On The Interpretation of Near Random-Walk Behavior in GNP

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Several recent papers have studied the univariate time-series process for U.S. GNP, including John Campbell and Gregory Mankiw, 1986; Peter Clark, 1986a, b; John Cochrane, 1986; Charles Nelson and Charles Plosser, 1982; Danny Quah, 1986; James Stock and Mark Watson, 1986; and Watson, 1986. A major focus of these papers has been the extent to which GNP movements are well approximated by a process with a unit root with drift, as opposed to stationary movements around a time trend. The empirical evidence on this is mixed. Campbell and Mankiw, 1986; Nelson and Plosser, 1982; and Stock and Watson, 1986, conclude that the random-walk (unit-root) approximation is quite good. Clark, 1986a, b; Cochrane, 1986, and, perhaps, Quah, 1986; and Watson, 1986, say that it is not.

Campbell and Mankiw, 1986, and Nelson and Plosser, 1982, both argue that if the random-walk approximation in fact is reasonable, there are important implications for business cycle theory. This is because movements in random walks are permanent: a shock today has an infinitely long-lived effect. The concept of a stationary natural rate, Campbell and Mankiw note, has little utility if a GNP shock is, on average, never offset by a return to some trend rate of GNP. Nelson and Plosser suggest that monetary disturbances are unlikely to be an important source of GNP fluctuations, since monetary shocks are typically thought to have no permanent effect. Both conclude that if the random-walk characterization is accurate, an implication is that fluctuations in GNP are unlikely to be driven by nominal demand shocks.[1] Similar inferences appear to be drawn by Rene Stultz and Walter Wasserfallen, 1985; Angus Deaton, 1986; and Olivier Blanchard and Quah, 1987.

Campbell and Mankiw, 1986, and Nelson and Plosser, 1982, of course recognize that their random-walk characterization is only a convenient approximation. In any finite sample, it will not be possible to discriminate between a unit root (random walk) and a root arbitrarily near, but below, unity (what this paper calls a "near random walk"). This is potentially a practical problem. The Monte Carlo evidence in David Dickey and Wayne Fuller, 1981, indicates that with Nelson and Plosser's 1982 sample size (less than 100), Nelson and Plosser's test of a unit-root null is not very likely to reject even when the true process is stationary, with autoregressive coefficients whose sum is as low as .8. Coefficients of this size and larger are suggested by studies that assume the GNP process is stationary. An AR(2) of log real GNP around trend fitted to annual U.S. data 1948–1985, for example, yields coefficients whose sum is .83; since the estimate of this sum is sharply downward biased for processes with near unit roots (Fuller, 1976), the .83 point estimate is suggestive of a sum even closer to unity.[2]

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[1]Nelson and Plosser, 1982, p. 166, conclude that "assigning a major portion of variance in output to the innovation in [a] nonstationary component gives an important role for real factors in output fluctuations and places limits on the importance of monetary theories of the business cycle." Campbell and Mankiw, 1986, p. 24, state that their results are "inconsistent with many prominent theories in which output fluctuations are primarily caused by shocks to aggregate demand...[including] models based on long-term nominal contracts."

[2]Throughout, I assume annual rather than quarterly data, for two reasons. The first is for consistency with some of the relevant studies, including John Taylor, 1980b, and Nelson and Plosser, 1982. The second is that the two-period contract length that, for simplicity, will

The aims of this paper are twofold. The first is to point out that it is dangerous to use a single country's univariate GNP process to draw structural inferences concerning the stability of the natural rate, or of the importance of nominal shocks in business cycles, given that in practice one cannot discriminate between random walk and near random-walk behavior. The second is to emphasize that simple natural rate models with nominal shocks are as capable as simple real business cycle models (for example, Robert King et al., 1987) in generating a highly persistent process for GNP.

The paper uses a variant of John Taylor's (1980a, b) overlapping wage contracts model, which maintains a stationary natural rate.[3] In my variant (unlike Taylor's) the only source of instability—the only reason GNP ever deviates from the natural rate—is shocks to monetary policy. Thus, monetary policy is the only important factor in the business cycle. It is shown that near random-walk behavior in GNP can result from monetary policy of the sort often attributed to the U.S. Federal Reserve.

