A STANDARD MONETARY MODEL AND THE VARIABILITY OF
THE DEUTSCHEMARK-DOLLAR EXCHANGE RATE

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Received January 1986, revised version received December 1986

This paper uses a novel test to see whether the Meese (1985) and Mohn (1985) models are consistent with the variability of the deutsche mark-dollar exchange rate 1974-84. The answer, perhaps surprisingly, is yes. Both models, however, explain the month-to-month variability as caused by a critical way from unobservables shocks to money demand and purchasing power parity. It would therefore be of interest in future work to model one or both of these shocks as explicit functions of economic variables.

I. Introduction

The implications of rationality and market efficiency for the variability of floating exchange rates have long been debated. Some thirty years ago, Friedman (1953) argued that speculation in a free market would stabilize exchange rates. Others [Viner (1956), cited in Solmen (1969)]: argued, at least implicitly, that this might not be the case. The observed fluctuations of exchange rates in recent years do not appear to have created a consensus view. Some believe these fluctuations consistent with rational responses to news about basic economic variables [Frenkel (1981), Frenkel and Musser (1980)]; others are doubtful [Haug (1981), Meese (1985)].

Formal evidence on whether exchange rates are in some well-defined sense exclusively variable is of interest for two reasons. The first, and perhaps more obvious, is that ifsofar as excess variability is prima facie evidence of market inefficiency, the implications for economic policy may be profound. See, for example, Tobin (1958). The second reason, emphasized by Shiller (1985) in connection with stock market studies, is that variability tests can produce very useful diagnostics. A rejection of a model by a variability test may provide guidance for future research: if a model cannot explain the variability

* I thank Wing T. Wong and the participants at the 1985 NBER Summer Institute on The Determinants of Asset Price Variability for helpful comments and discussion, an anonymous referee for an exceptionally careful and detailed report, and the National Science Foundation for financial support. Responsibility for remaining errors is mine. This paper was revised while I was a National Fellow at the Hoover Institution.

of exchange rates, then clearly in future research we should look for factors that will make exchange rates variable.

This paper extends the variability test I developed and applied to stock market data in West (1986). I evaluate whether the variability of the dollar-deutschmark exchange rate is consistent with the monetary models developed in Meese (1985) and Woo (1985). The answer, perhaps surprisingly, is yes. The 1974-84 variability in this exchange rate is consistent with these models. The shocks to money demand and purchasing power parity assumed present by Meese and Woo play a key role in this result. If the shocks are instead assumed absent, as in, for example, Huang (1981), the models are no longer consistent with the 1974-84 variability. The models therefore explain the month-to-month fluctuations in the deutschmark-dollar exchange rate as responses not only to news about basic economic variables, but also to shocks to money demand and purchasing power parity. Such an explanation certainly is logically coherent. But it is in my opinion not completely satisfactory, at least insofar as exchange rates are plausibly thought to move mainly in response to news about basic economic variables. It therefore would be of interest in future work using a monetary model to model one or both of these shocks explicitly, as functions at least in part of observable economic variables. This applies especially to shocks to purchasing power parity, which are in either model the entire explanation of deviations from purchasing power parity. Further work on sticky price models such as Driskell (1981) and Frankel (1979) are therefore of interest.

The extent to which the apparent consistency of the models with the variability of exchange rates may be considered evidence against irrationalities, inefficiencies and speculative bubbles is limited at best to the extent one believes the models correctly explain the exchange rate. Given the well-documented difficulty in developing structural exchange rate models [Meese and Rogoff (1983a, 1983b)], most economists, including me, would probably be hesitant to endorse without reservation any structural model, even one as carefully developed as Meese's or Woo's. Consequently, I do not believe a strong case can be made that the results here argue against speculative bubbles or against the notion that exchange rate models should take into account potential shifts in policy that do not occur [see Flood and Hodrick (1986) and Obstfeld and Rogoff (1985) on this important point]. Instead, the results have the natural interpretation of providing a constructive suggestion about future exchange rate modelling, as described in the previous paragraph. In this connection, it is worth emphasizing that while the estimation technique, sample period and data used here are different than in Meese and Woo, the models are precisely as in those papers. The models are presented only briefly and somewhat uncritically. In particular, no attempt is made to argue for either Woo or Meese when the two make contradictory assump-
tions (for example, whether secular drift is deterministic or stochastic; My aim is to establish a robust result. More extensive discussion of the models, as well as references to similar models, may be found in the original Mese and Wool papers.

The plan of the paper is as follows. Section 2 reviews the models and develops the variability test. Section 3 presents empirical results. Section 4 has conclusions. An appendix has some technical details.

2. The exchange rate models

Two models are used. those of Wool (1985) and Mese (1985). Both models combine a money demand equation, an interest parity condition, and a purchasing power parity condition. The observable shocks that Wool and Mese add to certain of the equations are temporarily suppressed for expositional ease; these shocks will be restored later in this section.

In Wool, U.S. and German money demand are given by

\[
m_t^m - p_t^m = a_0 + a_1 y_t^m + a_2 m_{t-1}^m - p_{t-1}^m, \tag{1}
\]

\[
m_t^g - p_t^g = a_0 + a_1 y_t^g + a_2 m_{t-1}^g - p_{t-1}^g, \tag{2}
\]

where \( m \) is the log of the money stock, \( p \) the log of the price level, \( y \) log income, and \( i \) a nominal interest rate. The \( a_j \)’s are positive parameters with \( a_2 \) less than one. \( \gamma \) ‘s superscript denotes the United States, an ‘1’ Germany. Wool (1985, pp. 2-3) states that direct tests of this money demand specification suggest that it is satisfactory, at least for 1974-81. To make it less likely that the basic results of this paper are explained by a shift in money demand during the larger sample period used here (1974-84), the empirical work applies the variability test to a subsample that falls within the 1974-81 period.

Subtracting (2) from (1) gives:

\[
m_t - p_t = a_0 + a_1 (y_t^m - y_t^g) + a_2 m_{t-1}^m - m_{t-1}^g, \tag{3}
\]

where \( m = m_t^m - m_t^g \). \( p = p_t^m - p_t^g \). Mese uses a special case of (1)-(3), setting \( a_1 = a_2 = 0; \)

\[
m_t - p_t = -a_0 (y_t^m - y_t^g) + \gamma_t, \tag{3'}
\]

where \( \gamma_t = p_t^m - p_t^g \).

