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A STANDARD MONETARY MODEL AND THE VARIABILITY OF THE DEUTSCHEMARK–DOLLAR EXCHANGE RATE

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This paper uses a novel test to see whether the Meese (1985) and Woo (1985) models are consistent with the variability of the deutschemark–dollar exchange rate 1974–84. The answer, perhaps surprisingly, is yes. Both models, however, explain the month-to-month variability as resulting in a critical way from unobservable shocks to money demand and purchasing power parity. It would therefore be of interest in future work to model one or both of these shocks as explicit functions of economic variables.

1. Introduction

The implications of rationality and market efficiency for the variability of floating exchange rates have long been debated. Some thirty years ago, Friedman (1953) argued that speculation in a free market would stabilize exchange rates. Others [Viner (1956), cited in Sohmen (1969)] argued, at least implicitly, that this might not be the case. The observed fluctuations of exchange rates in recent years do not appear to have created a consensus view. Some believe these fluctuations consistent with rational responses to news about basic economic variables [Frenkel (1981), Frenkel and Mussa (1980)], others are doubtful [Huang (1981), Meese (1985)].

Formal evidence on whether exchange rates are in some well-defined sense excessively variable is of interest for two reasons. The first, and perhaps more obvious, is that insofar as excess variability is prima facie evidence of market inefficiency, the implications for economic policy may be profound. See, for example, Tobin (1978). The second reason, emphasized by Shiller (1981) in connection with stock market studies, is that variability tests can produce very useful diagnostics. A rejection of a model by a variability test may provide guidance for future research: if a model cannot explain the variability

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of exchange rates, then clearly in future research we should look for factors that will make exchange rates variable.

This paper extends the variability test I developed and applied to stock market data in West (1986). I evaluate whether the variability of the dollar–deutschemark exchange rate is consistent with the monetary models developed in Meese (1985) and Woo (1985). The answer, perhaps surprisingly, is yes. The 1974–84 variability in this exchange rate *is* consistent with these models. The shocks to money demand and purchasing power parity assumed present by Meese and Woo play a key role in this result. If the shocks are instead assumed absent, as in, for example, Huang (1981), the models are no longer consistent with the 1974–84 variability.

The models therefore explain the month-to-month fluctuations in the deutschemark–dollar exchange rate as responses not only to news about basic economic variables, but also to shocks to money demand and purchasing power parity. Such an explanation certainly is logically coherent. But it is in my opinion not completely satisfactory, at least insofar as exchange rates are plausibly thought to move mainly in response to news about basic economic variables. It therefore would be of interest in future work using a monetary model to model one or both of these shocks explicitly, as functions at least in part of observable economic variables. This applies especially to shocks to purchasing power parity, which are in either model the entire explanation of deviations from purchasing power parity. Further work on sticky price models such as Driskell (1981) and Frankel (1979) are therefore of interest.

The extent to which the apparent consistency of the models with the variability of exchange rates may be considered evidence against irrationalities, inefficiences and speculative bubbles is limited at best to the extent one believes the models correctly explain the exchange rate. Given the well-documented difficulty in developing structural exchange rate models [Meese and Rogoff (1983a, 1983b)], most economists, including me, would probably be hesitant to endorse without reservation any structural model, even one as carefully developed as Meese's or Woo's. Consequently, I do not believe a strong case can be made that the results here argue against speculative bubbles or against the notion that exchange rate models should take into account potential shifts in policy that do not occur [see Flood and Hodrick (1986) and Obstfeld and Rogoff (1985) on this important point]. Instead, the results have the natural interpretation of providing a constructive suggestion about future exchange rate modelling, as described in the previous paragraph. In this connection, it is worth emphasizing that while the estimation technique, sample period and data used here are different than in Meese and Woo, the models are precisely as in those papers. The models are presented only briefly and somewhat uncritically. In particular, no attempt is made to argue for either Woo or Meese when the two make contradictory assumptions (for example, whether secular drift is deterministic or stochastic). My aim is to establish a robust result. More extensive discussion of the models, as well as references to similar models, may be found in the original Meese and Woo papers.

The plan of the paper is as follows. Section 2 reviews the models and develops the variability test. Section 3 presents empirical results. Section 4 has conclusions. An appendix has some technical details.

2. The exchange rate models

Two models are used, those of Woo (1985) and Meese (1985). Both models combine a money demand equation, an interest parity condition, and a purchasing power parity condition. The observable shocks that Woo and Meese add to certain of the equations are temporarily suppressed for expositional ease; these shocks will be restored later in this section.

In Woo, U.S. and German money demand are given by

$$m_t^{\mathrm{u}} - p_t^{\mathrm{u}} = -a_0 i_t^{\mathrm{u}} + a_1^{\mathrm{u}} y_t^{\mathrm{u}} + a_2(m_{t-1}^{\mathrm{u}} - p_{t-1}^{\mathrm{u}}), \tag{1}$$

$$m_t^{\mathrm{f}} - p_t^{\mathrm{f}} = -a_0 i_t^{\mathrm{f}} + a_1^{\mathrm{f}} y_t^{\mathrm{f}} + a_2(m_{t-1}^{\mathrm{f}} - p_{t-1}^{\mathrm{f}}), \tag{2}$$

where m is the log of the money stock, p the log of the price level, y log income, and i a nominal interest rate. The a_i's are positive parameters, with a_2 less than one. A 'u' superscript denotes the United States, an 'f', Germany. Woo (1985, pp. 2–3) states that direct tests of this money demand specification suggest that it is satisfactory, at least for 1974–81. To make it less likely that the basic results of this paper are explained by a shift in money demand during the larger sample period used here (1974–84), the empirical work applies the variability test to a subsample that falls within the 1974–81 period.

Subtracting (2) from (1) gives:

$$m_t - p_t = -a_0(i_t^{\mathrm{u}} - i_t^{\mathrm{f}}) + a_1^{\mathrm{u}} y_t^{\mathrm{u}} - a_1^{\mathrm{f}} y_t^{\mathrm{f}} + a_2(m_{t-1} - p_{t-1}), \tag{3}$$

where $m_t = m_t^{\mathrm{u}} - m_t^{\mathrm{f}}$, $p_t = p_t^{\mathrm{u}} - p_t^{\mathrm{f}}$. Meese uses a special case of (1)–(3), setting $a_1^{\mathrm{u}} = a_1^{\mathrm{f}}$, $a_2 = 0$:

$$m_t - p_t = -a_0(i_t^{\mathrm{u}} - i_t^{\mathrm{f}}) + a_1 y_t, \tag{3'}$$

where $y_t = y_t^{\mathrm{u}} - y_t^{\mathrm{f}}$.

