Kenneth D. West
"Sources of Fluctuations in Aggregate Inventories and GNP"

Additional Appendix

This appendix contains additional details on the empirical work that were omitted from the main body of the paper to save space. There are eight items:

I. Growth

II. Production versus sales variability

III. Tests for a common deterministic trend in Q and H

IV. Estimation of infinite horizon variance decompositions

V. Regressions to scale data

VI. Estimates of the β_i

VII. Asymptotic standard errors for the g_{ij}

VIII. Asymptotic standard errors for the π_{ij}, ϕ_c=ϕ_d=1, both scaled and unscaled data

IX. Plots of impulse response functions for ϕ_c=.969,ϕ_d=.997.

I. Growth

As noted in the text, exactly the same first order conditions, and, therefore, exactly the same reduced form, is delivered by the observationally equivalent model in which the economy maximizes the excess of consumers surplus W_t* over production costs,

$$\text{(A3)}\quad \max \lim_{T\to\infty} E_0 \sum_{t=0}^{T} b^t (W_t* - C_t) \qquad \text{s.t. } Q_t = S_t + \Delta H_t,$$

where in the model in the paper $W_t* = -g_{0S}S_t^2+2U_{dt}S_t$. It is algebraically convenient to work with the (A3) statement of the model. Let h_t, q_t, s_t, u_{dt}, and u_{ct} be the original data and shocks in levels, with H_t, Q_t, S_t, U_{dt}, and U_{ct} the scaled data (e.g., $H_t=h_t/g^t$). Let W_t*-C_t be

$$\begin{aligned}\text{(A4)}\quad & k_t - g_{0S}s_t^2 - 2m_{0St}s_t + 2u_{dt}s_t \\ & - 2g_{0Ht}(h_t - m_{0Ht} - g_{HS}E_t s_{t+1}) - 2g_{0Qt}(q_t - m_{0Qt}) - 2g_{1Qt}(\Delta q_t - m_{1Qt}) \\ & - g_{0H}(h_{t-1} - m_{0Ht} - g_{HS}s_t)^2 - g_{0Q}(q_t - m_{0Qt})^2 - g_{1Q}(\Delta q_t - m_{1Qt})^2 \\ & - 2(h.h_t + q_t)u_{ct}.\end{aligned}$$

k_t is a purely deterministic term that grows no faster than g^{2t}. The deterministic m_{ijt} shift the bliss level and minimum costs points, $m_{ijt} = g^t m_{ij0}$. Also, $g_{ijt} = g^t g_{ij0}$. Let $y_t=(h_t,q_t)$, $u_t=(u_{ct},u_{dt})$. Substituting (A4) into (A3), using $q_t = s_t + \Delta h_t$ to substitute out for s_t, differentiating with respect to q_t and h_t and then dividing by two yields

$$E_t(\, bA_1'y_{t+1} + A_0y_t + A_1y_{t-1} + m_t - [D_0u_t + bD_1u_{t+1}] \,) = 0,$$

where $m_t=(m_{Ht},m_{Qt})'$, A_0, A_1, D_0 and D_1 are defined in equation (8), and, e.g., $m_{Ht} = -m_{0St} + bm_{0St+1} + g_{0Ht} + g_{0H}g_{HS}m_{0Ht-1} + bg_{0H}(1-g_{HS})m_{0Ht}$, with m_{Qt} defined similarly. Dividing through by g^t and rearranging yields

$$\text{(A5)}\quad E_t(\, bgA_1'Y_{t+1} + A_0Y_t + g^{-1}A_1Y_{t-1} + M + DU_t \,) = 0,$$

where $D=D_0+bgD_1$ and $M=(M_H,M_Q)'$, with, e.g., $M_H = -(1-bg)m_{0S0} + g_{0H0} + g_{0H}g_{HS}m_{0H0} + bg_{0H}(1-g_{HS})m_{0H0}$. Equation (10) is a version of equation (A5) with the constant term and growth factor g suppressed.

By mimicking the argument in Hansen and Sargent (1981), it can be established that $bA_1'L^{-1}+A_0+A_1L = (C_0+bC_1L^{-1})'(C_0+C_1L)$, with C_0+C_1L a stable polynomial, $C_0+bC_1L^{-1}$ an unstable polynomial. It follows that $bgA_1'L^{-1}+A_0+g^{-1}A_1L = (C_0+bgC_1L^{-1})'(C_0+g^{-1}C_1L)$. Since $g\geq 1$, $C_0+g^{-1}C_1L$ is a stable polynomial. As long as $bg<1$, $C_0+bgC_1L^{-1}$ is an unstable polynomial. The rule of solving stable roots backwards, unstable roots forwards leads to the solution in the text.