In both models, uncovered interest parity is assumed to hold:

\[
E_t y_{t+1} - b_t = q_t - \gamma_t, \tag{4}
\]
where \( x_{t+1} \) is the log of the spot rate (dollars per deutsche mark) and \( E_t \) denotes the market’s expectation conditional on the market’s period \( t \) information. There is considerable evidence against (4) [Hansen and Hodrick (1983), Hodrick and Srivastava (1984)]. It seems reasonable, nonetheless, to maintain (4), at least when one wants to explain the sources of fluctuations in exchange rate movements. This is because it is plausible that the variance of deviations from uncovered interest parity is small compared to the variance of the left-hand side of (4). The arguments in Frankel (1985, pp. 211–215) suggest that small deviations are to be expected a priori, at least in Frankel’s portfolio balance model, and the low \( R^2 \)’s in even the unconstrained regressions in Hansen (1983) and Hodrick and Srivastava (1984) are consistent with this.

Finally, purchasing power parity (PPP) is assumed to hold:

\[
x_t = \rho x_{t-1}
\]

(5)

PPP certainly does not hold instantaneously, as assumed in (5), nor, perhaps, even in the long run. A suitable disturbance will be added to (5) below to provide a more realistic relation between the exchange rate and relative price levels.

A solution of the model requires substitution of (4) and (5) into (3) or (3) to eliminate \( \zeta_t - \zeta_{t-1}, \zeta_t \) and \( x_{t-1} \). Rearranging terms gives:

\[
a_0 E_t x_{t+1} = (1 + \alpha_0) x_t + a_2 x_{t-1} - a_1 \zeta_t - a_1 \zeta_{t-1} + a_0 m_{t-1},
\]

(6)

\[
a_0 E_t x_{t+1} = (1 + \alpha_0) x_t - m_t + a_1 \zeta_t
\]

(6)

A solution of eq. (6) is found as follows. Let \( \mathcal{L} \) denote the lag operator. Since \( a_2 > 0, 0 < a_0 < 1 \), the polynomial \( a_0[1 - (1 + 2a_0a_2)2\mathcal{L} + a_2a_1^2(1 - 2\mathcal{L})] \) may be factored as \( a_0(1 - \gamma \mathcal{L})[1 - (1/\gamma)2\mathcal{L}] \), where \( 0 < \gamma < 1 < 1/\gamma = \{(1 + a_0 + [(1 + a_0)^2 - 4a_0a_2])1/2/2a_0 \}. \) Solve the stable root \( \gamma \) backwards, the unstable root \( \lambda \) forwards to obtain:

\[
x_t = x_{t-1} + \rho E_t \left( \sum_{j=0}^{\infty} \lambda^j \zeta_{t+j} \right) = x_{t-1} + \rho \zeta^*.
\]

(7)

where \( \zeta = \frac{a_0}{2\lambda^2}(m_t - a_1 \zeta_{t-1}) \).

A solution to (6) is a special case of the solution to (7), with \( \gamma = 0 \) and the discount factor \( \lambda = (1 + a_0)^{-1} \lambda \), (say) \( \lambda \):

\[
x_t = E_t \left( \sum_{j=0}^{\infty} \lambda^j \zeta_{t+j} \right) = E_t \zeta^*.
\]

(7)

where \( \zeta = \frac{a_0}{2\lambda^2}(m_t - a_1 \zeta_{t-1}) \).
The variability test requires calculation of the variance of the innovation to the expected present discounted value of fundamentals, that is, the variance of the innovation to $E_{t^*}x_t$ or $E_{t^*}y_t$. This variance must be calculated relative to two information sets, the market's and another set $H_t$ or $H_{t^*}$. $H_t$ is an information set consisting of all of the fundamentals' variables $m_t$, $y'_t$ and $j'_t$; $H_{t^*}$ is the same for $m_t$ and $j_t$. The basic inequality exploited in this paper is

\[\sigma_t^2 = \mathbb{E}(E_{t^*}x_t - E_{t-1}x_t^2) \leq \mathbb{E}(E_{t^*}x_t^2 | H_{t-1})^2, \tag{8}\]

\[\sigma_{t^*}^2 = \mathbb{E}(E_{t^*}y_t - E_{t-1}y_t^2) \leq \mathbb{E}(E_{t^*}y_t^2 | H_{t-1})^2. \tag{8'}\]

It is shown in eq. (9) below that $\sigma_t^2$ is just the variance of the innovation in the exchange rate, under the model (7). The same is true for $\sigma_{t^*}^2$ under the model (7). The inequalities are established in West (1986). They say that forecasts made with a subset of the market's information set have a larger innovation variance than actual forecasts.

One may use (8) to test the model (7) as follows. From (7):

\[E[(x_t - E_{t-1}x_t)^2] = E[(x_t - m_t + E_{t-1}x_t - E_{t-1}x_t^2)^2]
\]

\[= E(E_{t^*}x_t^2 - E_{t-1}x_t^2)^2 = \sigma_t^4. \tag{9}\]

The left-hand side of (8) is thus simply the variance of the innovation in the exchange rate. One way to estimate this left-hand side is then as follows. Begin by rewriting (6) as

\[s_t - m_t = (1 + a_1)[a_0(E_{t-1}x_t - m_0) - a_1s_{t-1}] + n_t. \tag{10}\]

Write (10) in estimable form, by following McCallum (1976) and replacing the unobservable expectation $E_{t-1}x_t$ with the ex post value $s_{t-1}$:

\[s_t - m_t = (1 + a_0)[a_0(E_{t-1}x_t - m_0) - a_1s_{t-1}] + n_{t+1}. \tag{11}\]

where \(\mu_{t+1} = -b(s_{t+1} - E_{t+1}x_t)\) and, as in eq. (7), \(b = (1 + a_0)^{-1}a_0\). Eq. (11) may be estimated by instrumental variables. Potential instruments include current and lagged values of all the right-hand-side variables except $s_{t+1}$, which is not a legitimate instrument since $s$ is correlated with the disturbance. One can retrieve parameters of interest by simple arithmetic on the regression coefficients. For example, $\hat{\mu}_t = \hat{\beta}_1(1 - \hat{\beta})$, where $\hat{\beta}_1$ is the e-
simulated coefficient on \( y \). More importantly, one can obtain an estimate of the left-hand side of (8) using \( \Delta^2 \hat{\sigma}^2 \).

