Additional Appendix

West, Edison and Cho, "A Utility Based Comparison of Some Models of Exchange Rate Volatility"

This not-for-publication appendix contains results omitted from the body of the paper to save space. Following are:

I. Plots of annualized interest rate differentials

II. Proof of proposition

III. Notes on one week interest rate data.

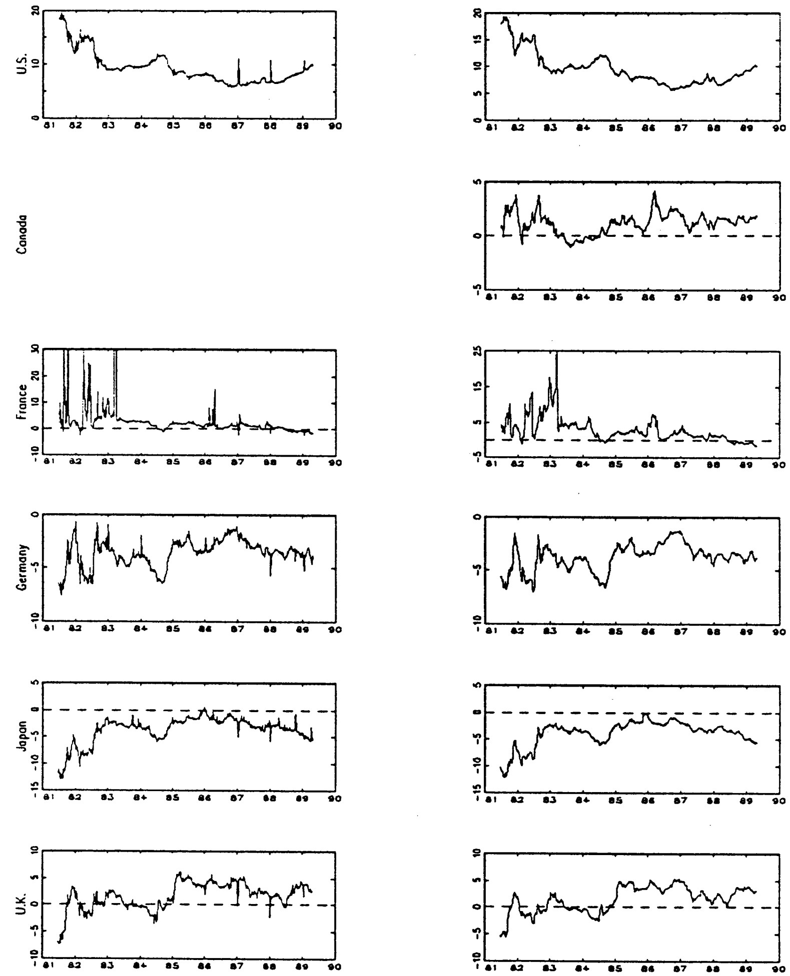
IV. Details of the results underlying summary of utility based results for alternative specifications.

V. Details of the results underlying summary of mean squared error results.

Much additional information on the exchange rate data and on the estimates of the models is in West and Cho (1992) and the additional appendix to West and Cho (1992).

Annualized Interest Rate Differentials (percent)

A. Weekly Rates B. Quarterly Rates



Non-U.S. rates are expressed as an excess over U.S. rate. Weekly data for Canada are not available. Data are described in the text.

II. Proof of Proposition

Assume exponential utility, with all assets are risky. The proof for quadratic utility is similar. For either utility function, the proof when there is a riskless asset follows as a special case. To simplify notation, all time subscripts are dropped.

Let the gross return on asset i be R_i, $i=1,\ldots,k$. Let $H=[H_{ij}]$ be the corresponding (kxk) full rank variance covariance matrix, assumed known for the moment. Let $r_i=R_i-R_k$ be the return in excess of the return on asset i for $i\le k-1$, $r=(r_1,\ldots,r_{k-1})'$, $\mu=Er$ and $\Omega=E(r-\mu)(r-\mu)'=QHQ'$, where the (k-1)xk matrix Q has 1 in row i, column i, for $i\le k-1$, -1 in all rows in column k, and 0 elsewhere. Let ω be the (k-1)x1 vector of covariances of r with R_k, $\omega=QHq$, where the kx1 vector q has 1 in the k'th row and zero's elsewhere. Let f_i be the fraction put in asset i, $i=1,\ldots,k-1$, with $1-f_1-\ldots-f_{k-1}$ the fraction in asset k. The problem is to maximize $E\{-\exp[-\theta W(f'r+R_k)]\} = -\exp[-\theta WE(f'r+R_k) +.5\theta^2W^2\text{var}(f'r+R_k)] = -\exp[-\theta Wf'\mu-\theta WER_k +.5\theta^2W^2(f'\Omega f+2f'\omega+H_{kk})] = -c.\exp[-\theta Wf'\mu +.5\theta^2W^2(f'\Omega f+2f'\omega)]$, $c=\exp(-\theta WER_k+.5\theta^2W^2H_{kk})>0$; the first equality follows since returns are normally distributed. Then $f = \Omega^{-1}[(\mu/\theta W)-\omega]$.