In both models, uncovered interest parity is assumed to hold:

$$\mathrm{E}_t s_{t+1} - s_t = i_t^{\mathrm{u}} - i_t^{\mathrm{f}}, \tag{4}$$

where s_{t+1} is the log of the spot rate (dollars per deutschemark) and E_t denotes the market's expectation conditional on the market's period t information. There is considerable evidence against (4) [Hansen and Hodrick (1983), Hodrick and Srivastava (1984)]. It seems reasonable, nonetheless, to maintain (4), at least when one wants to explain the sources of fluctuations in exchange rate movements. This is because it is plausible that the variance of deviations from uncovered interest parity is small compared to the variance of the left-hand side of (4). The arguments in Frankel (1985, pp. 211–215) suggest that small deviations are to be expected a priori, at least in Frankel's portfolio balance model, and the low R^2's in even the unconstrained regressions in Hansen and Hodrick (1983) and Hodrick and Srivastava (1984) are consistent with this.

Finally, purchasing power parity (PPP) is assumed to hold:

$$s_t = p_t. \tag{5}$$

PPP certainly does not hold instantaneously, as assumed in (5), nor, perhaps, even in the long run. A suitable disturbance will be added to (5) below to provide a more realistic relation between the exchange rate and relative price levels.

A solution of the model requires substitution of (4) and (5) into (3) or (3′) to eliminate $i_t^{\mathrm{u}} - i_t^{\mathrm{f}}$, p_t and p_{t-1}. Rearranging terms gives:

$$a_0\,\mathrm{E}_t s_{t+1} - (1+a_0)s_t + a_2 s_{t-1} = -m_t + a_1^{\mathrm{u}} y_t^{\mathrm{u}} - a_1^{\mathrm{f}} y_t^{\mathrm{f}} + a_2 m_{t-1}, \tag{6}$$

$$a_0\,\mathrm{E}_{t+1} s_{t+1} - (1+a_0)s_t = -m_t + a_1 y_t^{\cdot} \tag{6′}$$

A solution of eq. (6) is found as follows. Let L denote the lag operator. Since $a_0 > 0$, $0 < a_2 < 1$, the polynomial $a_0[1-(1+a_0)a_0^{-1}L + a_2 a_0^{-1}L^2]$ may be factored as $a_0(1-\gamma L)[1-(1/\lambda)L]$, where $0<\gamma<1<1/\lambda = \{[1+a_0+[(1+a_0)^2 - 4a_0a_2]^{1/2}\}/2a_0$. Solve the stable root γ backwards, the unstable root λ forwards to obtain:

$$s_t = \gamma s_{t-1} + \mathrm{E}_t\left(\sum_{i=0}^{\infty} \lambda^i z_{t+i}\right) = \gamma s_{t-1} + \mathrm{E}_t z_t^*, \tag{7}$$

where $z_t = \lambda a_0^{-1}(m_t - a_2 m_{t-1} - a_1^{\mathrm{u}} y_t^{\mathrm{u}} + a_1^{\mathrm{f}} y_t^{\mathrm{f}})$.

A solution to (6′) is a special case of the solution to (7), with $\gamma = 0$ and the discount factor $\lambda = (1+a_0)^{-1}a_0 =$ (say) b:

$$s_t = \mathrm{E}_t\left(\sum_{i=0}^{\infty} b^i \tilde{z}_{t+1}\right) = \mathrm{E}_t \tilde{z}_t^*, \tag{7′}$$

where $\tilde{z}_t = b a_0^{-1}(m_t - a_1 y_t)$.

The variability test requires calculation of the variance of the innovation to the expected present discounted value of fundamentals, that is, the variance of the innovation to $E_t z_t^*$ or $E_t \tilde{z}_t^*$. This variance must be calculated relative to two information sets, the market's and another set H_t or $\tilde{H}_t$. H_t is an information set consisting of all current and lagged values of the fundamentals' variables m_t, y_t^u and y_t^f; $\tilde{H}_t$ is the same for m_t and y_t. The basic inequality exploited in this paper is

$$\sigma_\varepsilon^2 = E(E_t z_t^* - E_{t-1} z_t^*)^2 \leqq E(E z_t^* \mid H_t - E_{t-1} z_t^* \mid H_{t-1})^2, \tag{8}$$

$$\tilde{\sigma}_\varepsilon^2 = E(E_t \tilde{z}_t^* - E_{t-1} \tilde{z}_t^*)^2 \leqq E(E \tilde{z}_t^* \mid \tilde{H}_t - E_{t-1} \tilde{z}_t^* \mid \tilde{H}_{t-1})^2. \tag{8'}$$

It is shown in eq. (9) below that σ_ε^2 is just the variance of the innovation in the exchange rate, under the model(7). The same is true for $\tilde{\sigma}_\varepsilon^2$, under the model (7′). The inequalities are established in West (1986). They say that forecasts made with a subset of the market's information set have a larger innovation variance than actual forecasts.

One may use (8) to test the model (7) as follows. From (7)

$$\begin{aligned} E(s_t - E_{t-1} s_t)^2 &= E(\gamma s_{t-1} - \gamma E_{t-1} s_{t-1} + E_t z_t^* - E_{t-1} z_t^*)^2 \\ &= E(E_t z_t^* - E_{t-1} z_t^*)^2 = \sigma_\varepsilon^2. \end{aligned} \tag{9}$$

The left-hand side of (8) is thus simply the variance of the innovation in the exchange rate. One way to estimate this left-hand side is then as follows. Begin by rewriting (6) as

$$s_t - m_t = (1+a_0)^{-1}[a_0(E_t s_{t+1} - m_t) - a_1^u y_t^u + a_1^f y_t^f - a_2(m_{t-1} - s_{t-1})]. \tag{10}$$

Write (10) in estimable form by following McCallum (1976) and replacing the unobservable expectation $E_t s_{t+1}$ with the ex post value s_{t+1}:

$$\begin{aligned} s_t - m_t = {} & (1+a_0)^{-1}[a_0(s_{t+1} - m_t) - a_1^u y_t^u + a_1^f y_t^f - a_2(m_{t-1} - s_{t-1})] \\ & + \mu_{t+1}, \end{aligned} \tag{11}$$

where $\mu_{t+1} = -b(s_{t+1} - E_t s_{t+1})$ and, as in eq. (7′), $b = (1+a_0)^{-1} a_0$. Eq. (11) may be estimated by instrumental variables. Potential instruments include current and lagged values of all the right-hand-side variables except s_{t+1}, which is not a legitimate instrument since it is correlated with the disturbance. One can retrieve parameters of interest by simple arithmetic on the regression coefficients. For example, $\hat{a}_1^u = -\hat{\beta}_1/(1-\hat{b})$, where $\hat{\beta}_1$ is the estimated coefficient on y_t^u. More importantly, one can obtain an estimate of the left-hand side of (8) using $\hat{b}^{-2}\hat{\sigma}_\mu^2$.