The basic idea is as follows. In practically any model, including Taylor's, serial correlation in movements of the money stock puts serial correlation in movements in prices. In Taylor's model, prices do not adjust instantaneously to movements in money. Additional persistence in prices is induced by the overlapping wage contracts. Movements in real interest rates and real balances therefore are serially correlated, and this induces serial correlation in aggregate demand and GNP. The degree of the serial correlation depends on the monetary authority's money supply rule and the model's basic parameters. Stylized versions of simple money supply rules, and plausible values for the model's basic parameters, suggest near random-walk behavior in GNP. The implied autoregressive root is about .8 to .99.

Near random-walk behavior, then, is perfectly consistent with Taylor's natural rate model. This is, of course, implicitly a message in Taylor (1980a, b), since it is argued there that, at least in the presence of supply shocks, the model is capable of tracking observed movements in GNP. The present paper generalizes Taylor's result in two ways. First, I show that near random-walk behavior results even in a model with purely nominal shocks. In light of Campbell and Mankiw's 1986, and Nelson and Plosser's 1982 interpretation of their results, this seems important to establish. Second, I show that near random-walk behavior results even in a version of Taylor's model extended to include standard IS and LM curves, with a monetary policy rule of targeting the interest rate. Given the widespread use of such an aggregate demand apparatus (at least in textbooks), this seems to be an useful generalization.

Before deriving my results, let me emphasize what I am *not* arguing. I am not arguing that destablizing monetary policy is the sole, or even most important, source of U.S. GNP's near random-walk behavior. I am suppressing the supply and demand shocks present in Taylor (1980a, b) not because I doubt their importance, but to make my point as cleanly and emphatically as possible. I am also not arguing that the unit-root approximation is a bad one. It is probably appropriate for forming simple ARIMA forecasts (Bennett McCallum, 1986a), for example. It may even be appropriate for structural estimation and inference: I am not arguing for a stationary monetary theory of the business cycle against, say, a nonstationary real theory. My point, rather, is that the stylized facts about GNP are perfectly consistent with Taylor's widely used model. Simple analysis of a given country's univariate process for GNP therefore is unlikely to be particularly helpful in distinguishing stationary from nonstationary theories, or between models in which nominal shocks are very important from those in which they have negligible effects.

be assumed in Section I below, is implausibly short for quarterly but not for annual data. In addition, and again for consistency, all empirical estimates are taken from studies using post-World War II U.S. data.

[3] I follow Taylor (1980a, b) in interpreting his model as a natural rate one. Bennett McCallum, undated, 1987, argues otherwise.

I. Near Random Walk for GNP

The aggregate supply curve (Phillips curve) is borrowed from Taylor (1980a, b). There are staggered two-period wage contracts. In each period, one half of the labor force fixes its nominal wage for the next two periods.

$$(1) \qquad x_t = .5x_{t-1} + .5_{t-1}x_{t+1} + .5\gamma({}_{t-1}y_t + {}_{t-1}y_{t+1}),$$

$$(2) \qquad p_t = .5(x_t + x_{t-1}),$$

$$(3) \qquad y_t = -\delta(i_t - {}_tp_{t+1} + p_t),$$

$$(4) \qquad m_t - p_t = \theta_1 y_t - \theta_2 i_t,$$

$$(5) \qquad i_t = \lambda i_{t-1} + u_t.$$

The variables are x_t = log nominal contract wage, y_t = log GNP, i_t = nominal interest rate, p_t = log price level, m_t = log money supply, u_t = a serially uncorrelated shock. A "$t-1$" subscript, as a prefix, denotes expectations at time $t-1$. All variables are zero-mean deviations from trend. Trend GNP is by definition potential or natural rate GNP.