Now let $\hat{H}$ be an estimate of the variance covariance matrix, $\hat{f} = \hat{\Omega}^{-1}[(\mu/\theta W)-\hat{\omega}]$, $\hat{\Omega}=Q\hat{H}Q'$, $\hat{\omega}=Q\hat{H}q$. Expected utility, then, is

$$\text{(A1)}\quad -c.\exp[-\theta W\hat{f}'\mu + .5\theta^2W^2(\hat{f}'\Omega\hat{f}+2\hat{f}'\omega)]$$
$$= -c.\exp[.5(\mu-\theta W\hat{\omega})'\hat{\Omega}^{-1}\Omega\hat{\Omega}^{-1}(\mu-\theta W\hat{\omega}) - (\mu-\theta W\hat{\omega})'\hat{\Omega}^{-1}(\mu-\theta W\omega)].$$

Let V be positive semidefinite, $\hat{H}_1=H+V$, $\hat{H}_2=H-V$. We wish to show that (A1) is larger when $\hat{\Omega}=\hat{\Omega}_1=Q\hat{H}_1Q'=\Omega+QVQ'$, $\hat{\omega}=\omega+QVq=\omega+v$ than when $\hat{\Omega}=\hat{\Omega}_2=Q\hat{H}_2Q'$, $\hat{\omega}=\omega-QVq=\omega-v$. Let $\Omega^{1/2}$ be a square root of Ω, $\Omega=\Omega^{1/2}\Omega^{1/2\prime}$. Then $\hat{\Omega}_1 = \Omega^{1/2}(I+\Omega^{-1/2}QVQ'\Omega^{-1/2\prime})\Omega^{1/2\prime}$, $\hat{\Omega}_2 = \Omega^{1/2}(I-\Omega^{-1/2}QVQ'\Omega^{-1/2\prime})\Omega^{1/2\prime}$. Since $\Omega^{-1/2}QVQ'\Omega^{-1/2\prime}$ is symmetric and positive semidefinite, it can be written as $P\Lambda P'$, where $PP'=I$ and $\Lambda=\text{diag}(\lambda_1,\ldots,\lambda_{k-1})$ is

the diagonal matrix of its eigenvalues. For future reference, note that it may be shown that since V is positive semidefinite, Q is of rank k-1 and $\hat{H}_2=H-V$ positive definite, $0\le\lambda_i<1$.

Let a_i' be the 1x(k-1) i'th row of $P'\Omega^{-1/2}$. Since $\hat{\Omega}_1 = \Omega^{1/2}P(I+\Lambda)P'\Omega^{1/2\prime}$, we have

$$\text{(A2)}\quad E(U_{t+1}|\hat{H}=H+V=\hat{H}_1) =$$
$$-c.\exp\{.5(\mu-\theta W\hat{\omega})'\hat{\Omega}_1^{-1}\Omega\hat{\Omega}_1^{-1}(\mu-\theta W\hat{\omega}) - (\mu-\theta W\hat{\omega})'\hat{\Omega}_1^{-1}(\mu-\theta W\omega)] =$$
$$-c.\exp\{.5[\mu-\theta W(\omega+v)]'\Omega^{-1/2\prime}P(I+\Lambda)^{-2}P'\Omega^{-1/2}[\mu-\theta W(\omega+v)]$$
$$- [\mu-\theta W(\omega+v)]'\Omega^{-1/2\prime}P(I+\Lambda)^{-1}P'\Omega^{-1/2}(\mu-\theta W\omega)\} =$$
$$-c.\exp\{\ .5\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega+v)]'a_i/(1+\lambda_i)\}^2 -$$
$$\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega+v)]'a_i\}[(\mu-\theta W\omega)'a_i]/(1+\lambda_i)\ \}$$