Inference about the estimates of eq. (11) will be difficult if, as is assumed in Moese, the variables have unit roots. In this case a differenced version of (11) may be used:

\[ \Delta u_t = a_1 \Delta u_{t-1} + b(\Delta m_{t-1} - a_3 y_{t-1}) + \hat{\mu}_{t-1}, \quad (11) \]

where \( \hat{\mu}_{t-1} = -b(\Delta m_{t-1} - \hat{E}_{t-1} y_{t-1}) - \hat{\sigma}^2 \).

The restrictions \( a_3 = a_1 \) and \( a_2 = 0 \) have been imposed, in accordance with Moese. Eq. (11) is written in a fashion convenient for estimation when the income elasticity \( a_1 \) is imposed a priori, as was done in Moese and in the empirical work here. Lags of \( \Delta m_{t-1} \) and \( \Delta u_{t-1} \) may be used as instruments to obtain estimates of \( b \) and thus \( \hat{\mu}_{t-1} \). The left-hand side of (8) may now be estimated as \( 0.5b^2 \hat{\sigma}^2 \), where \( \hat{\sigma}^2 = \hat{E}_{t-1} \).

The right-hand side of (6) [or (8)] may be calculated from estimates of the multivariate process followed by the fundamentals' variables \( m_{t-1} \) and \( y_{t-1} \) (or \( \Delta m_{t-1} \) and \( \Delta y_{t-1} \)). The desired variance is an extremely complicated function of \( \lambda \) (or \( b \)), the multivariate ARIMA parameters and the variance-covariance matrix of the multivariate innovations. Details are given in the appendix.

In summary, for the models (7) and (7'), one tests:

\[ 0 \leq E(\hat{\Sigma}_{t-1}^{2} | H_{t-1}, \hat{\sigma}^2 | H_{t-1}) = b^2 \hat{\sigma}^2, \quad (12) \]
\[ 0 \leq E(\hat{\Sigma}_{t-1}^{2} | \hat{R}_{t-1}, \hat{\sigma}^2 | \hat{R}_{t-1}) = 0.5b^2 \hat{\sigma}^2, \quad (12') \]

If this inequality does not hold, the model (7) [or (7')] is not correct. Some factor or factors left out of the model are making exchange rates too variable to be consistent with the model. Two possibilities not necessarily mutually exclusive, are considered here. The first is that the left-out factors are rational bubbles, and the second is that they are the usual unobservable regression disturbances.

Consider bubbles first. These are otherwise extraneous variables that are added to the solution (7) [or (7')] that still yield an exchange rate process that satisfies eqs. (6) and (11) [or (6) and (11')]:

\[ s_{t} = \gamma_{t-1} + \hat{E}_{t-1} y_{t-1} + C_{t}, \quad (13) \]
\[ s_{t} = \hat{E}_{t-1} y_{t-1} + C_{t}. \quad (13') \]

The variable \( C_{t} \) is a bubble, and follows the stochastic process \( E_{t-1} C_{t} = \)
Example of stochastic processes for $C_t$ may be found in Blanchard and Watson (1982) and West (1985). It is easy to verify that adding $C_t$ to (7) [or (10)] yields a process for $x_t$ that satisfies (6) [or (9)]. If (13) is correct, $x_{t+1} - E_{t} x_{t+1} = \left[ \frac{1}{\sigma^2} \right] (E_{t} x_{t} - E_{t} x_{t}^2) = C_{t+1} - E_{t} C_{t+1} = \delta x_{t+1} + \alpha_s + \epsilon_{t+1}$, where $\alpha_s$ is the innovation in $C_{t+1}$. So $E_{t} x_{t+1} - E_{t} x_{t+1}^2 = \sigma^2 + \delta^2 + \sigma^2$. Now, it is sometimes argued that financial markets tend to overreact to news about fundamentals, causing asset prices to jump excessively upon good news about fundamentals and to fall excessively upon bad news [Shiller (1984)]. If this overreaction is due to rational bubbles, this means that bubbles are positively correlated with fundamentals, i.e. $\alpha_s > 0$. In the presence of bubbles, then, it is plausible that $E_{t} x_{t+1} - E_{t} x_{t+1}^2 = \sigma^2$. That is, $E_{t} x_{t+1} - E_{t} x_{t+1}^2$ is larger than the variance of news about fundamentals. This would explain a failure of (12) to hold. The same applies to (12). Under the null hypothesis of no bubbles, (12) [or (12)] of course does hold, since in this case $x_{t+1} = x_{t+1} - E_{t} x_{t+1}$.

A second factor that might explain excess variability of the exchange rate is that $x_t$ is influenced not by a stochastic bubble, but by a disturbance of the sort often assumed present in regression equations. If a random shock $u_t$ is added to (11) and (17), the equations become:

$$ s_t - m_t = \left[ 1 + a_2 \right] \left[ \left[ a_2 (s_{t-1} - m_{t-1}) - a_2 \delta x_{t-1}^2 + a_2^2 \delta x_{t-1}^2 - a_2 (m_{t-1} - s_{t-1}) \right] + \eta_{t+1} \right] + \eta_{t+1}, $$

$$ d_t - dm_t = a_1 \left[ \left( d_{t-1} - dm_{t-1} \right) + \tilde{a} \right] + \eta_{t+1}, $$

(14)

(14')

where $\eta_{t+1} = \eta_{t+1} + u_t$, $\tilde{a}_{t+1} = \tilde{a}_{t+1} + u_t$.