II. Production versus sales variability

As in the previous section, let lower case letters denote the variables before scaling by g^t, uppercase letters denote scaled variables. Let $h_t/g^t = H_t = EH + H^{\sim}_t$, $EH^{\sim}_t = 0$, with similar notation for Q_t and S_t. (This is not the same $H^{\sim}_t$ and $Q^{\sim}_t$ that appear in Tables I and II.)

In the stationary model, one can derive the inequality var(Q)<var(S) in the presence of just demand shocks exactly as in West (1986). One compares the unconditional expectation of the infinite sum in equation (A3) under the optimal policy to that under the alternative policy that sets $h_t^A = Eh_t = g^tEH$, $s_t^A = s_t$, $q_t^A = s_t + E\Delta h_t$. One can derive the inequality var(Q)>var(S) in the presence of just cost shocks by performing the same comparison, this time with an alternative policy that sets $h_t^A = Eh_t$, $s_t^A = q_t - E\Delta h_t$, $q_t^A = q_t$.

In a model with unit roots, if there is no deterministic trend to the data (g=1), begin by noting that the period zero conditional expectation of the infinite sum in (A3) must be larger for the optimal policy than for any alternative. Consider the alternative that sets $h_t^A = Eh_t = g^tEH$, $s_t^A = s_t$, $q_t^A = s_t + E\Delta h_t$, in the presence of just demand shocks. Quadratic inventory costs ($H^{\sim}{}_t^2$) are strictly greater in the optimal policy. So a necessary condition for optimality is that the conditional expectation of the difference between the optimal and alternative values of (A3), exclusive of such costs, is nonnegative. Taking unconditional expectations of this difference implies $0 \leq E(S_t^2-Q_t^2)$. The argument for $0 \geq E(S_t^2-Q_t^2)$ in the presence of just cost shocks is similar.

That g>1 introduces some slight complications. If, for example, $q_t^A = s_t + E\Delta h_t$, $Q_t^A = q_t^A/g^t = S_t + EH - g^{-1}EH = Q_t - H_t + g^{-1}H_{t-1} + EH - g^{-1}EH \Longrightarrow (Q_t^A)^2 - Q_t^2 = -2Q_t\Delta H_t + \Delta H_t^2 + Q_t(1-g^{-1})H_{t-1}$, and the last of these three terms does

not have a finite unconditional expectation. Considering instead the policy $q_t^A = s_t + E\Delta h_t + [(g-1)/g](h_t-Eh_t)$, we find that $(Q_t^A)^2 - Q_t^2 = -2g^{-1}Q_t\Delta H_t + g^{-2}\Delta H_t^2$, which does have a finite expectation. It is an estimate of $cov(Q_t,\Delta H_t) + g^{-1}var(\Delta H_t)$ that is reported in Table 3.

III. Tests for a common deterministic trend in Q and H

For the stationary specification, an asymptotic test of whether the two unconstrained growth rates of .786 and .828 percent are significantly different from one another was performed. This did not reject the null of a common growth rate at conventional significance levels (t-statistic less than 1.43, using either fifth, tenth, or fifteenth order Newey and West (1987) corrections). For the nonstationary specification, a bootstrap test of the null of one deterministic and of the null of two stochastic trends was performed. The residuals for the AR(1) first difference specification were sampled with replacement, the scaled data were generated according to the estimated VAR parameters, and unscaled data were constructed using the estimated common growth rate of .807 percent. For each sample, I estimated and saved (a)the absolute value of the difference between the estimated growth rates of (i)inventories and (ii)GNP, and (b)the Durbin Watson of the regression of scaled inventories on scaled GNP, with an estimated common growth rate used for scaling.

Fewer than 60 percent of the estimates of the separate growth rates were less than .042 (=.828-.786) percent apart. The null of a common deterministic trend thus cannot be rejected. In addition, fewer than 50 percent of the Durbin-Watson statistics from the regression of scaled inventories on scaled GNP were smaller than .042, the figure for the actual data. Thus, the null of two stochastic trends cannot be rejected.

IV. Estimation of infinite horizon variance decompositions

For data assumed stationary these are just unconditional moments, calculated from the Yule-Walker equations. For differenced data these were calculated by computing $\lim_{n\to\infty} n^{-1} \operatorname{var}(Y_{t+n}-E_tY_{t+n}) = (I-\Pi)^{-1}(F\Omega_uF')(I-\Pi)^{-1}$, (easily established). This yields estimates of, say, θ_1 and θ_2, where $\lim_{n\to\infty} n^{-1} \operatorname{var}(Q_{t+n}-E_tQ_{t+n}) = \theta_1\sigma_c^2+\theta_2\sigma_d^2$. The infinite horizon fraction of the variance of Q_t due to cost shocks was then computed as $\theta_1\sigma_c^2/(\theta_1\sigma_c^2+\theta_2\sigma_d^2)$.