Inference about the estimates of eq. (11) will be difficult if, as is assumed in Meese, the variables have unit roots. In this case a differenced version of (11) may be used:

$$\Delta s_t - \Delta m_t - a_1 \Delta y_t = b(\Delta s_{t+1} - \Delta m_t - a_1 y_t) + \tilde{\mu}_{t+1}, \tag{11'}$$

where $\tilde{\mu}_{t+1} = -b[(s_{t+1} - \mathrm{E}_t s_{t+1}) - (s_t - \mathrm{E}_{t-1} s_t)]$. The restrictions $a_1^u = a_1^f$ and $a_2 = 0$ have been imposed, in accordance with Meese. Eq. (11′) is written in a fashion convenient for estimation when the income elasticity a_1 is imposed a priori, as was done in Meese and in the empirical work here. Lags of Δm_t and Δy_t may be used as instruments to obtain estimates of b and thus a_0. The left-hand side of (8′) may now be estimated as $0.5\hat{b}^{-2}\hat{\tilde{\sigma}}_\mu^2$, where $\tilde{\sigma}_\mu^2 = \mathrm{E}\tilde{\mu}_{t+1}^2$.

The right-hand side of (8) [or (8′)] may be calculated from estimates of the multivariate process followed by the fundamentals' variables m_t, y_t^u and y_t^f (or Δm_t and Δy_t). The desired variance is an extremely complicated function of λ (or b), the multivariate ARIMA parameters and the variance–covariance matrix of the multivariate innovations. Details are given in the appendix.

In summary, for the models (7) and (7′), one tests:

$$0 \leqq \mathrm{E}(\mathrm{E}z_t^* \mid H_t - \mathrm{E}_{t-1} z_t^* \mid H_{t-1})^2 - b^{-2}\sigma_\mu^2, \tag{12}$$

$$0 \leqq \mathrm{E}(\mathrm{E}\tilde{z}_t^* \mid \tilde{H}_t - \mathrm{E}_{t-1} \tilde{z}_t^* \mid \tilde{H}_{t-1})^2 - 0.5b^{-2}\tilde{\sigma}_\mu^2. \tag{12'}$$

If this inequality does not hold, the model (7) [or (7′)] is not correct. Some factor or factors left out of the model are making exchange rates too variable to be consistent with the model. Two possibilities, not necessarily mutually exclusive, are considered here. The first is that the left-out factors are rational bubbles, and the second is that they are the usual unobservable regression disturbances.

Consider bubbles first. These are otherwise extraneous variables that are added to the solution (7) [or (7′)] that still yield an exchange rate process that satisfies eqs. (6) and (11) [or (6′) and (11′)]:

$$s_t = \gamma s_{t-1} + \mathrm{E}_t z_t^* + C_t, \tag{13}$$

$$s_t = \mathrm{E}_t \tilde{z}_t^* + C_t. \tag{13'}$$

The variable C_t is a bubble, and follows the stochastic process $\mathrm{E}_{t-1} C_t =$

$\lambda^{-1}C_{t-1}$ in (13), $E_{t-1}C_t=b^{-1}C_{t-1}$ in (13′). Examples of stochastic processes for C_t may be found in Blanchard and Watson (1982) and West (1985).[1]

It is easy to verify that adding C_t to (7) [or (7′)] yields a process for s_t that satisfies (6) [or (6′)]. If (13) is correct, $s_{t+1}-E_ts_{t+1}=(E_{t+1}z^*_{t+1}-E_tz^*_{t+1})+(C_{t+1}-E_tC_{t+1})=\varepsilon_{t+1}+c_{t+1}$, where c_{t+1} is the innovation in C_{t+1}. So $E(s_{t+1}-E_ts_{t+1})^2=\sigma^2_\varepsilon+2\sigma_{\varepsilon c}+\sigma^2_c$. Now, it is sometimes argued that financial markets tend to overreact to news about fundamentals, causing asset prices to jump excessively upon good news about fundamentals and to fall excessively upon bad news [Shiller (1984)]. If this overreaction is due to rational bubbles, this means that bubbles are positively correlated with fundamentals, i.e. $\sigma_{\varepsilon c}>0$. In the presence of bubbles, then, it is plausible that $E(s_{t+1}-E_ts_{t+1})^2=\sigma^2_\varepsilon$. That is, $E(s_{t+1}-E_ts_{t+1})^2$ is larger than the variance of news about fundamentals. This would explain a failure of (12) to hold. The same applies to (12′). Under the null hypothesis of no bubbles, (12) [or (12′)] of course does hold, since in this case $\varepsilon_{t+1}=s_{t+1}-E_ts_{t+1}$.

A second factor that might explain excess variability of the exchange rate is that s_t is influenced not by a stochastic bubble, but by a disturbance of the sort often assumed present in regression equations. If a random shock u_t is added to (11) and (11′), the equations become:

$$s_t-m_t=(1+a_0)^{-1}[a_0(s_{t+1}-m_t)-a^{\mathrm{u}}_1y^{\mathrm{u}}_t+a^{\mathrm{f}}_1y^{\mathrm{f}}_t-a_2(m_{t-1}-s_{t-1})]$$
$$+\eta_{t+1}, \tag{14}$$

$$\Delta s_t-\Delta m_t-a_1\,\Delta y_t=b(\Delta s_{t+1}-\Delta m_t-a_1y_t)+\tilde{\eta}_{t+1}, \tag{14′}$$

where $\eta_{t+1}=\mu_{t+1}+u_t$, $\tilde{\eta}_{t+1}=\tilde{\mu}_{t+1}+u_t$.

Suppose, as in Meese and one of Woo's specifications, that u_t is white noise. Woo assumes that the u_t in (14) results from a white noise disturbance to the money demand equation (3). One can assume more generally that the u_t in (14) also reflects the sluggish deviations from PPP that are observed empirically. Meese assumes that the u_t in (14′) results from a random walk disturbance to the PPP equation (5). One can again assume something more general, namely that in (14′) u_t also reflects a random walk disturbance to the money demand equation (3′). A white noise shock to (14) and (14′), then, is

[1]It is appropriate to add a word on the theoretical question of whether bubbles are consistent with rationality, in light of the claims by Obstfeld and Rogoff (1985) and Diba and Grossman (1985) that they are not. The most rigorous and general paper that I am aware of that deals with this question is Tirole (1985). Tirole establishes that bubbles are perfectly consistent with rationality in a standard overlapping generations model, under suitable conditions. That Diba and Grossman (1985) and Obstfeld and Rogoff (1985) find bubbles inconsistent with their models appears to reflect the particular characteristics of the models they use rather than any general presumption against bubbles.

consistent with the sort of money demand and PPP disturbances that appear to be observed empirically.[2]

The composite disturbances η_{t+1} and $\tilde{\eta}_{t+1}$ are both MA(1). This means that current m_t should not be used as an instrument, since it is correlated with u_t. One also cannot use current values of other variables as instruments, insofar as money is determined simultaneously with these variables in equilibrium [Hodrick (1979)]. In any case, with suitable lags of variables as instruments, (14) and (14′) can be estimated. Note that the estimates are consistent in general under plausible identifying assumptions (e.g. that there are predetermined variables that shift the money supply but that do not appear in money demand). This is true whether or not the exchange rate and the money supply are endogenous, in either the sense of Granger causality or the usual simultaneous equations sense.

