Exam Formulas Page

Economics 310 **Spring 2009**

- Counting

  (Ordered, With Replacement) The total number of ways k objects can be drawn out of a set of n with replacement is n^k.

  (Ordered, Without Replacement) The total number of ways k objects can be drawn out of a set of n without replacement is $n \times (n-1) \times (n-2) \times \ldots \times (n-k+1) = n!/(n-k)!$.

  (Unordered, Without Replacement) The total number of ways k objects can be drawn out of a set of n without replacement is $n!/(k!(n-k)!)$.

  (Unordered, With Replacement) The total number of ways k objects can be drawn out of a set of n with replacement is $(n+k-1)!/(k!(n-1)!)$.

- $P(A \cup B) + P(A \cap B) = P(A) + P(B)$

- Total Probability Rule: Let $B_1, \ldots, B_n$ be a partition of S. For any event $A \subset S$,

$$P(A) = \sum_{i=1}^{n} P(A \cap B_i)$$

  Bayes Rule: If these events all have positive probability, then

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)}$$

- Covariance for discrete r.v.'s X, Y:

$$Cov(X,Y) = \sum_x \sum_y (x - E(X))(y - E(Y))P(X = x, Y = y)$$

- $Cov(aX + b, cY + d) = ac\,Cov(X,Y)$

- Two asset portfolio choice - Backward bending feasible set condition:

$$Cov(R_1, R_2) < \min\{Var(R_1), Var(R_2)\}$$

- Binomial Distribution: $X \sim Bin(n,p)$; $E(X) = np$, $Var(X) = np(1-p)$

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad \text{for } x = 0, 1, \ldots, n$$

- Poisson Distribution: $Y \sim Poisson(\lambda)$; $E(Y) = \lambda$, $Var(Y) = \lambda$

$$\Pr(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!} \quad \text{for } y = 0, 1, \ldots, n$$

- Uniform Distribution: $U \sim Unif[a,b]$: $E(U) = (a+b)/2$, $Var(U) = (b-a)^2/12$

$$\Pr(U \leq t) = \frac{t-a}{b-a} \quad \text{for } a \leq t \leq b$$

- Exponential Distribution: $T \sim Exp(\lambda)$; $E(T) = 1/\lambda$, $Var(T) = 1/\lambda^2$

$$\Pr(T > t+s | T > s) = \Pr(T > t) = e^{-\lambda t} \quad \text{for } s,t \geq 0$$

- Standard Normal Distribution: $Z \sim N(0,1)$

$$\Pr(Z \leq 1.645) = .95, \quad \Pr(Z \leq 1.96) = .975, \quad \Pr(Z \leq 2.33) = .99, \quad \Pr(Z \leq 2.576) = .995$$

- Chebyshev's Inequality: For a random variable W with mean μ_W and variance σ_W^2 and any $c > 0$,

$$\Pr(|W - \mu_W| \geq c) \leq \frac{\sigma_W^2}{c^2}$$

- Let $Y_1, Y_2, \ldots$ be a sequence of random variables and let Z be a continuous random variable. The sequence $Y_1, Y_2, \ldots$ *converges in distribution* to Z if for all c, $\Pr(Y_n \leq c) \longrightarrow \Pr(Z \leq c)$ as $n \longrightarrow \infty$.

- LLN/CLT: Let $X_1, X_2, \ldots$ be an i.i.d. sequence of random variables with mean $\mu = E(X_i)$ and variance $\sigma^2 = Var(X_i)$.
  LLN: Then for every $\varepsilon > 0$, $\Pr(\mu - \varepsilon < \bar{X}_n < \mu + \varepsilon) \longrightarrow 1$ as $n \longrightarrow \infty$.
  CLT: Then $\sqrt{n}(\bar{X}_n - \mu)/\sigma$ converges in distribution to $Z \sim N(0,1)$ as $n \longrightarrow \infty$.

- Let (T, Y) be discrete random variables. Fix a value t such that $\Pr(T = t) > 0$. The *conditional expectation* of Y given $T = t$ is

$$E(Y|T=t) = \sum_y y \Pr(Y = y | T = t).$$

- Law of Iterated Expectation: $E(Y) = E[E(Y|T)]$

- The α quantile of the random variable X is defined as the value q such that $\Pr(X \leq q) = \alpha$ (for $0 < \alpha < 1$).

- In hypothesis testing, Type I Error is rejecting the null hypothesis when the null is true. A Type II Error is not rejecting the null hypothesis when the null is false. The significance level is the maximum allowable probability of Type I Error.