Similarly

$$\text{(A3)}\quad E(U_{t+1}|\hat{H}=H-V=\hat{H}_2) =$$
$$-c.\exp\{\ .5\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega-v)]'a_i/(1-\lambda_i)\}^2 -$$
$$\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega-v)]'a_i\}[(\mu-\theta W\omega)'a_i]/(1-\lambda_i)\ \}.$$

Thus,

$$\text{(A4)}\quad E(U_{t+1}|\hat{H}=H+V) \ge E(U_{t+1}|\hat{H}=H-V) \iff$$
$$.5\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega+v)]'a_i/(1+\lambda_i)\}^2 -$$
$$\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega+v)]'a_i\}[(\mu-\theta W\omega)'a_i]/(1+\lambda_i)\} + .5\Sigma_{i=1}^{k-1}[(\mu-\theta W\omega)'a_i]^2$$
$$\le\ .5\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega-v)]'a_i/(1-\lambda_i)\}^2 -$$
$$\Sigma_{i=1}^{k-1}\{[\mu-\theta W(\omega-v)]'a_i\}[(\mu-\theta W\omega)'a_i]/(1-\lambda_i) + .5\Sigma_{i=1}^{k-1}[(\mu-\theta W\omega)'a_i]^2$$
$$\iff .5\Sigma_{i=1}^{k-1}\{\ [\mu-\theta W(\omega+v)]'a_i/(1+\lambda_i) - (\mu-\theta W\omega)'a_i\ \}^2 \le$$
$$.5\Sigma_{i=1}^{k-1}\{\ [\mu-\theta W(\omega-v)]'a_i/(1-\lambda_i) - (\mu-\theta W\omega)'a_i\ \}^2.$$

It is easily verified that since $0\le\lambda_i<1$, the inequality holds for each i and thus for the sum as well.

That equality holds in the proposition if and only if the two estimates yield the optimal fraction follows since it may be shown that

(A5) $E(U_{t+1}|\hat{H}=H+V) = E(U_{t+1}|\hat{H}=H-V)$ <==>

$\theta W v = -QVQ'\Omega^{-1}(\mu-\theta W\omega)$ <==>

$(\Omega+QVQ')^{-1}[(\mu/\theta W)-\omega-v] = \Omega^{-1}[(\mu/\theta W)-\omega] = (\Omega-QVQ')^{-1}[(\mu/\theta W)-\omega+v]$,

where the three expressions on the last line are the vectors of fractions chosen if $\hat{H}=H+V$, $\hat{H}=H$ and $\hat{H}=H-V$. The second line follows from the first by noting that the first line requires that the i'th term on the left hand side of the final expression in (A4) be the same as the i'th on the right hand side for all i, writing these k-1 equalities in matrix form and manipulating the resulting expression; the third line in (A5) follows from the second by straightforward algebraic manipulation.

III. Notes on one week interest rate data.

The raw data had both bid and asked rates. Some observations had bid higher > asked. We checked all of these against microfiche copies of The London Financial Times, and corrected five errors. We then rounded off both bid and asked to two digits, and then, as noted in the text, we averaged the two.

We had no one week interest rates for France the entire week of 10/8/84-10/12/84. So for 10/10/84, we simply used the quarterly rate.

IV. Details of the results underlying summary of utility based results for alternative specifications.

The format of the following tables is the same as that of Table III, except that there are no parentheses around asymptotic standard errors. Except when otherwise note, the CRRA is set to 1.

WEEKLY HORIZON

6/17/81 - 5/8/85

	homo	(1,1)	ig	e2AR	$\|e\|AR$	nonp	$\chi^2(5)$
FR	1751.137	182.757	312.280	491.861	109.895	0.000	2.594
	1940.082	532.339	790.502	882.192	342.864		0.762
GE	39.477	85.531	0.000	580.862	299.194	142.211	10.950
	45.437	75.656		343.289	143.503	90.396	0.052
JA	83.348	0.000	72.101	69.655	294.727	300.874	9.614
	340.482		73.101	357.724	177.966	412.574	0.087
UK	314.885	0.000	83.908	357.181	413.877	825.967	4.720
	186.219		69.683	235.567	297.481	700.345	0.451

5/15/85 - 4/5/89

	homo	(1,1)	ig	e2AR	$\|e\|AR$	nonp	$\chi^2(5)$
FR	0.529	0.000	2.206	24.010	132.444	25.299	3.134
	15.756		4.481	29.869	130.203	19.699	0.679
GE	45.519	8.567	0.000	57.188	118.587	471.335	5.515
	43.197	7.674		36.058	63.504	449.778	0.356
JA	0.000	8.701	16.965	64.720	51.907	1083.573	5.143
		11.967	16.233	50.794	33.777	865.387	0.399
UK	98.051	3.528	0.000	471.013	58.881	141.971	5.642
	61.974	10.272		390.191	35.912	78.621	0.343