The solutions to (14) and (14′) are

$$s_t = \gamma s_{t-1} + \mathrm{E}_t z_t^* + C_t + \lambda b^{-1} \mathrm{E}_t \sum_{i=0}^{\infty} \lambda^i u_{t+i} = \gamma s_{t-1} + \mathrm{E}_t z_t^* + C_t + \lambda b^{-1} u_t, \tag{15}$$

$$s_t = \mathrm{E}_t \tilde{z}_t^* + C_t + \mathrm{E}_t \sum_{i=0}^{\infty} b^i U_{t+i} = \mathrm{E}_t \tilde{z}_t^* + C_t + (1-b)^{-1} U_t, \tag{15′}$$

where U_t is a random walk shock whose innovation is u_t, $U_t - \mathrm{E}_{t-1} U_t = u_t$.

Our aim is still to use inequalities (8) and (8′) to see whether we must resort to bubbles to explain the variability of exchange rates. This will turn out to be much more complicated than when the usual regression disturbance is assumed absent. With u_t present in (14) and (14′), a violation of eq. (12) or (12′) can no longer be taken as evidence of bubbles. This is

[2]The disturbance u_t is a linear combination of a shock to the money demand equation (3), say, u_{1t}, and a shock to the PPP equation (5), say, u_{2t}. In principle u_t could depend on a shock to the interest parity condition (4) as well. But as far as I know, such a shock has not been assumed present in previous empirical work. Simple arithmetic yields $u_t = (1+a_0)^{-1}(-u_{1t} + u_{2t} - a_2 u_{2t-1})$ in (14), $u_t = (1+a_0)^{-1}(-\Delta u_{1t} + \Delta u_{2t})$ in (14′). For u_t to be white noise in (14) requires that u_{1t} be white noise and/or u_{2t} be $AR(1)$ with parameter a_2. For u_t to be white noise in (14′) requires that u_{1t} and/or u_{2t} be a random walk.

These requirements appear to be roughly consistent with existing empirical evidence. See, for example, Goldfeld (1976) or Mankiw and Summers (1984) on the disturbances to the money demand equation. See Adler and Lehmann (1983), Hakkio (1984) and Roll (1979) for evidence that deviations from PPP have a serial correlation coefficient quite near one. Since a_2 also appears to be quite near one [Goldfeld (1976), Woo (1985) and the estimates presented here], the assumption that $u_{2t} - a_2 u_{2t-1}$ is white noise is probably reasonable.

Technically, Woo and Meese cannot both be correct. As stated in the introduction, however, the aim of this paper is to establish a robust result concerning the monetary model. I will therefore not attempt to reconcile the technically contradictory assumptions of Woo and Meese concerning these shocks.

because even in the absence of bubbles σ_η^2 and $\tilde{\sigma}_\eta^2$ will depend not only on the variance of news about fundamentals but also on the variance of u_t and on the covariance between u_t and the news about fundamentals.

Nevertheless, inequalities (8) and (8′) can still be used to test for bubbles. The basic idea for the Woo specification is as follows; details are in the appendix. Under the null hypothesis of no bubbles, the two nonzero moments of the MA(1) disturbance η_{t+1} – $\mathrm{E}\eta_{t+1}^2$ and $\mathrm{E}\eta_{t+1}\eta_t$ – depend on the three unknowns σ_ε^2, $\sigma_{\varepsilon u}$ and σ_u^2. The two nonzero moments can be combined with a third piece of information to put bounds on the three unknowns, including, in particular, σ_ε^2. The Cauchy–Schwarz inequality, which states that $(\sigma_{\varepsilon u})^2 \leqq \sigma_\varepsilon^2\sigma_u^2$, is this third bit of information. That is to say, $\mathrm{E}\eta_{t+1}^2$, $\mathrm{E}\eta_{t+1}\eta_t$ and the Cauchy–Schwarz inequality suffice under the null hypothesis of no bubbles to identify an upper and lower bound to σ_ε^2. They do not, unfortunately, suffice to identify a point estimate of σ_ε^2. Similarly, in the Meese specification, an upper and lower bound to $\tilde{\sigma}_\varepsilon^2$ can be identified from the moments of $\tilde{\eta}_t$ and the Cauchy–Schwarz inequality.

Even with a u_t shock present, the right-hand side of (8) or (8′) can be calculated as before, as a complicated function of the parameters of the multivariate ARIMA process followed by the fundamentals' variables. In the presence of a white noise disturbance u_t, then, one can compare the lower bound estimates of σ_ε^2 or $\tilde{\sigma}_\varepsilon^2$ to the calculated value of the right-hand side of (8) or (8′). In the absence of bubbles, this lower bound should satisfy (8) or (8′).

Before turning to the empirical results, it is important to note two aspects of the procedure that might not be immediately obvious. The first relates to the procedure's implicit assumption that the estimates of the ARIMA process for the fundamentals yields an accurate estimate of the right-hand sides of (8) and (8′). One circumstance in which this will probably not be the case is when this process has shifted during the sample used in estimation or has been expected by the market to shift during or after the sample. This will happen if there are changes in policy rules [Flood and Hodrick (1986), Obstfeld and Rogoff (1985)]. This very real possibility is difficult (at least for me) to incorporate into the null. A partial solution is to obtain separate estimates for different sample periods if there is theoretical or empirical evidence of a midsample process shift. This will not, however, help if agents expected a shift that did not or has yet to occur. Consequently, a rejection of the null can be interpreted equally well as evidence of bubbles or as evidence of expected or actual shifts of the fundamentals process.

The second feature to note is that as long as the ARIMA process is stable, the procedure is legitimate whether or not there is feedback from the exchange rate or other variables to the fundamentals' variables. Inequalities (8) and (8′) hold so long as money and real income follow and are expected to follow a stable process. Any other variables that help determine money and real income in equilibrium have been implicitly solved out in the process of forecasting money and income.