Suppose, as in Meese and one of Woo's specifications, that $u_t$ is white noise. Woo assumes that the $x_t$ in (14) results from a white noise disturbance to the money demand equation (3). One can assume more generally that the $u_t$ in (14) also reflects the sluggish deviations from PPP that are observed empirically. Meese assumes that the $u_t$ in (14) results from a random walk disturbance to the PPP equation (5). One can again assume something more general, namely that in (14) $u_t$ also reflects a random walk disturbance to the money demand equation (3). A white noise shock to (14) and (14'), then, is

(1) It is appropriate to add a word on the theoretical question of whether bubbles are consistent with rationality, in light of the claims by Obstfeld and Rogoff (1985) and DiBos and Grossman (1985) that they are not. The most rigorous and general plane that I am aware of that deals with this question is Tirole (1985). Tirole establishes that bubbles are perfectly consistent with rationality in a weighted overlapping generations model, under realistic conditions. That DiBos and Grossman (1985) and Obstfeld and Rogoff (1985) find bubbles inconsistent with their models appears to reflect the particular characteristics of the models they use rather than any general presumption against bubbles.
consistent with the sort of money demand and PPP disturbances that appear to be observed empirically.\footnote{The disturbance \( \eta \) is a linear combination of a shock to the money demand equation (3), say \( \delta_\eta \), and a shock to the PPP equation (5), say \( \delta_p \). In principle, \( \delta_\eta \) could depend on a shock to the interest parity condition (4) as well. But as far as I know, such a shock has not been assumed present in previous empirical work. Simple arithmetic yields \( \eta = (1 + \rho_\eta) \cdot \delta_\eta + \rho_\eta \cdot \delta_p \). For \( \rho_\eta \) to be white noise, \( \delta_\eta \) requires that \( \rho_\eta \) be white noise with \( \rho_\eta \) be AR(1) with parameter \( \rho_\eta \). For \( \rho_\eta \) to be white noise in (4) requires that \( \rho_\eta \) and/or \( \rho_p \) be a random walk. These requirements appear to be roughly consistent with existing empirical evidence. See, for example, \text{Goldfeld} (1976) or \text{Mankiw and Summers} (1984) on the disturbances to the money demand equation. See \text{Adler and Lehmann} (1983), \text{Hakko} (1984) and \text{Roth} (1979) for evidence that deviations from PPP have a serial correlation coefficient quite near one. Since \( \rho_\eta \) also appears to be quite near one \( \text{Goldfeld} (1976) \), \text{Woo} (1985) and the estimates presented here, the assumption that \( \rho_\eta \cdot \rho_p \) is white noise is probably reasonable.}

The composite disturbances \( \tilde{\eta}_{t-1} \) and \( \overline{\eta}_{t-1} \) are both MA(1). This means that current \( m_t \) should not be used as an instrument, since it is correlated with \( \tilde{\eta}_t \). One also cannot use current values of other variables as instruments, insofar as money is determined simultaneously with these variables in equilibrium [\text{Hodrick} (1979)]. In any case, with suitable lags of variables as instruments, (14) and (14) can be estimated. Note that the estimates are consistent in general under plausible identifying assumptions (e.g. that there are predetermined variables that shift the money supply but that do not appear in money demand). This is true whether or not the exchange rate and the money supply are endogenous, in either the sense of Granger causality or the usual simultaneous equations sense.

The solutions to (14) are

\[
s_t = \gamma_{t-1} + \sum_{j=0}^{\infty} \beta^j u_{t+j} = \gamma_{t-1} + \sum_{j=0}^{\infty} \beta^j (1 - \beta)^{-1} u_t,
\]

where \( U_t \) is a random walk shock whose innovation is \( u_t \). Our aim is to use inequalities (8) and (8) to see whether we must resort to bubbles to explain the variability of exchange rates. This will turn out to be much more complicated than when the usual regression disturbance is assumed absent. With \( u_t \) present in (14) and (14), a violation of eq. (12) or (12) can no longer be taken as evidence of bubbles. This is
because even in the absence of bubbles $\sigma_2^2$ and $\sigma_3^2$ will depend not only on the variance of news about fundamentals but also on the variance of $v_t$ and on the covariance between $u_t$ and the news about fundamentals.

Nevertheless, inequalities (8) and (8') can still be used to test for bubbles. The basic idea for the Woo specification is as follows: details are in the appendix. Under the null hypothesis of no bubbles, the two nonzero moments of the MA(1) disturbance $\eta_{t-1} = \alpha \eta_{t-1} + \varepsilon_t$ depend on the three unknowns $\sigma_1^2$, $\alpha$, and $\sigma_3^2$. The two nonzero moments can be combined with a third piece of information to put bounds on the three unknowns, including, in particular, $\sigma_1^2$. The Cauchy-Schwarz inequality, which states that $(\alpha^2 + \sigma_3^2) \leq \sigma_1^2$, is this third bit of information. That is to say, $\alpha^2 \leq \sigma_1^2 + \sigma_3^2$, and the Cauchy-Schwarz inequality suffice under the null hypothesis of no bubble to identify an upper and lower bound to $\sigma_1^2$. They do not, unfortunately, suffice to identify a point estimate of $\sigma_1^2$. Similarly, in the Messe specification, an upper and lower bound to $\sigma_2^2$ can be identified from the moments of $\eta_t$ and the Cauchy-Schwarz inequality.

Even with a $u_t$ shock present, the right-hand side of (8) or (8') can be calculated as before, as a complicated function of the parameters of the multivariate ARIMA process followed by the fundamentals’ variables. In the presence of a white noise disturbance $u_t$, then, one can compare the lower bound estimates of $\sigma_2^2$ or $\sigma_3^2$ to the calculated value of the right-hand side of (8) or (8'). In the absence of bubble, this lower bound should satisfy (8) or (8').

Before turning to the empirical results, it is important to note two aspects of the procedure that might not be immediately obvious. The first relates to the procedure’s implicit assumption that the estimates of the ARIMA process for the fundamentals yield an accurate estimate of the right-hand side of (8) and (8'). One circumstance in which this will probably not be the case is when this process has shifted during the sample used in estimation or has been expected by the market to shift during or after the sample. This will happen if there are changes in policy rules [Flood and Hoptick (1986), Obstfeld and Rogoff (1990)]. This very real possibility is difficult (at least for me) to incorporate into the null. A partial solution is to obtain separate estimates for different sample periods if there is theoretical or empirical evidence of a non-sample process shift. This will not, however, help if agents expected a shift that did not or has yet to occur. Consequently, a rejection of the null can be interpreted equally well as evidence of bubbles or as evidence of expected or actual shifts of the fundamentals process.

The second feature to note is that as long as the ARIMA process is stable, the procedure is legitimate whether or not there is feedback from the exchange rate or other variables to the fundamentals’ variables. Inequalities (8) and (8') hold so long as money and real income follow and are expected to follow a stable process. Any other variables that help determine money
and real income in equilibrium have been implicitly solved out in the process of forecasting money and income.

3. Empirical results

3.1. Data

The raw data were monthly and seasonally unadjusted, 1974:1 to 1984:5. Data from 1973 and 1984:6 were used for lags and leads. Data on industrial production, money stock (M1) and the spot exchange rate (dollar- deutschmark) were kindly supplied by Richard Meese; a detailed description of this data set may be found in Meese and Rogoff (1983b).