6/17/81 - 5/25/83

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
FR	3526.629	383.410	637.188	1008.032	227.282	0.000	3.083
	3834.682	1059.816	1578.587	1754.576	678.977		0.687
GE	71.979	147.479	0.000	966.968	295.125	194.531	4.983
	78.653	149.410		684.046	192.316	185.282	0.418
JA	569.818	0.000	22.601	595.079	560.294	1063.167	13.557
	297.299		23.391	426.326	315.797	493.083	0.019
UK	158.673	0.000	148.392	347.664	279.983	102.352	9.173
	130.092		131.015	305.930	247.311	62.896	0.102

6/1/83 - 5/8/85

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
FR	4.492	5.276	10.991	0.000	15.379	22.449	4.162
	10.169	10.725	10.909		13.190	26.723	0.526
GE	6.961	23.557	0.000	194.595	303.265	89.869	11.727
	29.214	39.742		92.183	189.164	37.601	0.039
JA	58.474	461.643	583.178	5.841	490.556	0.000	26.752
	30.253	632.970	787.473	77.652	625.298		0.000
UK	471.158	0.000	19.399	366.701	547.822	1549.860	3.348
	352.324		40.753	321.381	515.428	1401.215	0.646

5/15/85 - 4/22/87

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
FR	0.000	1.269	7.905	48.309	265.692	51.034	3.546
		31.488	33.997	56.015	258.062	48.855	0.616
GE	73.886	2.569	0.000	108.861	217.659	925.882	6.404
	83.146	10.918		62.307	110.320	889.855	0.269
JA	0.000	5.302	33.942	25.335	53.423	425.173	5.015
		10.022	33.049	16.607	30.358	415.898	0.414
UK	190.507	10.173	0.000	880.053	113.118	275.791	6.154
	112.633	19.998		766.484	71.327	139.564	0.292

4/29/87 - 4/5/89

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
FR	4.552	2.224	0.000	3.204	2.686	3.057	9.361
	2.063	1.532		1.680	1.722	1.444	0.096
GE	17.165	14.562	0.000	5.540	19.563	17.011	5.357
	27.959	12.401		23.418	16.981	26.462	0.374
JA	0.003	12.102	0.000	104.092	50.395	1741.686	4.378
	0.008	22.888		98.492	61.114	1674.298	0.496
UK	8.681	0.000	3.119	64.941	7.743	11.220	10.014
	17.989		8.405	36.643	16.290	16.302	0.075

6/17/81 - 4/5/89, CRRA=10

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
FR	20.543	1.763	3.313	5.722	2.547	0.000	2.987
	23.327	6.435	9.522	10.634	4.504		0.702
GE	1.034	1.138	0.000	7.728	5.104	7.531	10.891
	0.768	0.933		4.378	1.988	5.606	0.054
JA	0.916	0.000	0.983	1.547	4.142	16.931	10.218
	4.490		1.014	4.677	2.054	11.818	0.069
UK	4.970	0.000	0.968	10.004	5.681	11.718	7.481
	2.703		0.755	5.392	3.634	9.022	0.187

QUARTERLY HORIZON

6/17/81 - 5/8/85

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	15.603	4.322	139.083	18.355	43.080	0.000	5.256
	11.870	9.737	101.215	9.493	34.093		0.385
FR	344.415	0.000	12.976	285.598	402.948	334.139	9.935
	218.848		44.353	172.948	215.682	187.619	0.077
GE	70.997	39.507	0.000	93.694	110.580	100.381	2.000
	71.493	31.008		77.605	84.401	90.123	0.849
JA	266.587	18.975	0.000	101.474	108.471	321.186	12.191
	139.527	30.773		76.188	100.040	154.527	0.032
UK	161.066	0.000	17.428	55.026	46.625	145.511	5.525
	103.315		10.310	44.737	39.232	99.987	0.355

5/15/85 - 4/5/89

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	0.000	121.510	1934.938	51.618	157.890	12.190	14.345
		61.635	1393.196	29.767	122.610	12.508	0.014
FR	17.962	0.000	3.987	11.173	17.639	26.030	3.889
	17.338		6.997	12.225	17.905	19.418	0.565
GE	22.344	4.881	35.028	0.000	20.665	23.238	6.076
	20.926	9.700	41.116		11.421	15.728	0.299
JA	0.000	10.468	80.920	8.451	70.836	11.683	14.010
		6.869	57.827	9.685	29.690	9.853	0.016
UK	0.000	95.152	241.086	40.338	65.523	14.502	6.781
		58.775	140.311	34.395	49.417	7.207	0.237