Equation (1) says that the nominal wage depends on actual and expected wages, as well as expected demand pressure. The latter is measured by expected deviations of GNP from trend. Equation (2) is a price markup equation. Equation (3) is a standard IS curve, relating GNP to the *ex ante* real interest rate. Equation (4) is a standard LM curve, expressing the demand for money as a function of the nominal interest rate and GNP. As noted in the introduction, the supply and demand shocks that quite plausibly are present in equations (1) to (4) are suppressed, to emphasize the potential role for monetary policy in output fluctuations.

Equation (5) is the money supply rule, with $0 \leq \lambda < 1$ and u_t a serially uncorrelated shock. The monetary authority is thus assumed to smooth movements in interest rates. Empirical evidence that i_t followed a near random walk in the postwar period (λ is near one) may be found in Eugene Fama and Michael Gibbons (1982). A theoretical argument why the Fed might have set nominal interest rates to follow a near random walk may be found in Mankiw (1986).

It is straightforward, though tedious, to solve the model.[4] Let

$$(6) \qquad a = -(1+\gamma^{-1}\delta^{-1}) + \gamma^{-1}\delta^{-1}(4\gamma\delta+1)^{1/2},$$

$$b = 2\gamma\delta(\lambda+\lambda^2)/\{(\lambda-1)(a\gamma\delta+\lambda\gamma\delta+2+2\gamma\delta)\}.$$

The Appendix shows that the contract wage x_t obeys

$$(7) \qquad (1-aL)(1-\lambda L)x_t = bu_t,$$

where L is the lag operator. In conjunction with the price markup equation (2), equation (7) can be used to solve for the stochastic process for p_t. When this is plugged into the IS curve (3), one can calculate the stochastic process for y_t. Output in general follows an ARMA (2,2) process,

$$(8) \qquad y_t = [\eta(L)/\rho(L)]u_t,$$

$$\rho(L) \equiv (1-aL)(1-\lambda L),$$

$$\eta(L) \equiv \delta[(-1+.5b)+aL-.5bL^2]$$

$$= (-\delta+.5b\delta)[1+(-1+.5b)^{-1}aL - (-1+.5b)^{-1}(.5b)L^2]$$

$$\equiv d_0(1+d_1L+d_2L^2).$$

To see how the properties of (8), the univariate process y_t, depend on the monetary policy rule (depend on λ), consider two

[4] The well-known indeterminacy of rational expectations models under interest rate rules (Thomas Sargent, 1979; McCallum, 1986b), applies here as well. The rule (5) is interpreted as in McCallum (1986b) as the limit of a certain non-interest rate rule that yields a unique stationary solution for y_t. The restriction $\lambda \neq 1$ is imposed because for $\lambda = 1$ this solution technique breaks down (a divide by zero is implied). See the formula for b in equation (6) below.

cases. The first is $\lambda = 0$, $i_t = u_t$. Since $\lambda = 0$ implies $b = 0$ (see equation (6)), we have from (8) that

$$y_t = -\delta u_t. \tag{9}$$

So if the monetary authority takes care that the current nominal interest rate is independent of past shocks, deviations of output from the natural rate are serially uncorrelated.

Consider instead the case when λ is near unity. From (6), as $\lambda \to 1$, $b \to -\infty$. It follows that for λ arbitrarily near one, d_1 (defined in (8)) will be arbitrarily near zero and d_2 arbitrarily near -1. Thus for λ very near 1, $\eta(L)$ will factor as $(-\delta + .5b\delta)(1 + \eta_1 L)(1 - \eta_2 L)$, $\eta_1 \approx 1$, $\eta_2 \approx 1 \approx \lambda$. Since the $1 - \eta_2 L$ factor approximately cancels the $1 - \lambda L$ autoregressive factor, y_t will behave very much like the ARMA(1,1) process that results when these factors are canceled,[5]

$$y_t \approx d_0[(1+L)/(1-aL)]u_t. \tag{10}$$

It follows that y_t will behave much like a variable with a unit root if $\underline{a}$ is near one.