The raw data appeared to require some transformations to induce stationarity. It is well known that detrending and differencing a variable are not asymptotically equivalent, whether the variable's secular drift is deterministic or stochastic [Nelson and Plosser (1982)]. Rather than get sidetracked into analysis of the source of the pronounced upward movement of some of the variables (especially \( y_t \) and \( y_{t-1} \)), I decided to handle such apparent non-stationarity as did Woo and Meese. The actual data used in my test of the Woo specification therefore were the residuals from a regression of levels of variables on seasonal dummies and a linear trend, because Woo assumed that secular drift is deterministic. The data used in the Meese specification were the residuals from a regression of differences of variables on seasonal dummies, because Meese assumed that secular drift is stochastic.1 Separate detrending regressions were run for each of the subsamples described below.

Since all estimation was linear, the estimates of regression coefficients are identical to those that would have been obtained had the trend and seasonal terms been included in the regressions. These preliminary regressions were done to cut down the otherwise enormous size of the variance-covariance matrix of the parameters.

Estimates were obtained for 1974:1 to 1984:5, and for two subsamples as well, 1974:1 to 1979:9 and 1979:10 to 1984:5. The subsample estimates were obtained because, as noted in the previous section, the procedure for estimating the requisite innovation variances tacitly assumes that the fundamentals variables follow a stable ARIMA process over the entire sample period, and there is some evidence that they did not. The tests in Meese (1985), for example, suggest that the Fed's October 1979 change in operating procedures resulted in a shift to the ARIMA process of \( \delta m \) and/or \( \delta y \). Woo (1985), on the other hand, found that the Fed's change did not result in such

1Note that it follows from eqs. (11) and (15) that under the null hypothesis of no hubble, the endogenous drift in the exchange rate (if any) will be deterministic under Woo's assumptions about shocks and fundamentals' variables. Similarly, the exchange rate has a unit root under Meese's assumptions.
a shift. While neither paper uses precisely my sample period nor my specification for the fundamentals process, and each uses different (seasonally adjusted) data, the data are similar enough that the hypothesis of stability seems debatable. I therefore also estimated and tested the model using not only the entire sample period, but also the pre- and post-October 1979 subsamples. Note that the use of these subsamples implicitly assumes that the market instantaneously caught on to any such shift by the Fed, and, as noted in the previous section, that the market did not expect such a shift.

3.2. Estimation technique

For the Woo specification, four regression equations were estimated: eq. (14), and a three-variable vector autoregression for the fundamentals variables $m_t$, $y_t$, and $f_t$. The lag length for the autoregression was set at four when the whole sample was used, two when a subsample was used. For the whole sample regressions, there were 12 variables (12 = 3 variables × 4 lags per variable) on the right-hand side of each of the three autoregressions. The corresponding figure for the subsample regressions was six. A shorter lag length was used in the subsamples to preserve degrees of freedom. Diagnostic tests such as $Q$ statistics suggested that the lag lengths were adequate, for the whole sample and both subsamples. Some experimentation, summarized in footnote 9 below, indicated that the results are not sensitive to choice of lag length.

For the Meese specification, three regression equations were estimated: eq. (14), and a two-variable vector autoregression for the fundamentals variables $d_m$, and $d_y$. Lag lengths were chosen as in the Woo specification.

Let $\theta$ denote the vector of parameters that must be estimated to calculate the innovation variances of interest. The vector $\theta$ consists of the coefficients on the right-hand-side variables in (14) or (14'); $E_{t-1} \sigma$, and $E_{t-1} \sigma$, the first and second autocovariances of the disturbance to (14) or (14'), the coefficients on the right-hand-side variables in the fundamentals autoregressions; and the elements of the variance-covariance matrix of the innovations in the fundamentals. In the Woo specification, for example, $\theta$ contains 48 elements, when estimating with the entire sample period: four coefficients on the right-hand side of (14); $E_{t-1} \sigma$, and $E_{t-1} \sigma$, 36 coefficients on the right-hand side of the autoregressions; and the six independent elements of the variance-covariance matrix of the disturbances to the fundamentals' autoregressions.

The elements of $\theta$ were estimated as follows. The right-hand-side variables in (14) and (14') were estimated by two-stage least squares, with the right-hand-side variables of the autoregressions used as instruments. The moments $E_{t-1} \sigma$, and $E_{t-1} \sigma$, were estimated from the moments of the two-stage least squares residuals. The autoregression parameters were estimated by OLS. The elements of the variance-covariance matrix of the autoregression dis-
turbances were estimated from the OLS residuals, with the usual degree of freedom adjustment.

Calculation of the asymptotic covariance matrix of \( \theta \) is described in the appendix. It suffices to make three remarks here. First, the standard errors on the coefficients in (14) and (14) allow for the MA(1) serial correlation that \( u_t \) displays if there is a \( u_t \) shock present. They are, however, still consistent if \( u_t \) is serially uncorrelated when \( u_t \) is absent. Second, standard errors on all regression coefficients were calculated to allow for arbitrary heteroskedasticity conditional on the instruments (i.e., conditional on the right-hand-side variables in the autoregressions). Third, proper account was taken not only of the uncertainty in the estimates of the regression coefficients, but also of (a) the uncertainty in the estimates of the variances and covariances such as \( \text{E}\{ u_t \} \), and (b) the correlation of the estimates of the various elements of \( \theta \).

The innovation variances in eqs. (8) and (8)' are complicated functions of \( \theta \). Let \( f(\theta) \) denote one of these variances. The standard error on \( f(\theta) \) was calculated as \( \left[ \left| f(\theta) \right| f(\theta) \right]^{-1/2} \), where \( V \) is the variance-covariance matrix of \( \theta \). The derivatives of \( f(\theta) \) of all such functions were calculated numerically.