V. Regressions to scale data

A. The first regression is that of log(H) on a constant and a time trend, the second that of log(Q) on a constant and a time trend, the third that of log(H) and log(Q) on constants and a time trend, constraining the coefficient on the time trend but not the constant to be the same for both log(H) and log (Q).

DEPENDENT VARIABLE 22 LOGH
FROM 1947: 1 UNTIL 1986: 4
OBSERVATIONS 160 DEGREES OF FREEDOM 158
R**2 .98259523 RBAR**2 .98248507
SSR .41431915 SEE .51208135E-01
DURBIN-WATSON .03435775
Q(36)= 1203.30 SIGNIFICANCE LEVEL .000000

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	6.898694	.8134820E-02	848.0451
2	TREND	24	0	.8278275E-02	.8765125E-04	94.44560

DEPENDENT VARIABLE 23 LOGQ
FROM 1947: 1 UNTIL 1986: 4
OBSERVATIONS 160 DEGREES OF FREEDOM 158
R**2 .98548817 RBAR**2 .98539632
SSR .31069384 SEE .44344297E-01
DURBIN-WATSON .06198289
Q(36)= 818.569 SIGNIFICANCE LEVEL .000000

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	7.016528	.7044444E-02	996.0372
2	TREND	24	0	.7862309E-02	.7590264E-04	103.5841

DEPENDENT VARIABLE 25 LOGHQ
FROM 1947: 1 UNTIL 2026: 4
OBSERVATIONS 320 DEGREES OF FREEDOM 317
R**2 .98351946 RBAR**2 .98341548
SSR .75454189 SEE .48787891E-01
DURBIN-WATSON .04651863
Q(51)= 2824.53 SIGNIFICANCE LEVEL .000000

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	6.915436	.6121456E-02	1129.704
2	CON2A	27	0	.8434927E-01	.5454652E-02	15.46373
3	TREND2	26	0	.8070292E-02	.5904949E-04	136.6700

VI. Estimates of the β_i

95 percent confidence intervals, from the bootstrap, are in parentheses.

ϕ_c, ϕ_d	β_1	β_2	β_3	β_4
1.0,1.0	-.39	.93	-.34	-.40
	(-.68,-.15)	(.41,1.52)	(-.39,-.21)	(-.63,-.18)
.949,.949	-.32	.77	-.37	-.33
	(-.61,-.11)	(.29,1.40)	(-.42,-.25)	(-.58,-.17)
.997,.969	-.37	.87	-.35	-.33
	(-.66,-.13)	(.30,1.47)	(-.48,-.20)	(-.61,-.13)

VII. Asymptotic standard errors for the g_{ij}

ϕ_c, ϕ_d	g_{0Q}	g_{0S}	g_{0H}	g_{HS}	g_{1Q}
1.0,1.0	-.072	.392	.145	-.040	.344
	(.128)	(.152)	(.064)	(.352)	(.050)

VIII. Asymptotic standard errors for the π_{ij}, $\phi_c=\phi_d=1$, both scaled and unscaled data

Scaled data:

DEPENDENT VARIABLE 30 ΔH
FROM 1947: 3 UNTIL 1986: 4
TOTAL OBSERVATIONS 158 SKIPPED/MISSING 0
USABLE OBSERVATIONS 158 DEGREES OF FREEDOM 155
R**2 .48732169 RBAR**2 .48070648
SSR 7262.5946 SEE 6.8451040
DURBIN-WATSON 2.43771175
Q(36)= 49.5592 SIGNIFICANCE LEVEL .656635E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	π_{11}	30	1	.5000198	.6675202E-01	7.490707
2	π_{12}	31	1	.2307379	.5219937E-01	4.420319
3	CONSTANT	0	0	-.1975552	.5453114	-.3622796

DEPENDENT VARIABLE 31 ΔQ
FROM 1947: 3 UNTIL 1986: 4
TOTAL OBSERVATIONS 158 SKIPPED/MISSING 0
USABLE OBSERVATIONS 158 DEGREES OF FREEDOM 155
R**2 .17745930 RBAR**2 .16684587
SSR 19076.316 SEE 11.093827
DURBIN-WATSON 2.19384641
Q(36)= 29.4534 SIGNIFICANCE LEVEL .771581