We have $a \to 1$ as $\gamma\delta \to 0$, that is, as the aggregate supply and/or IS curves become horizontal. Taylor (1980b) estimated γ to be about .05 to .10. Jeffrey Sachs's (1980) Phillips curve regressions suggest that γ is about .01 to .07.[6] Benjamin Friedman's (1977) estimates suggest $\delta = .17$;[7] Taylor (1985) indicates that δ is less than .125. If we take .01 to .10 as the range for γ, .1 to .2 as the range for δ, then $\gamma\delta$ is about .001 to .02. This yields a range for $\underline{a}$ of about .96 to .998. See Table 1, Part A. Plausible parameter values therefore suggest that the near random-walk characterization will be quite good if the monetary authority attempts to stabilize interest rates by setting ${}_{t-1}(i_t - \lambda i_{t-1})$ to zero for λ near one. With $\underline{a}$ this near unity, it will be difficult to reject the null hypothesis of a unit root, in sample sizes typically available.[8]

The intuition to the effect of λ on the univariate y_t process is as follows. With $\lambda = 0$, the contract wage and price level are nonstochastic: $x_t = x_{t-1} = 0$ is the only stationary (constant mean) solution to (7) with $b = 0$. So the IS curve (3) implies $y_t = -\delta i_t$,

TABLE 1—IMPLIED AUTOREGRESSIVE ROOT FOR GNP

A. Interest Rate Target	
Range for structural parameters:	
sensitivity of wages to excess demand (γ):	.01 to .10
slope of IS curve (δ):	.1 to .2
Range for implied AR root:	.96 to .998
B. Money Supply Target	
Range for structural parameters:	
sensitivity of wages to excess demand (γ):	.01 to .10
degree of monetary accommodation (g):	.3
Range for implied AR root:	.78 to .93

Note: The model for panel A consists of equations (1) to (5); for panel B of equations (1), (2), (11), and (12). In each case λ is assumed at or near unity.

[5] This illustrates the possibility that approximate cancellation of common autoregressive and moving average factors may help explain Andrew Rose's (1987) result that univariate time-series have simpler ARMA representations than are suggested by multiequation structural models. See Rose, 1987, pp. 27–29.

[6] Rewrite (1) as $x_t - x_{t-1} = {}_{t-1}(x_{t+1} - x_t) + \alpha y_t + e_t$, $\alpha \equiv 2\gamma$, $e_t \equiv -(x_t - {}_{t-1}x_t) - \gamma(y_t - {}_{t-1}y_t + y_t - {}_{t-1}y_{t+1})$. This is in the usual Phillips curve form, inflation = expected inflation + α * excess demand + shock. The .01 to .07 range reported in the text is one half of Sachs's (1980) estimates of α (that is) one half of his post-war estimates of the coefficients that he calls β_1 and ϕ in his Tables 3 and 4).

[7] This is Friedman's (1977, p. 322) implied estimate of the long-run elasticity of real spending with respect to the nominal interest rate, from regressions using quarterly data. [The short-run (single quarter) elasticity is .09.] Friedman (1977, p. 323) notes that in his 1961–76 sample period, nominal and expected real yields are likely to be very highly correlated, which suggests that his estimates are appropriate for an IS curve that depends on the expected real rate.

[8] To make the argument in the preceding two paragraphs concrete, it may help to calculate y_t's ARMA parameters for specific λ and a. Suppose that $a = .96$, $\lambda = .96$. (The value for λ is Fama and Gibbons's (1982, Table 2) point estimate of the first-order serial correlation coefficient of monthly T-bill rates, 1953–77; Fama and Gibbons do not report figures for annual interest rates.) Then one can grind through the formulas in the text to show that y_t is moving average polynomial factors as $(1 + .32L)(1 - .98L)$. Output will therefore behave much like an ARMA(1,1) variable with a single-autoregressive unit root of .96.

and, with $\lambda=0$, i_t is serially uncorrelated. By contrast, when $\lambda \neq 0$, the autoregressive root of $1-\lambda L$ in the monetary authority's control variable puts the same root in the wage and price processes; the long-run properties of the money supply rule of course are reflected in wages and prices. But that is not all. As Taylor (1980a) has emphasized, overlapping contracts can be an endogenous source of persistence. The serial correlation in the money supply induces serial correlation in wages and prices above and beyond that directly produced by the $1-\lambda L$ root. So expected inflation, ${}_tp_{t+1}-p_t$, does not move instantaneously, and one to one, with i_t. The real interest rate is serially correlated, and, therefore, as per the IS curve (3), so is GNP.