3.3. Empirical results

Table 1 reports the estimates of the basic regression regression parameters.\(^4\)

<table>
<thead>
<tr>
<th>Parameter estimates.</th>
<th>( a_0 )</th>
<th>( a_1^* )</th>
<th>( d_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) 1974:1-1984:5</td>
<td>1.3340</td>
<td>0.1968</td>
<td>0.2851</td>
<td>0.9035</td>
</tr>
<tr>
<td></td>
<td>(0.2122)</td>
<td>(0.1635)</td>
<td>(0.2771)</td>
<td>(0.0922)</td>
</tr>
<tr>
<td>(2) 1974:1-1979:9</td>
<td>0.1759</td>
<td>0.1835</td>
<td>0.3225</td>
<td>1.4410</td>
</tr>
<tr>
<td></td>
<td>(0.2152)</td>
<td>(0.2025)</td>
<td>(0.2383)</td>
<td>(0.1747)</td>
</tr>
<tr>
<td>(3) 1979:10-1984:5</td>
<td>0.5319</td>
<td>0.4944</td>
<td>1.3771</td>
<td>0.8662</td>
</tr>
<tr>
<td></td>
<td>(0.2789)</td>
<td>(0.2119)</td>
<td>(0.4020)</td>
<td>(0.1804)</td>
</tr>
<tr>
<td>Eq. (14)'</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) 1974:1-1984:5</td>
<td>0.4687</td>
<td>0.5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.3291)</td>
<td>(0.3321)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) 1974:1-1979:9</td>
<td>0.4921</td>
<td>0.5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2557)</td>
<td>(0.2557)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) 1979:10-1984:5</td>
<td>0.2379</td>
<td>0.5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.5407)</td>
<td>(0.5407)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Asymptotic standard errors in parentheses. Symbols defined in the text.

\(^4\)The estimates of the fundamentals processes are not reported, to conserve space. An appendix containing these estimates, as well as those of the initial regressions to induce stationarity, is available on request.
Consider first the estimates of (14), in lines (1) to (3). About half the estimates are significantly different from zero at the 5 percent level, and almost all are more or less reasonable. Consider first the interest semielasticity $a_0$. Its estimates vary somewhat from sample to sample, but are roughly consistent with the estimates in Woo and Meese. The estimates of both the U.S. and German income elasticities, $a_1$ and $a_1'$, are also roughly consistent with the slightly higher estimates in Woo. The estimates of $a_2$ are, again, similar to those in Woo. One estimate [line (2)] exceeded its theoretical upper bound of unity, as did one of Woo's estimates [Woo (1985, p. 8)]. Combining $a_1'$ and $a_1$, or $a_1$ and $a_2$, for lines (1) and (3) yields, as in Woo, a somewhat high long-run income elasticity of two or more.

Now consider the estimates of (14) to lines (4) to (6) of table 1. The estimates of $a_0$ were obtained by imposing $a_0 = 0.5$ as did Meese. Results and estimates for $a_1 = 0.4$ and $a_2 = 0.3$, the other two imposed values of $a$, for which Meese reported results, were almost identical. The estimates of $a_0$ are somewhat lower than in Meese but, perhaps, not implausibly so. One of the three estimates is significantly different from zero at the 5 percent level [line (5)].

In sum, then, the regression results suggest that the Woo specification is quite acceptable, the Meese specification less so. In addition, it is reassuring that the use of different sample periods, estimation techniques and, in the case of Woo, different data, lead to qualitatively similar parameter estimates.

Let us now turn to the variability test. Column (1) of table 2 presents the estimates of the right-hand sides of (8) and (8). Column (2) reports the estimate of the variance of $\sigma^2$ if the unobservable shock $u$ is assumed absent. Under the null hypothesis that bubbles are also absent, the column (2) estimate should be less than the column (1) estimate. It is not, for any of the six specifications. Column (2) is anywhere from five to two hundred times as large as column (1). It is significantly larger (at the 5 percent level) in two specifications [lines (1) and (5)]. See column (3). My variability test, inequalities (8) and (8), therefore indicates, as did Huang's (1981), that a standard monetary model with neither bubbles nor the usual regression

---

1Allowing $a_0$ to be estimated freely did not generate similar results. In this case, in two of the three samples, either $a_0$ or $a_1$, or both, were wildly implausible ($a_0 > 100$, or $a_1 = 0$, respectively). Therefore did not even calculate the variability test. It is not clear to me why un constrained estimates were not sensible. A rette has commented that this suggests a specification error in Meese's model, and, citing Cuthbert and Ochfield (1986), has suggested that one possible culprit is Meese's assumption the deviations from PPP are a random walk.

2It may help in interpreting all the figures in table 2 to note that the 1973:3 to 1984:5 variance is $\sigma^2 = 11.53$ (i.e., $10^{-2}$). The in-sample variance of the news about fundamentals (line (1), column (1), thus, is about one-eighth of what would be the out-of-sample error variance from forecasting the spot rate as a random walk.

3For the Meese specification in lines (4) to (6), the column (2) estimate may be calculated as either $-0.36 \sigma_{z_{1}}$ or $-0.36 \sigma_{z_{2}}$. The former is reported in table 2. The latter yields values lower than those in column (2), but still much larger than those in column (1).
Table 2
Variability measures.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^2_{e1}$</td>
<td>$\sigma^2_{e2}$</td>
<td>$\sigma^2_{e3}$</td>
<td>$\sigma^2_{e4}$</td>
<td>$\sigma^2_{e5}$</td>
</tr>
<tr>
<td>R.h.s. of (8)</td>
<td>$\sigma^2_{u1}$</td>
<td>$\sigma^2_{u2}$</td>
<td>$\sigma^2_{u3}$</td>
<td>$\sigma^2_{u4}$</td>
<td>$\sigma^2_{u5}$</td>
</tr>
<tr>
<td>(1)</td>
<td>1.995</td>
<td>75.830</td>
<td>14.934</td>
<td>1.556</td>
<td>0.339</td>
</tr>
<tr>
<td>(2)</td>
<td>2.584</td>
<td>650.400</td>
<td>91.975</td>
<td>1.950</td>
<td>0.856</td>
</tr>
<tr>
<td>(3)</td>
<td>9.665</td>
<td>53.015</td>
<td>33.554</td>
<td>4.856</td>
<td>0.684</td>
</tr>
<tr>
<td>(4)</td>
<td>7.710</td>
<td>55.610</td>
<td>91.060</td>
<td>0.014</td>
<td>0.096</td>
</tr>
<tr>
<td>(5)</td>
<td>2.026</td>
<td>38.438</td>
<td>37.512</td>
<td>3.582</td>
<td>1.345</td>
</tr>
<tr>
<td>(6)</td>
<td>3.113</td>
<td>131.500</td>
<td>126.387</td>
<td>0.216</td>
<td>2.902</td>
</tr>
</tbody>
</table>