3. Empirical results

3.1. Data

The raw data were monthly and seasonally unadjusted, 1974:1 to 1984:5. Data from 1973 and 1984:6 were used for lags and leads. Data on industrial production, money stock (M1) and the spot exchange rate (dollar–deutschemark) were kindly supplied by Richard Meese; a detailed description of this data set may be found in Meese and Rogoff (1983b).

The raw data appeared to require some transformations to induce stationarity. It is well known that detrending and differencing a variable are not asymptotically equivalent, whether the variable's secular drift is deterministic or stochastic [Nelson and Plosser (1982)]. Rather than get sidetracked into analysis of the source of the pronounced upward movement of some of the variables (especially y_t^u and y_t), I decided to handle such apparent non-stationarity as did Woo and Meese. The actual data used in my test of the Woo specification therefore were the residuals from a regression of levels of variables on seasonal dummies and a linear trend, because Woo assumed that secular drift is deterministic. The data used in the Meese specification were the residuals from a regression of differences of variables on seasonal dummies, because Meese assumed that secular drift is stochastic.[3] Separate detrending regressions were run for each of the subsamples described below. Since all estimation was linear, the estimates of regression coefficients are identical to those that would have been obtained had the trend and seasonal terms been included in the regressions. These preliminary regressions were done to cut down the otherwise enormous size of the variance–covariance matrix of the parameters.

Estimates were obtained for 1974:1 to 1984:5, and for two subsamples as well, 1974:1 to 1979:9 and 1979:10 to 1984:5. The subsample estimates were obtained because, as noted in the previous section, the procedure for estimating the requisite innovation variances tacitly assumes that the fundamentals variables follow a stable ARIMA process over the entire sample period, and there is some evidence that they did not. The tests in Meese (1985), for example, suggest that the Fed's October 1979 change in operating procedures resulted in a shift to the ARIMA process of Δm_t and/or Δy_t. Woo (1985), on the other hand, found that the Fed's change did not result in such

[3]Note that it follows from eqs. (15) and (15′) that under the null hypothesis of no bubbles, the endogenous drift in the exchange rate (if any) will be deterministic under Woo's assumptions about shocks and fundamentals' variables. Similarly, the exchange rate has a unit root under Meese's assumptions.

a shift. While neither paper uses precisely my sample period nor my specification for the fundamentals process, and Woo uses different (seasonally adjusted) data, the data are similar enough that the hypothesis of stability seems debatable. I therefore also estimated and tested the model using not only the entire sample period, but also the pre- and post-October 1979 subsamples. Note that the use of these subsamples implicitly assumes that the market instantaneously caught on to any such shift by the Fed, and, as noted in the previous section, that the market did not expect such a shift.

3.2. Estimation technique

For the Woo specification, four regression equations were estimated: eq. (14), and a three-variable vector autoregression for the fundamentals variables m_t, y_t^u and y_t^f. The lag length for the autoregression was set at four when the whole sample was used, two when a subsample was used. For the whole sample regressions, then, there were 12 variables ($12 = 3$ variables $\times 4$ lags per variable) on the right-hand side of each of the three autoregressions. The corresponding figure for the subsample regressions was six. A shorter lag length was used in the subsamples to preserve degrees of freedom. Diagnostic tests such as Q statistics suggested that the lag lengths were adequate, for the whole sample and both subsamples. Some experimentation, summarized in footnote 9 below, indicated that the results are not sensitive to choice of lag length.

For the Meese specification, three regression equations were estimated: eq. (14′), and a two-variable vector autoregression for the fundamentals' variables Δm_t and Δy_t. Lag lengths were chosen as in the Woo specification.

Let θ denote the vector of parameters that must be estimated to calculate the innovation variances of interest. The vector θ consists of the coefficients on the right-hand-side variables in (14) or (14′); $E\eta_{t+1}^2$ and $E\eta_{t+1}\eta_t$, the first and second autocovariances of the disturbance to (14) or (14′); the coefficients on the right-hand-side variables in the fundamentals autoregressions; and the elements of the variance–covariance matrix of the innovations in the fundamentals. In the Woo specification, for example, θ contains 48 elements, when estimating with the entire sample period: four coefficients on the right-hand side of (14); $E\eta_{t+1}^2$ and $E\eta_{t+1}\eta_t$; 36 coefficients on the right-hand side of the autoregressions; and the six independent elements of the variance–covariance matrix of the disturbances to the fundamentals' autoregressions.

The elements of θ were estimated as follows. The right-hand-side variables in (14) and (14′) were estimated by two-stage least squares, with the right-hand-side variables of the autoregressions used as instruments. The moments $E\eta_{t+1}^2$ and $E\eta_{t+1}\eta_t$ were estimated from the moments of the two-stage least squares residuals. The autoregression parameters were estimated by OLS. The elements of the variance–covariance matrix of the autoregression disturbances were estimated from the OLS residuals, with the usual degree of freedom adjustment.

Calculation of the asymptotic covariance matrix of θ is described in the appendix. It suffices to make three remarks here. First, the standard errors on the coefficients in (14) and (14′) allow for the MA(1) serial correlation that η_t displays if there is a u_t shock present. They are, however, still consistent if η_t is serially uncorrelated when u_t is absent. Second, standard errors on all regression coefficients were calculated to allow for arbitrary heteroskedasticity conditional on the instruments (i.e. conditional on the right-hand-side variables in the autoregressions). Third, proper account was taken not only of the uncertainty in the estimates of the regression coefficients, but also of (a) the uncertainty in the estimates of the variances and covariances such as $E\eta_{t+1}^2$ and (b) the correlation of the estimates of the various elements of θ.

The innovation variances in eqs. (8) and (8′) are complicated functions of θ. Let $f(\theta)$ denote one of these variances. The standard error on $f(\theta)$ was calculated as $[(\partial f/\partial\theta)V(df/\partial\theta)']^{1/2}$, where V is the variance–covariance matrix of θ. The derivatives of $\partial f/\partial\theta$ of all such functions were calculated numerically.

3.3. Empirical results

Table 1 reports the estimates of the basic regression regression parameters.[4]

Table 1

Parameter estimates.

	a_0	a_1^u	a_1^f	a_2
Eq. (14):				
(1) 1974:1–1984:5	1.3340	0.1968	0.2951	0.9035
	(0.2122)	(0.1635)	(0.2771)	(0.0922)
(2) 1974:1–1979:9	0.1759	0.1853	0.3253	1.4410
	(0.2152)	(0.2025)	(0.2383)	(0.1747)
(3) 1979:10–1984:5	0.5319	0.4944	1.3371	0.8662
	(0.2789)	(0.2119)	(0.4203)	(0.1804)
Eq. (14′)				
(4) 1974:1–1984:5	0.4687	0.5000		
	(0.3321)			
(5) 1974:1–1979:9	0.4921	0.5000		
	(0.2557)			
(6) 1979:10–1984:5	0.2379	0.5000		
	(0.5407)			

Notes: Asymptotic standards errors in parentheses. Symbols defined in the text.