More generally, for any λ between 0 and 1, there will also be persistence in GNP. If λ is near zero, y_t will behave much like the serially uncorrelated variable defined in (9). The closer λ is to unity, the more y_t will behave like the serially correlated process defined in (10).

It is worth noting that a similar result obtains if, as in Taylor (1980a, b) the money supply rule involves targeting the money supply instead of the interest rate. To analyze this type of rule, it is convenient to follow Taylor and replace the IS and LM curves with a simple quantity equation,

$$y_t+p_t=m_t. \tag{11}$$

Unlike Taylor, I have set to zero shocks to velocity (deviations of velocity from trend), as explained in the introduction. Also, replace the interest rate target (5) with a money supply target,

$$m_t=gp_t+\lambda(m_{t-1}-gp_{t-1})+u_t, \tag{12}$$

where u_t is a serially uncorrelated shock and $0 \leq \lambda \leq 1$.

To understand (12), consider first the case when $\lambda, u_t \equiv 0$. Then, as in Taylor (1980a), the parameter g measures how accommodative monetary policy is. (For my purposes one could have the monetary authority look directly at y_t as well as, or instead of, p_t; only p_t appears in the money supply rule for consistency with Taylor (1980a, b).) The shock u_t is not present in Taylor (1980a, b). It is intended to reflect shocks to the money supply resulting from, say, random movements in the money multiplier. The $\lambda(m_{t-1}-gp_{t-1})$ term is present to capture a tendency of the monetary authority to absorb previous control errors. If $\lambda=1$, previous u_t's are never offset and are carried through to all future money supplies. Such random-walk behavior ("base drift") has been argued to characterize Federal Reserve policy in the United States, at least in recent years (see Carl Walsh, 1986, and the references cited therein).

The model may be solved as in Taylor (1980a, b); the details are omitted to save space. Let $\beta=1-g$; $c=(1+.5\beta\gamma)(1-.5\beta\gamma)^{-1}$; $a=c-(c^2-1)^{1/2}$, if $c>1$; $a=c+(c^2-1)^{1/2}$, if $c<-1$; $b=.5\gamma(\lambda+\lambda^2)/[1+.5\beta\gamma-.5(1-.5\beta\gamma)(a+\lambda)]$. Then

$$\begin{aligned} x_t &= ax_{t-1}+b(m_{t-1}-gp_{t-1}), \\ y_t &= -.5\beta b[(1+L)/(1-aL)] \\ &\quad \times(m_{t-1}-gp_{t-1}) \\ &\quad +\lambda(m_{t-1}-gp_{t-1})+u_t. \end{aligned} \tag{13}$$

Consider first the case where $\lambda=0$, $m_t=gp_t+u_t$. Since $\lambda=0$ implies $b=0$ (see the formula for b above equation (13)), we have $y_t=u_t$, and y_t is serially uncorrelated. Suppose instead that $\lambda=1$, and $m_t=gp_t+(m_{t-1}-gp_{t-1})+u_t$. Then it is straightforward but tedious to show that (13) reduces to

$$y_t=ay_{t-1}+u_t+.5(1-a)u_{t-1}.$$

So $y_t \sim$ ARMA (1,1). In any finite sample, y_t will look arbitrarily like a random walk for $\underline{a}$ arbitrarily close to unity. Now, $a \to 1$ as $\beta\gamma=(1-g)\gamma \to 0$. As was just noted, γ is about .01 to .10. Taylor (1980b) estimated β to be about .3. This indicates that $\beta\gamma$ is about .003 to .03, yielding an implied $\underline{a}$ of .78 to .93. See Table 1, Part B.