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	π_{21}	30	1	-.2988348	.1081847	-2.762266
2	π_{22}	31	1	.4889196	.8459927E-01	5.779242
3	CONSTANT	0	0	-.2419853	.8837836	-.2738060

Unscaled (raw) data:

DEPENDENT VARIABLE 30 ΔH
FROM 1947: 3 UNTIL 1986: 4
TOTAL OBSERVATIONS 158 SKIPPED/MISSING 0
USABLE OBSERVATIONS 158 DEGREES OF FREEDOM 155
R**2 .48683366 RBAR**2 .48021216
SSR 30186.405 SEE 13.955322
DURBIN-WATSON 2.38573786
Q(36)= 45.0114 SIGNIFICANCE LEVEL .144211

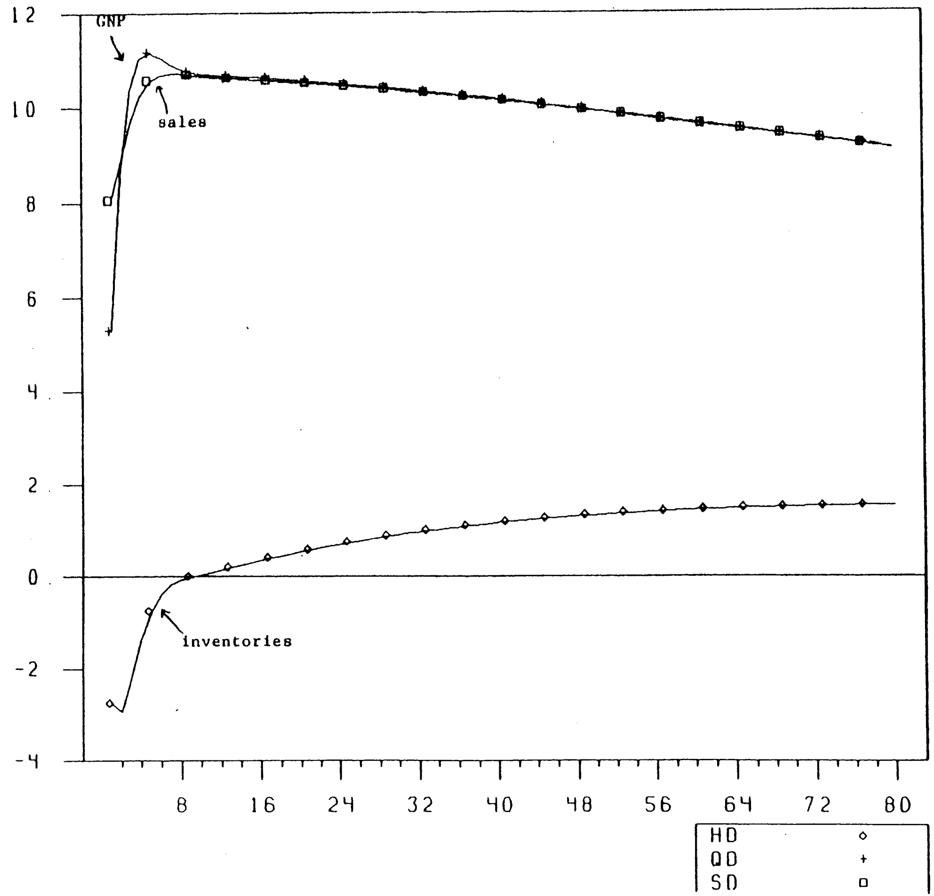
NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTI
***	*******	***	***	************	************	***********
1	π_{11}	30	1	.5329307	.6298521E-01	8.461203
2	π_{12}	31	1	.2232077	.4883594E-01	4.570563
3	CONSTANT	0	0	3.200179	1.471144	2.175300

DEPENDENT VARIABLE 31 ΔQ
FROM 1947: 3 UNTIL 1986: 4
TOTAL OBSERVATIONS 158 SKIPPED/MISSING 0
USABLE OBSERVATIONS 158 DEGREES OF FREEDOM 155
R**2 .16822391 RBAR**2 .15749132
SSR 80816.213 SEE 22.834074
DURBIN-WATSON 2.18309959
Q(36)= 46.2777 SIGNIFICANCE LEVEL .117221

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTI
***	*******	***	***	************	************	***********
1	π_{21}	30	1	-.3528995	.1030581	-3.424277
2	π_{22}	31	1	.4339241	.7990668E-01	5.430386
3	CONSTANT	0	0	14.74241	2.407125	6.124491

IX. Plots of impulse response functions for $\phi_c = .969$, $\phi_d = .997$

RESPONSES TO DEMAND SHOCK



RESPONSES TO COST SHOCK