Correlation vs. Causation

- College students living in co-ed dorms are more likely to binge drink

  $Y =$ Binge drinking frequency;
  $T =$ Living in a co-ed dorm

- Cities with larger police forces tend to have higher crime rates

  $Y =$ City crime rate;
  $T =$ Size of police force

- Grade school students who eat dinner with their family often are more likely to have higher grades

  $Y =$ Student grades;
  $T =$ Frequency of eating dinner with family

- Those with higher education are more likely to have greater lifetime earnings

  $Y =$ Lifetime earnings;
  $T =$ Educational attainment

- The more firemen sent to a fire the greater the fire damage

  $Y =$ Fire damage;
  $T =$ Number of firemen sent to a fire

- Those who ride to work in the back of limousines tend to have higher salaries

  $Y =$ Salary;
  $T =$ Indicator of riding to work in limo

Modeling Causality

Counterfactual Notion of Causality:

The statement that

Smoking causes cancer

is relative to what would have happened if Not Smoking.

Canonical Example:

Medical intervention/Drug Treatment

Let r.v. T denote *treatment status*:

$$T = \begin{cases} 1 & \text{if received treatment (e.g. drug)} \\ 0 & \text{if did not receive treatment (e.g. placebo)} \end{cases}$$

Let r.v.'s Y_0 and Y_1 denote *potential outcomes*:

$$\begin{aligned} Y_0 &= \text{outcome if no treatment} \quad (T = 0 \text{ case}) \\ Y_1 &= \text{outcome if treated} \quad (T = 1 \text{ case}) \end{aligned}$$

For an individual i, treatment and potential outcomes will be denoted: $T_i, Y_{0,i}, Y_{1,i}$

Definition. For individual i, the <u>*causal effect*</u> of treatment T is

$$Y_{1,i} - Y_{0,i}$$

Fundamental challenge to finding causal effects and determining causation:

We never observe *both* $Y_{1,i}$ and $Y_{0,i}$.

Let r.v. Y_i denote the *observed outcome* for individual i:

$$Y_i = \begin{cases} Y_{0,i} & \text{if } T_i = 0 \\ Y_{1,i} & \text{if } T_i = 1 \end{cases}$$

So, $Y_i = T_i Y_{1,i} + (1 - T_i) Y_{0,i}$.

Observe: (T_i, Y_i)

Unobserved: $Y_{1,i} - Y_{0,i}$

Consider

$$\textit{Average Causal Effect} \;=\; E(Y_{1,i} - Y_{0,i})$$

Note $E(Y_{1,i} - Y_{0,i}) \;=\; E(Y_{1,i}) \;-\; E(Y_{0,i})$

Can the Average Causal Effect be determined given the distribution of the observables (Y_i, T_i)?

Given the distribution of the observables (Y_i, T_i), we could determine

$$E(Y_i|T_i = 1) - E(Y_i|T_i = 0)$$

How does this compare to $E(Y_{1,i}) - E(Y_{0,i})$?

Recall:

$$E(Y_i|T_i = 1) - E(Y_i|T_i = 0) = Corr(Y_i, T_i) \times \frac{\sqrt{Var(Y_i)}}{\sqrt{P(T_i = 1)P(T_i = 0)}}$$

Example

Do college scholarships improve school performance?

$Y_i =$ freshman GPA for student i

$$T_i = \begin{cases} 1 & \text{student } i \text{ receives college scholarship} > \$1000 \\ 0 & \text{otherwise} \end{cases}$$

Interpret:

- $Y_{1,i}, \quad Y_{0,i}, \quad Y_{1,i} - Y_{0,i}$
- $E(Y_{1,i}), \quad E(Y_{0,i}), \quad E(Y_{1,i} - Y_{0,i})$

Observable outcome

$$Y_i = \begin{cases} Y_{1,i} & \text{if } i \text{ receives scholarship } (T_i = 1) \\ Y_{0,i} & \text{if } i \text{ does not receive scholarship } (T_i = 0) \end{cases}$$

Interpret:

- $E(Y_i|T_i = 1), \quad E(Y_i|T_i = 0)$

Compare:

- $E(Y_{1,i}), \quad E(Y_{0,i})$

Suppose

$$Y_{1,i} - Y_{0,i} = .5 \qquad \text{for all } i$$

Then,

- Causal Effect on i is 0.5
- Average Causal Effect is $E(Y_{1,i} - Y_{0,i}) = \mathbf{0.5}$