The intuition to the effect of λ on the univariate y_t process is similar to that for the previous money supply rule. With $\lambda = 0$, the contract wage and price level are nonstochastic: $x_t = x_{t-1} = 0$ is the only stationary solution to (13) with $b = 0$. So the aggregate demand equation (11) implies $y_t = m_t$, and with $\lambda = 0$, m_t is serially uncorrelated. By contrast, when $\lambda = 1$, the unit root in the money supply first of all puts a unit root in wages and prices; the long-run properties of wages and prices are governed by the money supply. But because of the staggered contracts, real balances, the difference between m and p, have additional persistence: prices do not move instantaneously, and one for one, with money supplies. This persistence is transmitted directly into GNP by the aggregate demand equation (11).

II. Conclusions

Neither stationarity of the natural rate nor nominal shocks playing an important role in the business cycle are inconsistent with a root very near to unity being present in the GNP process. In Taylor's (1980a, b) stationary natural rate model, extreme persistence in GNP movements is precisely what is predicted, given stylized versions of money supply rules often attributed to the Fed, and plausible values for the model's basic parameters. The model also predicts that different money supply rules would result in dramatically less persistent movements in GNP.

This is not to argue that, in fact, the business cycle in the United States is purely, or even largely, monetary in origin, or that natural rate theory is to be preferred to nonnatural rate theory. Rather, detailed study of the univariate process for a single country's GNP is unlikely to be particularly helpful in deciding some important business cycle issues. Potentially more helpful are comparative studies of GNP processes across various countries and various time periods. The evidence here is mixed. Stultz and Wasserfallen (1985) and Campbell and Mankiw (1987) conclude that during the postwar period the random-walk approximation is reasonable for a number of industrialized countries. This perhaps makes it less likely that GNP behavior could change dramatically with a change in policy regime. On the other hand, Stock and Watson (1986) find that for the United States, the random-walk approximation is reasonable only in the post-1919 period. This is consistent with the present paper's model: the near random-walk behavior of the nominal interest rate appears to have begun around 1915–1920 (Mankiw et al., 1987), and inflation appears to have been more sensitive to excess demand pre-1929 than postwar (Sachs, 1980).[9] In any case, estimation of multivariate structural models is, of course, potentially still more helpful than estimation of univariate time-series.

Appendix

As stated in fn. 4, the money supply rule (5) is understood to be the limit of a certain non-interest rate rule. This rule is a simple generalization of the rule in Robert Driskell and Steven Sheffrin, 1986, and Bradford De Long and Lawrence Summers, 1986:

$$\text{(A1)} \quad m_t = \alpha(i_t - z_t), \quad z_t \equiv u_t/(1-\lambda L), \quad \alpha > 0.$$

Thus, if i_t is above (below) its target level z_t, m_t is increased (decreased). The rule (A1) yields a unique stationary solution for y_t for any finite α; the solution for y_t under the rule (5) is understood to be the one that results when one first solves using a finite α and then takes the limit as $\alpha \to \infty$.

Use (A1) to eliminate m_t from the LM curve (4) and rearrange to get

$$i_t = (\alpha + \theta_2)^{-1}(p_t + \alpha z_t + \theta_1 y_t).$$

Substitute the above into the IS curve (3) and rearrange to get

(A2)

$$\begin{aligned} y_t = {} & \left[1 + (\alpha+\theta_2)^{-1}\delta\theta_1\right]^{-1} \\ & \times \left\{ -\delta\alpha(\alpha+\theta_2)^{-1} z_t \right. \\ & \left. + \delta_t p_{t+1} - \delta\left[1 + (\alpha+\theta_2)^{-1}\right] p_t \right\} \\ \equiv {} & -\delta_0 z_t + \delta_{1t} p_{t+1} - \delta_2 p_t. \end{aligned}$$

[9]Stock and Watson (1986) suggest that the seeming stationarity of pre-1919 GNP may instead be an artifact of the way these data were constructed.

