such lags appear in the reduced form [Hansen (1985)].

IV estimators are asymptotically invariant to choice of left-hand-side variable (provided the function of the a_i's that multiplies the variable moved to the left-hand side is nonzero), so the asymptotic analysis in section 4 proceeds without reference to such choice. Such invariance need not, however, obtain in finite samples, and, indeed, choice of left-hand-side variable seems to be important in practice: using various data sets, Ramey (1991), Kashyap and Wilcox (1993), and Krane and Braun (1991) found that estimated parameters sometimes varied widely for different choices. In the simulation analysis of section 5, we therefore experiment with alternative choices.

The second estimator that we consider in our asymptotic calculations is FIML. This estimator is applied to the two-equation system (3a)–(3b). Given an imposed value of b, it produces estimates of a_1/a_2, a_0/a_2, a_3, ϕ_1, and ϕ_2. See Blanchard (1983) and West (1993). FIML is more efficient than IV4, or, for that matter, IV with any number of lags of H_t and S_t as instruments. But it is much more complex computationally, and, unlike the instrumental variables estimators, it is inconsistent if the assumed structure of demand is incorrect.

4. Asymptotic approximation

Let $\hat{\beta}$ be the estimate from a given method, where $\beta = (a_0/a_2, a_1/a_2, a_3)'$. Then under the null of correct specification, $T^{1/2}(\hat{\beta} - \beta) \overset{A}{\sim} \mathrm{N}(0, V)$, for an asymptotic variance–covariance matrix V that can be computed in straightforward but tedious fashion. See West and Wilcox (1993b) and West (1986b).

IV versus FIML: For the 12 parameter estimates (12 = 4 DGP's times 3 parameters), the ratios of the asymptotic standard errors of IV4 to those of FIML (i.e., the ratios of the square roots of the diagonal elements of the respective V's) range from about 1.2 to nearly 3.0 (not reported in any table). This is consistent with the results reported in West (1986b), which found large asymptotic efficiency gains from use of FIML when data are highly serially correlated, as they are in our DGP's. It was noted in the previous section that use of additional lags of H_t and S_t as instruments results in efficiency gains. These gains often are substantial: when the number of instruments $q = 12$ and the instrument vector Z_t is $(H_{t-1}, S_{t-1}, \ldots, H_{t-6}, S_{t-6})'$, the ratio of the asymptotic standard errors of IV12 to those of FIML ranges from less than 1.1 to about 1.9, and is generally less than 1.3. The efficiency benefits of increased lags beyond the sixth (of $q > 12$) are small.

For our DGP's, then, the asymptotic efficiency gains from using FIML rather than IV occasionally are large, but often are modest, at least if a generous number of lags of H_t and S_t are used as instruments.

Since the IV estimator is easier to compute, and in empirical application does not require specification of demand, simulation evaluation of the finite sample properties of an estimator like IV12 is of interest, but we have yet to complete such an exercise.

Probability of correctly signed estimates: To get a handle on the precision of estimation one should expect in samples of typical size, we use the asymptotic theory to predict how likely one is to obtain a correctly signed parameter estimate in a sample of a given size. Specifically, we interpret the asymptotic approximation as telling us that for a sample of size T,

$$\hat{\beta} \approx \mathrm{N}(\beta, V/T)\,. \tag{8}$$

Let $\beta \equiv [\beta_i]$. If β_i is positive, the probability that $\hat{\beta}_i$ has the wrong sign is then

$$[\text{prob } \hat{\beta}_i < 0] = \Phi[-\beta_i/(V_{ii}/T)^{1/2}]\,, \tag{9}$$

where V_{ii} is the (i,i) element of V and $\Phi(x)$ is the probability that a standard normal is less than x; if β_i is negative, an analogous inequality is used to compute the probability. Given V and a sample size T, one can compute the probability that a given estimate will be negative. Under our null of correct specification, (9) will produce no bigger a value for FIML than for IV4 or any other IV estimators.

Table 2 has (9) for IV4 and FIML for each DGP, for $T = 300$, 300 being approximately the number of monthly observations currently available at the two-digit level in the United States. As one might expect, such a sample size is sufficiently large that there is often a negligible probability of a parameter estimate having a wrong sign (at least according to the asymptotic theory; we discuss finite sample simulation results in the next section). On the other hand, the number of nonzero entries is surprising to us. (Given rounding, the probability must be less than 0.005 for the entry in table 2 to be zero.) For a 95 percent confidence interval to exclude wrong-signed estimates, the probability must be less than 0.025, which in turn requires that when rounded to two digits, as in table 2, the probability is less than 0.03. For FIML, this happens in only six, or half, the entries (a_0/a_2, all four DGP's; a_1/a_2, DGP D; a_3, DGP B); for IV4 [and for IV12 (not reported in the table)], it happens for five, or fewer than half, the entries.