6/17/81 - 5/25/83

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	60.726	13.880	0.000	42.230	52.867	25.264	5.179
	86.938	70.346		81.143	102.734	76.702	0.394
FR	713.601	0.000	47.365	595.635	804.458	692.711	10.102
	406.752		90.169	308.543	380.108	332.015	0.072

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
GE	49.491	55.421	0.000	138.180	171.258	136.816	7.714
	122.291	58.650		145.794	162.620	172.304	0.173
JA	362.110	0.173	0.000	107.461	180.031	496.731	26.634
	254.191	23.338		125.391	193.741	284.949	0.000
UK	19.177	4.804	23.224	0.000	0.235	9.653	4.148
	28.066	8.646	28.324		7.582	20.328	0.528

6/1/83 - 5/8/85

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	0.000	24.662	309.504	24.217	63.159	4.473	7.721
		13.808	165.964	13.261	51.463	4.345	0.172
FR	0.511	21.005	0.000	0.159	27.081	0.726	17.920
	27.746	18.630		23.033	39.796	20.764	0.003
GE	92.614	23.511	0.000	48.980	49.592	63.760	11.025
	66.874	13.978		31.688	24.360	46.895	0.051
JA	170.393	37.910	0.000	95.446	36.410	144.410	24.256
	115.364	56.638		93.043	52.857	105.316	0.000
UK	309.118	0.000	16.425	115.411	98.294	287.477	3.328
	182.443		11.009	83.212	72.285	179.287	0.649

5/15/85 - 4/22/87

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	0.000	198.341	3514.622	85.613	290.788	20.972	15.011
		109.933	2594.820	52.230	220.644	22.083	0.010
FR	35.797	0.000	9.209	22.201	35.994	52.280	6.996
	30.426		14.178	22.341	33.727	34.049	0.221
GE	116.217	46.478	0.000	54.170	86.260	110.142	8.396
	48.885	21.951		24.567	37.858	40.494	0.136
JA	0.000	3.523	162.087	4.192	46.193	16.017	15.135
		6.145	103.114	6.764	21.484	11.841	0.010
UK	0.000	150.102	370.854	53.432	86.664	23.540	11.184
		107.984	255.008	64.723	91.644	13.496	0.048

4/29/87 - 4/5/89

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	0.000	44.405	349.608	17.500	24.516	3.376	10.745
		21.496	211.127	16.780	37.278	8.123	0.057
FR	1.333	1.247	0.000	1.367	0.487	0.964	3.711
	1.654	1.502		1.529	1.150	1.526	0.592
GE	0.000	34.660	141.106	17.259	26.502	7.837	17.394
		18.533	74.200	9.635	17.690	5.290	0.004
JA	0.013	17.407	0.000	12.711	95.421	7.374	12.256

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
	0.012	11.423		17.113	53.548	15.276	0.031
UK	0.000	40.035	110.924	27.205	44.318	5.436	6.437
		19.886	53.822	13.369	22.511	3.065	0.266

6/17/81 - 4/5/89, CRRA=10

	homo	(1,1)	ig	e2AR	|e|AR	nonp	$\chi^2(5)$
CA	0.000	1.095	19.259	0.524	1.736	0.046	9.155
		0.665	14.177	0.315	1.298	0.140	0.103
FR	2.552	0.000	0.139	1.964	2.747	2.617	7.828
	1.613		0.282	1.212	1.461	1.545	0.166
GE	0.410	0.036	0.000	0.350	0.744	0.634	6.203
	0.845	0.493		0.747	0.811	0.865	0.287
JA	3.132	0.000	0.471	0.755	1.437	2.826	12.137
	1.303		0.664	0.605	0.928	1.457	0.033
UK	0.502	0.051	1.492	0.000	0.164	0.510	5.028
	0.820	0.505	1.253		0.205	0.780	0.412

V. Details of the results underlying summary of mean squared error results.

Root mean squared errors, one week horizon, apart from a scale factor of 10**-4:

	HOMO	(1,1)	IG	|E|(12)	E2(12)	NONP
FRANCE	5.166	5.352	5.160	5.273	5.200	5.201
GERMANY	4.703	4.783	4.695	4.923	4.767	4.724
JAPAN	4.381	4.323	4.343	4.410	4.387	4.442
U.K.	5.745	5.632	5.562	6.020	5.725	6.537