Since ${}_{t-1}z_t = \lambda z_{t-1}, {}_{t-1}z_{t+1} = \lambda^2 z_{t-1}$, (A2) implies

$$\text{(A3)} \qquad {}_{t-1}y_t + {}_{t-1}y_{t+1} = -\delta_0(\lambda + \lambda^2)z_{t-1} + \delta_1({}_{t-1}p_{t+2} + {}_{t-1}p_{t+1}) - \delta_2({}_{t-1}p_{t+1} + {}_{t-1}p_t).$$

Use the price markup equation (2) to eliminate the price terms from (A3) and substitute the result into the supply equation (1). After some rearrangement, this becomes

$$\text{(A4)} \qquad (\gamma\delta_1)\,{}_{t-1}x_{t+2} + [2+\gamma(2\delta_1 - \delta_2)]\,{}_{t-1}x_{t+1} - [4+\gamma(2\delta_2 - \delta_1)]\,{}_{t-1}x_t + (2-\gamma\delta_2)x_{t-1} - 4(x_t - {}_{t-1}x_t) = 2\gamma(\lambda + \lambda^2)\delta_0 z_{t-1}.$$

For (A4) to hold, $x_t - {}_{t-1}x_t$ must be identically zero. It follows from Driskell and Sheffrin (1986) that the polynomial

$$\text{(A5)} \qquad (\gamma\delta_1)a^3 + [2+\gamma(2\delta_1 - \delta_2)]a^2 - [4+\gamma(2\delta_2 - \delta_1)]a + (2-\gamma\delta_2)$$

has exactly one stable and two unstable roots. Let the unique stable root be a_1. Since $z_t \sim$ AR(1), it follows that solving the unstable roots forward, the stable root backward leads to a solution of the form $x_t = a_1 x_{t-1} + b_1 z_{t-1}$. One can solve for b_1 by using $x_t = a_1 x_{t-1} + b_1 z_{t-1}$ to compute ${}_{t-1}x_{t+2}, {}_{t-1}x_{t+1}$ and ${}_{t-1}x_t$, and putting these into (A4). (The exact formula is not of interest.)

Let $\alpha \to \infty$. (The solution for y_t is the same whether one uses the present technique of solving for y_t using the x_t process that results for $\alpha \to \infty$, or solves for y_t for finite α and then lets $\alpha \to \infty$.) Then $\delta_0, \delta_1, \delta_2 \to \delta$ and (A5) reduces to

$$(a-1)[\gamma\delta a^2 + (2+2\gamma\delta)a - (2-\gamma\delta)].$$

Since $|a_1| < 1$ for finite α, then, by continuity, as $\alpha \to \infty$, a_1 approaches the stable root of $\gamma\delta a^2 + (2+2\gamma\delta)a - (2-\gamma\delta)$. This is $\underline{a} \equiv -(1+\gamma^{-1}\delta^{-1}) + \gamma^{-1}\delta^{-1}(4\gamma\delta + 1)^{1/2}$. Also $b_1 \to b$, where b is given in equation (6). Equation (7) now follows.

It is perhaps worth noting that one can derive the same result concerning near random-walk behavior of y_t by letting $\lambda = 1$, but (a) assuming that the α in equation (A1) is finite (but large), or (b) letting $\alpha \to -\infty$ rather than $\alpha \to +\infty$.

REFERENCES

Blanchard, Olivier J. and Quah, Danny, "The Dynamic Effects of Aggregate Demand and Supply Disturbances," unpublished manuscript, MIT, 1987.

Campbell, John Y. and Mankiw, N. Gregory, "Are Output Fluctuations Transitory?," unpublished manuscript, 1986.

______ **and** ______, "International Evidence on the Persistence of Economic Fluctuations," unpublished manuscript, Harvard University and Princeton University, 1987.

Clark, Peter K., (1986a) "The Cyclical Component of U.S. Economic Activity," unpublished manuscript, Stanford University, 1986.

______, (1986b) "The Shape of U.S. Business Cycles," unpublished manuscript, Stanford University, 1986.

Cochrane, John H., "How Big Is the Random Walk in GNP?," unpublished manuscript, University of Chicago, 1986.