Some of the indicated probabilities are substantial. The most striking is that for a_1/a_2, in DGP A. Even the FIML estimator has a nearly one-third probability of yielding an incorrectly signed estimate of this parameter, and the IV4 estimator, which is essentially that used in Eichenbaum (1988) and West (1986a), has an even higher probability. It may be shown that a sample size greater than 5000, or over 400 years of monthly observations, is required for the FIML probability to fall below 0.025.[2]

Now, it may not be of particular interest whether one or another parameter is estimated with the correct sign; it may be, for example, that one would not care much whether an estimate of a_1/a_2 is (say) -4.0 or 1.0 or 6.0. But those familiar with the inventory literature will recognize that the sign of a_1 is a matter of considerable debate, with Ramey (1991) contending that a_1 is negative, Eichenbaum (1989) that it is positive. And West and Wilcox (1993b) argue that in DGP A, estimates of a_1/a_2 from opposite ends of a two standard deviation interval around the true value of 1.0 have substantively different economic implications: those towards the lower end of this interval imply strongly procyclical inventory movements, in the sense of predicting a positive correlation between inventory investment and sales (and hence considerably greater variability in production than in sales) even in the absence of cost shocks; those towards the upper end do not.

In sum, conventional asymptotic theory suggests that, depending on the DGP, it may be difficult to obtain accurate estimates of the parameters of the model in sample sizes typically available.

5. Simulation evidence on instrumental variables estimators

For each data-generating process, West and Wilcox (1993b) constructed 1000 samples of size 300, assuming that the cost shock u_t and the demand shock ε_{St} are jointly iid normal and obtaining initial conditions on H_t and S_t by drawing from the relevant unconditional normal distribution. We then obtained estimates of IV4 for three choices of left-hand-side variable, yielding 36 sets of estimates altogether (36 = 4 DGP's times 3 parameters times 3 normalizations).

[2] Specifically, the FIML standard error on a_1/a_2 for $T = 300$ is about 2.2 [and correspondingly, the probability that a $N(1,2.2^2)$ r.v. is negative is about 0.32]. When this standard error falls to about 0.5, the implied probability will be about 0.025. This in turn requires a sample size $(2.2/0.5)^2 \approx 19$ times as large as that assumed in table 2.

In work currently in process, we are examining the finite sample behavior of other IV estimators.

The three choices of left-hand-side variable were as follows. The first was obtained as illustrated in section 2, by moving ba_2H_t to the left-hand side of eq. (2) and dividing by ba_2 [Ramey (1991)]; the second by moving $[a_0(1 + 4b + b^2) + a_1(1 + b) + ba_2]H_t$ and dividing by $a_0(1 + 4b + b^2) + a_1(1 + b) + ba_2$ [Kashyap and Wilcox (1993)]; the third by moving $[(1 + b)a_0 + a_1]\,(bQ_{t+1} - Q_t)$ and dividing by $(1 + b)a_0 + a_1$ [West (1986a)].

We divide our discussion of simulation results into two parts: parameter estimation and hypothesis testing.

Parameter estimation: Parameter estimates were usually centered reasonably well: in 28 of our 36 sets of estimates, the median across the 1000 estimates was within one asymptotic standard deviation of the true parameter. Seven of the eight exceptions occurred in the Ramey (1991) normalization, a fact that we will return to in the next paragraph. The estimates were slightly more variable than the asymptotic theory predicts; across the 36 sets, the median width of a 50 percent confidence interval constructed by dropping the 250 smallest and 250 largest estimates was 1.5 asymptotic standard deviations; for a normally distributed variable, the value is 1.4.

Perhaps the most controversial difference in the results summarized in table 1 of this paper is that a_1/a_2 is estimated to be negative by Ramey (1991), positive by West (1986a) and Eichenbaum (1989). The median bias noted in the previous paragraph included downward biases in estimates of a_1/a_2 for Ramey's (1991) choice of left-hand-side variable. In DGP *A*, 88.9 percent of the 1000 point estimates of this parameter were negative (compared to the 43 percent predicted by the asymptotic calculation reported in table 2); in DGP *C* the figure was 55.6 percent (compared to 26 percent according to the asymptotic calculation). The percentages that were incorrectly signed for DGP's *B* and *D* were quite close to the values in table 2. The figures for West's (1986a) left-hand-side variable, by contrast, were: DGP *A*, 48.0 percent; DGP *C*, 4.7 percent. Thus, part of the explanation for the table 1 results might be that Ramey's choice of left-hand-side variable sometimes is downward biased in its estimates of a_1, while West's sometimes is upward biased, although we repeat that in DGP *B*, which was calibrated to Ramey's estimates, there was little bias. [The normalization used by Kashyap and Wilcox (1993) rarely showed bias, for any DGP.]