[4]The estimates of the fundamentals processes are not reported, to conserve space. An appendix containing these estimates, as well as those of the initial regressions to induce stationarity, is available on request.

Consider first the estimates of (14), in lines (1) to (3). About half the estimates are significantly different from zero at the 5 percent level, and almost all are more or less reasonable. Consider first the interest semielasticity a_0. Its estimates vary somewhat from sample to sample, but are roughly consistent with the estiamtes in Woo and Meese. The estimates of both the U.S. and German income elasticities, a_1^u and a_1^f, are also roughly consistent with the slightly higher estimates in Woo. The estimates of a_2 are, again, similar to those in Woo. One estimate [line (2)] exceeded its theoretical upper bound of unity, as did one of Woo's estimates [Woo (1985, p. 8)]. Combining a_1^u and a_2, or a_1^f and a_2, for lines (1) and (3) yields, as in Woo, a somewhat high long-run income elasticity of two or more.

Now consider the estimates of (14′) in lines (4) to (6) of table 1. The estimates of a_0 were obtained by imposing $a_1=0.5$ as did Meese. Results and estimates for $a_1=0.4$ and $a_1=0.3$, the other two imposed values of a_1 for which Meese reported results, were almost identical.[5] The estimates of a_0 are somewhat lower than in Meese but, perhaps, not implausibly so. One of the three estimates is significantly different from zero at the 5 percent level [line (5)].

In sum, then, the regression results suggest that the Woo specification is quite acceptable, the Meese specification less so. In addition, it is reassuring that the use of different sample periods, estimation techniques and, in the case of Woo, different data, lead to qualitatively similar parameter estimates.

Let us now turn to the variability test. Column (1) of table 2 presents the estimates of the right-hand sides of (8) and (8′).[6] Column (2) reports the estimate of the variance of σ_ε^2 if the unobservable shock u_t is assumed absent. Under the null hypothesis that bubbles are also absent, the column (2) estimate should be less than the column (1) estimate. It is not, for any of the six specifications. Column (2) is anywhere from five to two hundred times as large as column (1). It is significantly larger (at the 5 percent level) in two specifications [lines (1) and (5)].[7] See column (3). My variability test, inequalities (8) and (8′), therefore indicates, as did Huang's (1981), that a standard monetary model with neither bubbles nor the usual regression

[5]Allowing a_1 to be estimated freely did not generate similar results. In this case, in two of the three samples, either a_0 or a_1 or both were wildly implausible ($a_0>100$, or a_1 negative). I therefore did not even calculate the variability test. It is not clear to me why unconstrained estimates were not sensible. A referee has commented that this suggests a specification error in Meese's model, and, citing Cumby and Obstfeld (1984), has suggested that one possible culprit is Meese's assumption that deviations from PPP are a random walk.

[6]It may help in interpreting all the figures in table 2 to note that the 1973:3 to 1984:5 variance in Δs_t is 11.53 (times 10^{-4}). The in-sample variance of the news about fundamentals in line (1), column (1), thus, is about one-sixth of what would be the out-of-sample error variance from forecasting the spot rate as a random walk.

[7]For the Meese specification in lines (4)–(6), the column (2) estimate may be calculated as either $0.5b^{-2}\mathrm{E}\eta_{t+1}^2$ or $-b^{-2}\mathrm{E}\eta_{t+1}\eta_t$. The former is reported in table 2. The latter yields values lower than those in column (2), but still much larger than those in column (1).

Table 2

Variability measures.

	(1) R.h.s. of (8)	(2) σ_ε^2 or $\tilde{\sigma}_\varepsilon^2, u_t \equiv 0$	(3) $(1)-(2)$	(4) $\min \sigma_\varepsilon^2$ or $\tilde{\sigma}_\varepsilon^2, u_t \neq 0$	(5) $(1)-(4)$
(1) 1974:1–1984:5	1.895	16.830	−14.934 (4.195)	1.556	0.339 (2.990)
(2) 1974:1–1979:9	2.584	650.340	−647.756 (1172.419)	82.751	−80.166 (240.921)
(3) 1979:10–1984:5	9.665	53.019	−43.354 (41.473)	4.885	4.781 (4.011)
(4) 1974:1–1984:5	3.710	55.610	−51.900 (42.530)	0.014	3.696 (0.998)
(5) 1974:1–1979:9	2.926	38.438	−35.512 (17.700)	1.582	1.345 (7.945)
(6) 1979:10–1984:5	3.113	131.500	−128.387 (297.648)	0.216	2.902 (4.662)

Notes: Lines (1)–(3) are based on eq. (14), lines (4)–(6) on eq. (14′), as described in the text. Asymptotic standard errors in parentheses. Symbols defined in the text. All figures are times 10^4.

disturbance is inconsistent with the variability of the dollar–deutschemark exchange rate.[8]

The monetary models are not, however, inconsistent with the data if one allows for the usual regression disturbance. Column (4) reports the minimum possible value of σ_ε^2, calculated as described in the appendix. Column (5) reports the difference between columns (1) and (4). With one exception [line (2)], the column (4) estimate is less than the column (1) value. The difference, unfortunately, is estimated rather imprecisely. In only one specification [line (4)] is the point estimate of the difference significantly different from zero at the 5 percent level. It is clear, nonetheless, that once regression disturbances are permitted, one cannot reject the null hypothesis that the variance of the innovation in the expected present discounted value of fundamentals is less when the market's information set is used (σ_ε^2) than when only past values of fundamentals are used [column (1)]. This result is robust to changes in the lag length of the fundamentals' autoregressions.[9]

[8]The figures reported in table 2 are based on estimates that used lags of fundamentals' variables as instruments. For lines (1)–(3), different (and more efficient) estimates of columns (1)–(3) may be obtained when there is no disturbance u_t by using current as well as lagged values of fundamentals as instruments. See the discussion in section 2. So I recalculated columns (1) and (2) using estimates obtained when current as well as lagged values were used. For all three sample periods, column (2) was greater than column (1).