Notes: Lines (1)-(3) are based on eq. (14), lines (4)-(6) on eq. (15), as described in the text. Asymptotic standard errors in parentheses. Symbols defined in the text. All figures are times 10$.^9$

Disturbance is inconsistent with the variability of the dollar-deutsche mark exchange rate.$^9$

The monetary models are not, however, inconsistent with the data if one allows for the usual regression disturbance. Column (4) reports the minimum possible value of $\sigma^2$, calculated as described in the appendix. Column (5) reports the difference between columns (1) and (4). With one exception [line (2)], the column (4) estimate is less than the column (1) value. The difference, unfortunately, is estimated rather imprecisely. In only one specification [line (4)] is the point estimate of the difference significantly different from zero at the 5 percent level. It is clear, nonetheless, that once regression disturbances are permitted, one cannot reject the null hypothesis that the variance of the innovation in the expected present discounted value of fundamentals is less when the market's information set is used ($\sigma^2$) than when only past values of fundamentals are used [column (1)]. This result is robust to changes in the lag length of the fundamentals autoregressions.$^9$

$^9$The figures reported in table 2 are based on estimates that used lags of fundamentals' variables as instruments. For lines (1)-(3), different and more efficient estimates of column (4)-

(5) may be obtained when there is no disturbance $\sigma^2$, by using current as well as lagged values of fundamentals as instruments. See the discussion in section 2. So I recalculated column (1) and

(2) using all variables obtained when current as well as lagged variables were used. For all three sample periods, column (2) was greater than column (1).  

$^9$To see whether the results were sensitive to choice of lag length of the fundamentals autoregression, I calculated two additional point estimates (but not standard errors) of each of the table 2 entries. These were for lag lengths $r=2$ and $3$, for the whole sample, $r=3$ and 4 for the subsamples. The Woold specication proved quite robust, with all additional estimates yielding a negative figure for column (3) in table 2, and a positive figure for column (5). The
The consistency does not, in my opinion, mean that the Woo and Meese models capture the variability in an entirely satisfactory fashion. It is often argued that the exchange rate is an asset price and ought to fluctuate as do many asset prices in response to news about economic variables [Frenkel (1981), Frenkel and Musa (1980)]. The empirical results suggest that is the Woo and Meese models these fluctuations result in an important way from shocks that have no explicit links to economic theory even to any economic variables (except, of course, tautologically, to the variables in the equations in which the shocks appear). Therefore, while it undoubtedly is desirable to allow for regression disturbances in exchange rate models, it appears that some nontrivial extensions to the Woo and Meese models are required, if one of these models is to explain the fluctuations basically as responses to news about observable economic variables. This may well be true of other monetary models as well. In any case, it would seem highly desirable to model deviations from PPP as functions, at least in part, of observable economic variables. Sticky price models such as Driskell (1981) and Frankel (1979) may be useful starting points.

3.4. Comparison with previous studies

The basic conclusion of this paper conflicts with previous studies on volatility and speculative bubbles [Huang (1981), Meese (1985)]. A reconciliation with these studies is therefore in order.

Reconciliation with Huang (1981) is simple. Huang followed some studies such as Bilson (1978) and assumed no regression disturbances in any of the basic equations. As was just noted, when this assumption is made here, the result is that the monetary model cannot explain the variability of the exchange rate. Surely, this argues more for allowing for the usual regression disturbance than for a basic failure of the monetary model. See Hodrick (1979) on the theoretical importance of allowing for the usual regression disturbances.

Reconciliation with Meese (1985) is not quite as straightforward. Meese applied to the exchange rate the test for speculative bubbles that I developed and applied to stock prices in West (1985). A general description of my specification test may be found in West (1985). For concreteness I will explain it here in the context of Meese’s application.

The specification test compares two estimates of \( h, b = (1 + \alpha) \gamma \alpha \), as in (7). One estimate of \( b \) is obtained from eq. (14) by instrumental variables, and is consistent even if there are bubbles. The second estimate is obtained

Meese specification was not as robust. Three samples produced implausible parameter estimates, such as negative \( b \) (whole sample, \( r = 2 \), and first subsample, \( r = 2 \) and \( r = 4 \)). The other three samples did, however, yield a positive figure in column (3), and all but \( r = 4 \), second subsample, yielded a negative figure for column (5).
from estimation of two types of equations: a closed form solution to the expected present discounted formula (7), and the fundamentals process. This second estimate is not consistent if there are bubbles. Meese compared the two estimates of $b$ and found them more different than is consistent with sampling error. The implication is that there are bubbles.

There are at least two possible explanations for the conflict between Meese's results and those of the present paper. Both, unsurprisingly, are econometric. The first is that the specification test may have more power. The second is that one test may have better finite sample properties. I suspect that the present paper's test is better in this respect, at least when there are in-sample shifts in the ARIMA process of the fundamentals' variables. This is because such shifts will obscure the link between the ARIMA process and the closed form solution to (7). This will potentially cause a strong bias in the second of the two estimates of $b$ that were described in the previous paragraph. By contrast, although the present variability test requires a fundamentals process that is stable and expected by the market to remain so, there appears to be no presumption that it is biased toward finding excess variability if the process in fact is unstable: both sides of (8) are likely to be estimated quite noisily.

Both of these possible explanations are quite tentative. Some further research is required to reconcile the fact that one of my bubble tests finds bubbles, the other does not.

4. Conclusions

Two basically standard monetary models appear to be consistent with the 1974-84 variability of the dollar-deutschmark exchange rate. As noted in the introduction, the extent to which this consistency may be interpreted as evidence against speculative bubbles or process switching is limited at best to the extent one believes the models correctly explain this exchange rate. But regardless of how enthusiastically one endorses either model, it is of note that shocks to money demand and PPP play a key role in the apparent consistency of the models with the data. It is therefore of interest in future work to model these shocks as functions at least in part of observable economic variables.