Test statistics: We examined nominal 5 percent tests of the hypothesis that a given variable was equal to the true parameter value. Unsurprisingly, such tests rejected much too frequently when the parameter estimate was biased; in DGP *A*, for example, with Ramey's normalization, 58.2 percent was the actual size of a nominal 5 percent test of H_0: $a_1/a_2 = 1$. Even absent such bias in the parameter estimate, tests on a_3 rejected too *in*frequently, for reasons that are not clear to us. Otherwise, the actual sizes of nominal 5 percent tests typically ranged from about 2 to about 8 percent. The overall implication is that hypothesis tests may be quite poorly sized in samples of size typically available, a conclusion also reached by studies such as Andrews and Monahan (1992) and Newey and West (1993).

6. Conclusions

For some plausible data-generating processes and sample sizes, asymptotic theory indicates that it may be difficult to obtain sharp estimates of the linear-quadratic inventory model, regardless of what estimator is used. For these and other DGP's, instrumental variables estimators that use a large number of lagged variables as instruments often are nearly as efficient asymptotically as full information maximum likelihood. Simulation evidence indicates that the finite sample distribution of instrumental variables estimators with a low dimension instrument vector usually, but not always, is well approximated by conventional asymptotic theory.

Priorities for future research include simulation evaluation of instrumental variables estimators with many lags, and asymptotic and simulation evaluation of estimators that gain efficiency by pooling data from various industries.

References

Andrews, Donald W.K. and Christopher J. Monahan, 1992, An improved heteroskedasticity and autocorrelation consistent covariance matrix, Econometrica 60, 953–966.

Blanchard, Olivier J., 1983, The production and inventory behavior of the American automobile industry, Journal of Political Economy 91, 365–400.

Blanchard, Olivier J. and Angelo Melino, 1986, The cyclical behavior of prices and quantities: The case of the automobile market, Journal of Monetary Economics 17, 379–408.

Eichenbaum, Martin S., 1989, Some empirical evidence on the production level and production cost smoothing models of inventory investment, American Economic Review 79, 853–864.

Hansen, Lars Peter, 1982, Large sample properties of generalized method of moments estimators, Econometrica 50, 1029–1054.

Hansen, Lars Peter, 1985, A method for calculating bounds on the asymptotic variance–covariance matrices of generalized method of moments estimators, Journal of Econometrics 30, 203–228.

Holt, Charles C., Franco Modigliani, John F. Muth, and Herbert A. Simon, 1960, Planning production, inventories and work force (Prentice-Hall, Englewood Cliffs, NJ).

Kashyap, Anil K and David W. Wilcox, 1993, Production and inventory control at the General Motors Corporation during the 1920s and 1930s, American Economic Review 83, 383–401.

Krane, Spencer D. and Steven N. Braun, 1991, Production smoothing evidence from physical product data, Journal of Political Economy 99, 558–581.

Newey, Whitney K. and Kenneth D. West, 1993, Automatic lag selection in covariance matrix estimation, Manuscript (University of Wisconsin, Madison, WI).

Ramey, Valerie A., 1991, Nonconvex costs and the behavior of inventories, Journal of Political Economy 99, 306–334.

West, Kenneth D., 1986a, A variance bounds test of the linear quadratic inventory model, Journal of Political Economy 94, 374–401.

West, Kenneth D., 1986b, Full versus limited information estimation of a rational expectations model: Some numerical comparisons, Journal of Econometrics 33, 367–386.
West, Kenneth D., 1990, The sources of fluctuations in aggregate inventories and GNP, Quarterly Journal of Economics CV, 939–972.
West, Kenneth D., 1993, Inventory models, in: M. Pesaran and M. Wickens, eds., Handbook of applied econometrics (Basil Blackwell, Oxford) forthcoming.
West, Kenneth D. and David W. Wilcox, 1993a, Finite sample behavior of alternative instrumental variables estimators of a dynamic linear model, Manuscript (University of Wisconsin, Madison, WI).
West, Kenneth D. and David W. Wilcox, 1993b, Some evidence on the finite sample behavior of an instrumental variables estimator of the linear quadratic inventory model, in: R. Fiorito, ed., Inventory cycles and monetary policy (Springer-Verlag, Berlin) forthcoming.