[9]To see whether the results were sensitive to choice of lag length of the fundamentals autoregression, I calculated two additional point estimates (but not standard errors) of each of the table 2 entries. These were for lag lengths $r=2$ and 3, for the whole sample, $r=3$ and 4 for the subsamples. The Woo specification proved quite robust, with all additional calculations yielding a negative figure for column (3) in table 2, and a positive figure for column (5). The

The consistency does not, in my opinion, mean that the Woo and Meese models capture the variability in an entirely satisfactory fashion. It is often argued that the exchange rate is an asset price and ought to fluctuate as do many asset prices in response to news about economic variables [Frenkel (1981), Frenkel and Mussa (1980)]. The empirical results suggest that in the Woo and Meese models these fluctuations result in an important way from shocks that have no explicit links to economic theory or even to any economic variables (except, of course, tautologically, to the variables in the equations in which the shocks appear). Therefore, while it undoubtedly is desirable to allow for regression disturbances in exchange rate models, it appears that some nontrivial extensions to the Woo and Meese models are required, if one of these models is to explain the fluctuations basically as responses to news about observable economic variables. This may well be true of other monetary models as well. In any case, it would seem highly desirable to model deviations from PPP as functions, at least in part, of observable economic variables. Sticky price models such as Driskell (1981) and Frankel (1979) may be useful starting points.

3.4. Comparison with previous studies

The basic conclusion of this paper conflicts with previous studies on volatility and speculative bubbles [Huang (1981), Meese (1985)]. A reconciliation with these studies is therefore in order.

Reconciliation with Huang (1981) is simple. Huang followed some studies such as Bilson (1978) and assumed no regression disturbances in any of the basic equations. As was just noted, when this assumption is made here, the result is that the monetary model cannot explain the variability of the exchange rate. Surely, this argues more for allowing for the usual regression disturbance than for a basic failure of the monetary model. See Hodrick (1979) on the theoretical importance of allowing for the usual regression disturbances.

Reconciliation with Meese (1985) is not quite as straightforward. Meese applied to the exchange rate the test for speculative bubbles that I developed and applied to stock prices in West (1985). A general description of my specification test may be found in West (1985). For concreteness I will explain it here in the context of Meese's application.

The specification test compares two estimates of b, $b \equiv (1+a_0)^{-1}a_0$ as in (7′). One estimate of b is obtained from eq. (14′) by instrumental variables, and is consistent even if there are bubbles. The second estimate is obtained

Meese specification was not as robust. Three samples produced implausible parameter estimates, such as negative b (whole sample, $r=2$, and first subsample, $r=3$ and $r=4$). The other three samples did, however, yield a positive figure for column (3), and all but $r=4$, second subsample, produced a negative figure for column (5).

from estimation of two types of equations: a closed form solution to the expected present discounted formula (7′), and the fundamentals process. This second estimate is not consistent if there are bubbles. Meese compared the two estimates of b and found them more different than is consistent with sampling error. The implication is that there are bubbles.

There are at least two possible explanations for the conflict between Meese's results and those of the present paper. Both, unsurprisingly, are econometric. The first is that the specification test may have more power. The second is that one test may have better finite sample properties. I suspect that the present paper's test is better in this respect, at least when there are in-sample shifts in the ARIMA process of the fundamentals' variables. This is because such shifts will obscure the link between the ARIMA process and the closed form solution to (7′). This will potentially cause a strong bias in the second of the two estimates of b that were described in the previous paragraph. By contrast, although the present variability test requires a fundamentals process that is stable and expected by the market to remain so, there appears to be no presumption that it is biased toward finding excess variability if the process in fact is unstable: *both* sides of (8′) are likely to be estimated quite noisily.

Both of these possible explanations are quite tentative. Some further research is required to reconcile the fact that one of my bubble tests finds bubbles, the other does not.

4. Conclusions

Two basically standard monetary models appear to be consistent with the 1974–84 variability of the dollar–deutschemark exchange rate. As noted in the introduction, the extent to which this consistency may be interpreted as evidence against speculative bubbles or process switching is limited at best to the extent one believes the models correctly explain this exchange rate. But regardless of how enthusiastically one endorses either model, it is of note that shocks to money demand and PPP play a key role in the apparent consistency of the models with the data. It is therefore of interest in future work to model these shocks as functions at least in part of observable economic variables.

Appendix

This appendix describes calculation of: (A.1) the right-hand side of (8); (A.2) a lower bound estimate to σ_ε^2 when there is a shock u_t; and (A.3) the variance–covariance matrix.

A.1. Right-hand side of (8)

Consider first when eq. (14′) is the appropriate specification. Let $\Delta x_t = [\Delta m_t, \Delta y_t]$ follow an $AR(r)$ process, $\Phi(L)\Delta x_t = \Delta x_t - \Phi_1 \Delta x_{t-1} - \cdots - \Phi_r \Delta x_{t-r} = v_t$. Each Φ_i is a (2×2) matrix. Using the formulas in Hansen and Sargent (1980), it may be shown that

$$\mathrm{E}\sum b^i x_{t+i} \,|\, \tilde{H}_t - \mathrm{E}\sum b^i x_{t+1} \,|\, \tilde{H}_{t-1} = (1-b)^{-1}\Phi(b)^{-1}(x_t - \mathrm{E}x_t \,|\, \tilde{H}_{t-1})$$

$$= (1-b)^{-1}\Phi(b)^{-1}v_t,$$

where $\Phi(b) = I - \Phi_1 b - \cdots - \Phi_r b^r$. Let $\tilde{\alpha} = ba_0^{-1}[1, -a_1]'$.

From eq. (7′),

$$\mathrm{E}\tilde{z}_t^* \,|\, \tilde{H}_t - \mathrm{E}\tilde{z}_t^* \,|\, \tilde{H}_{t-1} = \tilde{\alpha}'[\mathrm{E}\sum b^i x_{t+1} \,|\, \tilde{H}_t - \mathrm{E}\sum b^i x_{t+1} \,|\, \tilde{H}_{t-1}]$$

$$= (1-b)^{-1}\tilde{\alpha}'\Phi(b)^{-1}v_t.$$

Thus,

$$\mathrm{E}(\mathrm{E}\tilde{z}_t^* \,|\, \tilde{H}_t - \mathrm{E}\tilde{z}_t^* \,|\, \tilde{H}_{t-1})^2 = (1-b)^{-2}\tilde{\alpha}'\Phi(b)^{-1}\Omega\Phi(b)^{-1\prime}\tilde{\alpha}, \Omega \equiv \mathrm{E}v_t v_t'.$$

When $x_t = [m_t, y_t^{\mathrm{u}}, y_t^{\mathrm{f}}]$ follows an $AR(r)$ process, $\Phi(L)x_t = v_t$, as is consistent with (7), the comparable formula is

$$\mathrm{E}(\mathrm{E}z_t^* \,|\, H_t - \mathrm{E}z_t^* \,|\, H_{t-1})^2 = \alpha'\Phi(\lambda)^{-1}\Omega\Phi(\lambda)^{-1\prime}\alpha, \alpha \equiv \lambda a_0^{-1}[1 - a_2\lambda, -a_1^{\mathrm{u}}, a_1^{\mathrm{f}}]'.$$