Now, suppose

$$E(Y_{0,i}|T_i = 1) = 3 \quad \text{and} \quad E(Y_{0,i}|T_i = 0) = 2$$

Then,

$$E(Y_{1,i}|T_i = 1) = 3.5 \quad \text{and} \quad E(Y_{1,i}|T_i = 0) = 2.5$$

And,

$$\begin{aligned} E(Y_i|T_i = 1) - E(Y_i|T_i = 0) &= E(Y_{1,i}|T_i = 1) - E(Y_{0,i}|T_i = 0) \\ &= 3.5 - 2.0 = \mathbf{1.5} \end{aligned}$$

When does

$$\mathbf{E(Y_{1,i}) = E(Y_i|T_i = 1)}?$$

Recall $E(Y_i|T_i = 1) \;=\; E(Y_{1,i}|T_i = 1)$

$$\begin{aligned}
\mathbf{E(Y_{1,i})} &= E(Y_{1,i}|T_i = 1)P(T_i = 1) \;+\; E(Y_{1,i}|T_i = 0)P(T_i = 0) \\
&= E(Y_{1,i}|T_i = 1)(1 - P(T_i = 0)) \;+\; E(Y_{1,i}|T_i = 0)P(T_i = 0) \\
&= \mathbf{E(Y_{1,i}|T_i = 1)} - \underbrace{P(T_i = 0)\,[E(Y_{1,i}|T_i = 1) - E(Y_{1,i}|T_i = 0)]}_{= 0?}
\end{aligned}$$

$E(Y_{1,i}) = E(Y_i|T_i = 1)$ if

1. $P(T_i = 0) = 0$ (equivalently $P(T_i = 1) = 1$)

   OR

2. $E(Y_{1,i}|T_i = 1) \;=\; E(Y_{1,i}|T_i = 0)$

So, $E(Y_{1,i}) = E(Y_i|T_i = 1)$ if

1. $P(T_i = 0) = 0$ (equivalently $P(T_i = 1) = 1$)

   OR

2. $E(Y_{1,i}|T_i = 1) \;=\; E(Y_{1,i}|T_i = 0)$

Similarly, $E(Y_{0,i}) = E(Y_i|T_i = 0)$ if

1. $P(T_i = 1) = 0$ (equivalently $P(T_i = 0) = 1$)

   OR

2. $E(Y_{0,i}|T_i = 1) \;=\; E(Y_{0,i}|T_i = 0)$

Interpret:

$E(Y_{1,i}|T_i = 1) \;=\; E(Y_{1,i}|T_i = 0)$, and $E(Y_{0,i}|T_i = 1) \;=\; E(Y_{0,i}|T_i = 0)$

Conclusion:

IF

(a) $E(Y_{1,i}|T_i = 1) \;=\; E(Y_{1,i}|T_i = 0)$; and

(b) $E(Y_{0,i}|T_i = 1) \;=\; E(Y_{0,i}|T_i = 0)$,

THEN

$$E(Y_{1,i} \;-\; Y_{0,i}) \;=\; E(Y_i|T_i = 1) - E(Y_i|T_i = 0)$$

Experiments

Key to Treatment Assignment

$$(Y_{1,i}, Y_{0,i}) \text{ independent } T_i$$

Then,

$$\begin{aligned} E(Y_{1,i}|T_i = 1) &= E(Y_{1,i}|T_i = 0), \text{ and} \\ E(Y_{0,i}|T_i = 1) &= E(Y_{0,i}|T_i = 0) \end{aligned}$$

Hence,

$$E(Y_i|T_i = 1) - E(Y_i|T_i = 0) = E(Y_{1,i} - Y_{0,i})$$

Conditional Independence of Treatment Assignment:

$$(Y_{1,i}, Y_{0,i}) \text{ independent } T_i \mid X_i$$

Then,

$$\underbrace{E(Y_i|T_i = 1, X_i = x) - E(Y_i|T_i = 0, X_i = x)}_{\textit{Observable Distribution } (Y_i, T_i, X_i)} = \underbrace{E(Y_{1,i} - Y_{0,i}|X_i = x)}_{\textit{Conditional Average Causal Effect}}$$

suggests a "regression" approach (Econ 410!!)