Deaton, Angus, "Life Cycle Models of Consumption: Is the Evidence Consistent with the Theory?," NBER Working Paper No. 1910, 1986.

De Long, J. Bradford and Summers, Lawrence H., "Is Increased Price Flexibility Destabilizing?," *American Economic Review*, December 1986, *76*, 1031–44.

Dickey, David A. and Fuller, Wayne A., "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econo-*

metrica, June 1981, *49*, 1057–72.

Driskell, Robert A. and Sheffrin, Steven M., "Is Price Flexibility Destabilizing?," *American Economic Review*, September 1986, *76*, 802–07.

Fama, Eugene F. and Gibbons, Michael, "Inflation, Real Returns and Capital Investment," *Journal of Monetary Economics*, May 1982, *9*, 297–323.

Friedman, Benjamin M., "The Inefficiency of Short-Run Monetary Targets for Monetary Policy," *Brookings Papers on Economic Activity*, 1:1977, 293–335.

Fuller, Wayne A., *Introduction to Statistical Time Series*, New York: Wiley & Sons, 1976.

King, Robert, Plosser, Charles, Stock, James and Watson, Mark, "Stochastic Trends and Economic Fluctuations," NBER Working Paper No. 2229, 1987.

Mankiw, N. Gregory, "The Optimal Collection of Seignorage: Theory and Evidence," unpublished manuscript, Harvard University, 1986.

______, **Miron, Jeffrey A. and Weil, David N.**, "The Adjustment of Expectations to a Change in Regime: A Study of the Founding of the Federal Reserve," *American Economic Review*, June 1987, *77*, 358–74.

McCallum, Bennett T., "A Note on John Taylor's Macroeconomic Model," unpublished manuscript, Carnegie-Mellon University, undated.

______, (1986a) "On 'Real' and 'Sticky Price' Theories of the Business Cycle," *Journal of Money, Credit, and Banking*, November 1986, *28*, 397–414.

______, (1986b) "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine," *Journal of Monetary Economics*, January 1986, *17*, 135–60.

______, "Inflation: Theory and Evidence," unpublished manuscript, Carnegie-Mellon University, 1987

Nelson, Charles R. and Plosser, Charles I., "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics*, September 1982, *10*, 139–62.

Quah, Danny, "What Do We Learn from Unit Roots in Macroeconomic Time Series?," unpublished manuscript, MIT, 1986.

Rose, Andrew K., "The Autoregressivity Paradox," unpublished manuscript, University of California-Berkeley, 1987.

Sachs, Jeffrey, "The Changing Cyclical Behavior of Wages and Prices: 1890–1976," *American Economic Review*, March 1980, *70*, 78–90.

Sargent, Thomas J., *Macroeconomic Theory*, New York: Academic Press, 1979.

Stock, James H. and Watson, Mark W., "Does GNP have a Unit Root?," *Economic Letters* 1986, *22*, 147–51.

Stultz, Rene M. and Wasserfallen, Walter, "Macroeconomic Time Series, Business Cycles, and Macroeconomic Policies," in K. Brunner and A. Meltzer, eds., *Understanding Monetary Regimes*, Vol. 22, *Carnegie-Rochester Conference Series on Public Policy*, *J. Monet. Econ.*, Suppl. 1985, 9–55.

Taylor, John B., (1980a) "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, February 1980, *88*, 1–23.

______, (1980b) "Output and Price Stability: An International Comparison," *Journal of Economic Dynamics and Control*, February 1980, *2*, 109–32.

______, "What Would Nominal GNP Targeting Do to the Business Cycle?," in K. Brunner and A. Meltzer, eds., *Understanding Monetary Regimes*, Carnegie-Rochester Conference Series on Public Policy, *22*, Spring 1985, *J. Monet. Econ.* Suppl., 61–84.

Walsh, Carl, "In Defense of Base Drift," *American Economic Review*, September 1986, *76*, 692–700.

Watson, Mark W., "Univariate Detrending Methods with Stochastic Trends," *Journal of Monetary Economics*, July 1986, *18*, 49–76.