A.2. Lower bound σ_ε^2

The basic procedure for (14) is as follows. The procedure for (14′) is similar. We have from (15) that when there are no bubbles $s_{t+1} - \mathrm{E}_t s_{t+1} = \mathrm{E}_{t+1} z_{t+1}^* - \mathrm{E}_t z_{t+1}^* + \lambda b^{-1} u_{t+1}$. It may be shown that the minimum and maximum possible values of σ_ε^2 occur when u_t and ε_t are perfectly correlated, $u_t - h\varepsilon_t$ for some h. In such a case,

$$\mathrm{E}\eta_{t+1}^2 = \mathrm{E}[-b(s_{t+1} - \mathrm{E}_t s_{t+1}) + u_t]^2$$
$$= \mathrm{E}[-b(\varepsilon_{t+1} + \lambda b^{-1}h\varepsilon_{t+1}) + h\varepsilon_t]^2 \equiv f_1(\sigma_\varepsilon^2, h),$$

$$\mathrm{E}\eta_{t+1}\eta_t = \mathrm{E}[-b(\varepsilon_{t+1} + \lambda b^{-1}h\varepsilon_{t+1}) + h\varepsilon_t][-b(\varepsilon_t + \lambda b^{-1}h\varepsilon_t) + h\varepsilon_{t-1}]$$

$$\equiv f_2(\sigma_\varepsilon^2, h).$$

λ and b have been omitted as argumants in f_1 and f_2 since they may be identified from the regression parameters. f_1 and f_2 may be combined to eliminate σ_ε^2. The result is a quadratic equation in h. One of the two roots to this quadratic may be plugged back into f_1 or f_2 to obtain the minimum possible value of σ_ε^2.

A.3. Variance–covariance matrix

This explains the calculation of the variance–covariance matrix of the parameter vector θ, for the Woo specification. How the matrix was calculated for the Meese specification will be obvious from the description to follow.

Let $z_t=(m_{t-1},\ldots,m_{t-r},y^{\mathrm{u}}_{t-1},\ldots,y^{\mathrm{u}}_{t-r},y^{\mathrm{f}}_{t-1},\ldots,y^{\mathrm{f}}_{t-r})'$ be the $(3r\times 1)$ vector of instruments used; $r=4$ for the whole sample, $r=2$ for the subsamples. Write eq. (14) as $w_t=x_t'\beta+\eta_t$, $w_t\equiv s_t-m_t, x_t$ and β (4×1) defined in the obvious way. Let $A=(\sum x_t z_t')(\sum z_t z_t')^{-1}$ be the usual 2SLS weighting vector. Write the fundamentals autoregressions as $m_t=z_t'\delta_1+v_{1t}, y^{\mathrm{u}}_t=z_t'\delta_2+v_{2t}, y^{\mathrm{f}}_t=z_t'\delta_3+v_{3t}$. Finally, let T be the sample size.

One way of describing the estimation technique used is to note that the $(12+9r)\times 1$ parameter vector θ was chosen to satisfy an orthogonality condition. This orthogonality condition is

$$0=T^{-1}\sum h_t(\hat\theta)=\begin{bmatrix} T^{-1}A\sum z_t(w_t-x_t'\hat\beta)\\ \hat{\mathrm{E}}\eta_t^2-T^{-1}\sum(w_t-x_t'\hat\beta)^2\\ \hat{\mathrm{E}}\eta_t\eta_{t-1}-T^{-1}\sum(w_t-x_t'\hat\beta)(w_{t-1}-x_{t-1}'\hat\beta)\\ T^{-1}\sum z_t(m_t-z_t'\hat\delta_1)\\ T^{-1}\sum z_t(y^{\mathrm{u}}_t-z_t'\hat\delta_2)\\ \\ \mathrm{T}^{-1}\sum z_t(y^f_t-z_t'\hat\delta_3)\\ \hat{\mathrm{E}}v_{1t}^2-(T-3r)^{-1}\sum(m_t-z_t'\hat\delta_1)^2\\ \hat{\mathrm{E}}v_{1t}v_{2t}-(T-3r)^{-1}\sum(m_t-z_t'\hat\delta_1)(y^{\mathrm{u}}_t-z_t'\hat\delta_2)\\ \hat{\mathrm{E}}v_{1t}v_{3t}-(T-3r)^{-1}\sum(m_t-z_t'\hat\delta_1)(y^f_t-z_t'\hat\delta_3)\\ \hat{\mathrm{E}}v_{2t}^2-(T-3r)^{-1}\sum(y^{\mathrm{u}}_t-z_t'\hat\delta_2)^2\\ \hat{\mathrm{E}}v_{2t}v_{3t}-(T-3r)^{-1}\sum(y^{\mathrm{u}}_t-z_t'\hat\delta_2)(y^f_t-z_t'\hat\delta_3)\\ \hat{\mathrm{E}}v_{3t}^2-(T-3r)^{-1}\sum(y^f_t-z_t'\hat\delta_3)^2 \end{bmatrix}.$$

As stated in the text, then, $\hat\beta$ is estimated by 2SLS, $\hat{\mathrm{E}}\eta_t^2$ and $\hat{\mathrm{E}}\eta_t\eta_{t-1}$ from the moments of the 2SLS residuals, $\hat{\delta}_1$, $\hat{\delta}_2$ and $\hat{\delta}_3$ by OLS, $\hat{E}v_{it}v_{jt}(i,j=1,2,3)$ from the OLS residuals with a degrees of freedom correction.

Since $Eh_t(\theta^*)=0$, where θ^* is the true unknown θ, $T^{1/2}(\hat{\theta}-\theta^*)$ is the asymptotically normal with $(12+9r)\times(12+9r)$ covariance matrix $V\equiv(\text{plim}\ T^{-1}\sum h_{t\theta})^{-1}S\ (\text{plim}\ T^{-1}\sum h'_{t\theta})^{-1}$ [Hansen (1982)]. $h_{t\theta}$ is $\partial h_t/\partial\theta$ and was straightforward to calculate. $S=\sum_{i=-\infty}^{\infty}Eh_th'_{t-i}$ and was calculated as in Newey and West (1986), using three lags of $h_t(\hat{\theta})$. Newey and West (1986) show that the resulting positive semidefinite estimate of V is consistent, for arbitrary correlation between η_s and $v_{jt}(j=1,2,3)$, and arbitrary heteroskedasticity of η_t and v_{jt} conditional on the instruments.

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