Appendix

This appendix describes calculation of: (A.1) the right-hand side of (8); (A.2) a lower bound estimate to $\sigma^2$ when there is a shock $\omega$; and (A.3) the variance-covariance matrix.
A.1. Right-hand side of (8)

Consider first when eq. (14) is the appropriate specification. Let \( A, B = \{ A_{m}\}, \{ A_{k}\} \) follow an \( AR(r) \) process, \( \Phi(B)A_{t} = \Phi_{1}A_{t-1} + \cdots + \Phi_{r}A_{t-r} = \epsilon_{t} \). Each \( \Phi \) is a \( (2 \times 2) \) matrix. Using the formulas in Hansen and Sargent (1980), it may be shown that

\[
E[\sum b_{k}x_{k+1} | \bar{A}_{t}, \bar{E}[\sum b_{k}x_{k+1} | \bar{A}_{t-1}, (1-\rho)^{-1} \Phi(b)^{-1} (x_{t} - \bar{E}[x_{t} | \bar{A}_{t-1}])
\]

where \( \Theta(0) = \Phi(0) + \cdots + \Phi(0)^{b} \). Let \( \bar{z} = \bar{a}_{0}[1, -a_{1}] \).

From eq. (7),

\[
E[\sum b_{k}x_{k+1} | \bar{A}_{t}, \bar{E}[\sum b_{k}x_{k+1} | \bar{A}_{t-1}, \bar{E}[\sum b_{k}x_{k+1} | \bar{A}_{t-1}]]
\]

\[
= (1-\rho)^{-1} \bar{z} \Phi(b)^{-1} \bar{v}_{t},
\]

Thus,

\[
E[E[\sum b_{k}x_{k+1} | \bar{A}_{t}, \bar{E}[\sum b_{k}x_{k+1} | \bar{A}_{t-1}]] = (1-\rho)^{-1} \bar{z} \Phi(b)^{-1} \bar{v}_{t},
\]

When \( x_{t} = \{ m_{t}, \gamma_{t} \} \) follows an \( AR(r) \) process, \( \Phi(L)x_{t} = \epsilon_{t} \), as is consistent with (7), the comparable formula is

\[
E[\sum b_{k}x_{k+1} | H_{t}, \bar{E}[\sum b_{k}x_{k+1} | H_{t-1}]] = (1-\rho)^{-1} \bar{z} \Phi(b)^{-1} \bar{v}_{t},
\]

A.2. Lower bound \( \alpha^{2} \)

The basic procedure for (14) is as follows. The procedure for (14) is similar. We have seen that when there are no bubbles \( u_{t}, E_{t}u_{t-1} = E_{t}u_{t-1} = E_{t}x_{t-1} + \delta_{t-1} u_{t-1} \). It may be shown that the minimum and maximum possible values of \( \alpha^{2} \) occur when \( u_{t} \) and \( \epsilon_{t} \) are perfectly correlated, \( u_{t} \sim \delta_{t} \) for some \( h \). In such a case,

\[
E[u_{t}^{2}] = [E[-(b_{1}u_{t} + b_{1}u_{t-1}) + \delta_{t}^{2}]]^{2}
\]

\[
= [E[-b_{1}u_{t}^{2} + b_{1}^{2}u_{t}^{-1} + \delta_{t}^{2}]] = f_{1}(\alpha^{2}, h),
\]

\[
E[u_{t}^{2}] = [E[-(b_{1}u_{t} + b_{1}u_{t-1}) + \delta_{t}^{2}]]^{2}
\]

\[
= f_{2}(\alpha^{2}, h),
\]
A.3. Variance–covariance matrix

This explains the calculation of the variance–covariance matrix of the parameter vector \( \hat{\theta} \), for the Woot specification. How the matrix was calculated for the Woot specification will be obvious from the description to follow.

Let \( z_{it} = (m_{it}, \ldots, m_{it}, x_{it}, \ldots, x_{it}) \) be the \( (3r \times 1) \) vector of instruments used; \( r = 4 \) for the whole sample, \( r = 2 \) for the subsamples. Write eq. (14) as \( w_{it} = x_{it}' + n_{it}, w_{it} = x_{it} - m_{it} x_{it} \) and \( \beta \) \( (4 \times 1) \) defined in the obvious way. Let \( A = (\sum x_{it} x_{it}' \sum x_{it})^{-1} \) be the usual SLS weighting vector. Write the fundamentals autoregressions as \( m_{it} = z_{it} \beta + e_{it}, y_{it} = z_{it} \beta + e_{it}, \beta = z_{it} \beta + \nu_{it} \).

Finally, let \( T \) be the sample size.

One way of describing the estimation technique used is to note that the \((12 + 9r) \times 1\) parameter vector \( \hat{\theta} \) was chosen to satisfy an orthogonality condition. This orthogonality condition is

\[
0 = T^{-1} \sum j_{it}(\hat{\theta}) = \begin{bmatrix}
T^{-1} \sum z_{it}(x_{it}' - \hat{x}_{it}) \\
\hat{E}_{n_{it}} - T^{-1} \sum (w_{it} - x_{it}' \hat{\beta})' \\
\hat{E}_{y_{it} n_{it}} - T^{-1} \sum (w_{it} - x_{it}' \hat{\beta})(w_{it} - x_{it}' \hat{\beta})' \\
T^{-1} \sum (x_{it}' - \hat{z}_{it} \beta)' \\
T^{-1} \sum (y_{it}' - \hat{z}_{it} \beta)' \\
\end{bmatrix}
\]

As stated in the text, then, \( \hat{\theta} \) is estimated by 2SLS, \( \hat{E}_{n_{it}} \) and \( \hat{E}_{y_{it} n_{it}} \) from the
moments of the 2SLS residuals, $\delta_1$, $\delta_2$ and $\delta_3$ by OLS, $E_\theta \mu_i(\theta, j=1, 2, 3)$ from the OLS residuals with a degrees of freedom correction.

Since $E_h(\theta^2) = 0$, where $\theta^2$ is the true unknown $\theta$, $T^{1/2}(\theta - \theta^2)$ is asymptotically normal with $(12 + 99) \times (12 + 99)$ covariance matrix $V = (T - 1) \Sigma_h^{-1} S^{-1}$ (plim $T^{-1} \sum_h h_{ij}$) [Hansen 1982]. $S$ is $T \times T$ and was straightforward to calculate. $S = \sum_{t=1}^T E_h h_{it}$, and was calculated as in Newey and West (1986), using three lags of $h_{it}$. Newey and West (1986) show that the resulting positive semidefinite estimate of $V$ is consistent, for arbitrary correlation between $\pi_i$ and $\pi_j (j=1, 2, 3)$, and arbitrary heteroskedasticity of $\pi_i$ and $\pi_j$ conditional on the instruments.

References


