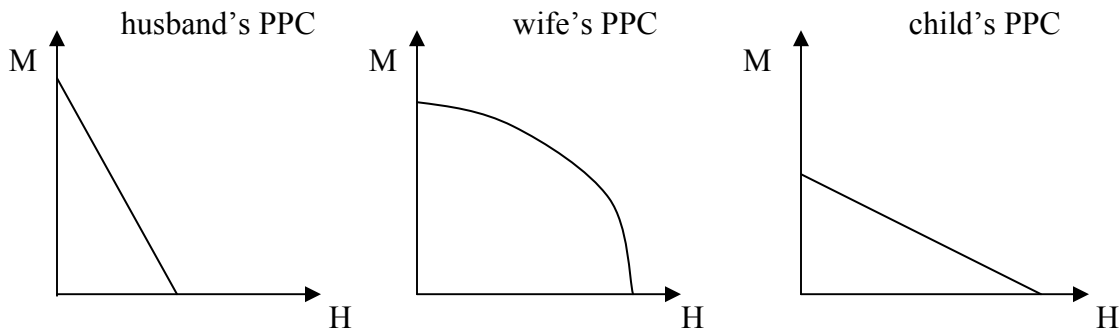


Answer all questions. 100 points possible.

1. [30 points] Consider a household composed of a husband, wife, and child. Each member of the household allocates time to produce market goods (M) and/or household goods (H) in order to maximize household utility  $U(M, H)$ . Assume that both the husband and the child have constant returns to production in each sector (with the husband having the comparative advantage at market work). Further assume that the wife has diminishing returns to production in each sector, and that her PPC is initially flat at the vertical intercept (with slope close to zero), eventually very steep at the horizontal intercept (with slope approaching  $-\infty$ ), and smooth (with continually changing slope and no “kink” points). Graphically, the individual PPCs are given by



Draw the combined PPC for the household, and briefly discuss its shape. [HINT: Obviously, you don't have numerical information about the individual PPCs. But you do have enough information to illustrate the qualitative features of the combined PPC. To receive full credit, your graph should be neat and appropriately labeled.] Then, assuming that the household is efficient, state whether each of the following outcomes is *possible* or *impossible*.

- husband and wife work only in market; child works only in household
- husband and wife work in both sectors; child works only in household
- husband works in both sectors; wife and child work only in household
- all 3 members work in both sectors

2. [15 points] Consider a household composed of a selfish wife with utility function  $U_w = u(Z_w)$  and an envious husband with utility function  $U_h = u(Z_h) - \eta u(Z_w)$ . [Following the notation from lecture, suppose that  $Z_w$  and  $Z_h$  are consumption levels, that  $u(Z)$  is a concave function of  $Z$ , and that  $\eta > 0$  is an envy parameter.] Further suppose that husband is endowed with income  $I_h$  while the wife is endowed with income  $I_w$ . Either spouse can transfer income to the other, but cannot enforce negative transfers. For this household, describe the equilibrium (after-transfer) consumption levels  $Z_h^*$  and  $Z_w^*$  and illustrate by drawing an indifference-curve diagram (or diagrams). If the wife could take an action that would increase her own income ( $I_w$ ) but decrease total income ( $I_w + I_h$ ), would she take this action? Briefly explain.

3. [30 points] Consider a **non-transferable-utility** matching model with 6 males and 6 females. The following matrix gives the payoffs received by each male and female in each potential match. [The matrix specifies the payoffs  $(m_{ij}, f_{ji})$  given a match between male  $i$  and female  $j$ . Assume that each individual receives 0 if he/she remains single.]

		females						
		1	2	3	4	5	6	
males	1	[	(2,3)	(3,6)	(1,7)	(4,4)	(5,8)	(7,5)
	2		(6,7)	(4,2)	(9,2)	(3,7)	(8,6)	(5,1)
	3		(5,5)	(7,4)	(6,5)	(8,3)	(2,2)	(3,4)
	4		(3,8)	(2,8)	(7,9)	(6,6)	(4,1)	(5,2)
	5		(9,4)	(6,7)	(7,3)	(3,8)	(4,4)	(5,3)
	6		(8,2)	(5,5)	(6,4)	(7,5)	(3,9)	(4,6)

a) Is the match structure  $\{M1-F6, M2-F5, M3-F4, M4-F3, M5-F2, M6-F1\}$  stable? If not, give one reason why.

b) Derive two stable match structures.

c) In the present example, for every possible stable match structure, will M1 will always be matched with the same partner? Are there more than two possible stable match structures? What theoretical result do you need to know in order to answer these questions? Briefly explain.

4. [25 points] Consider a **transferable-utility** matching model with 2 males and 2 females. Suppose that the surplus matrix is given by

		females		
		1	2	
males	1	[	1	3
	2		4	x

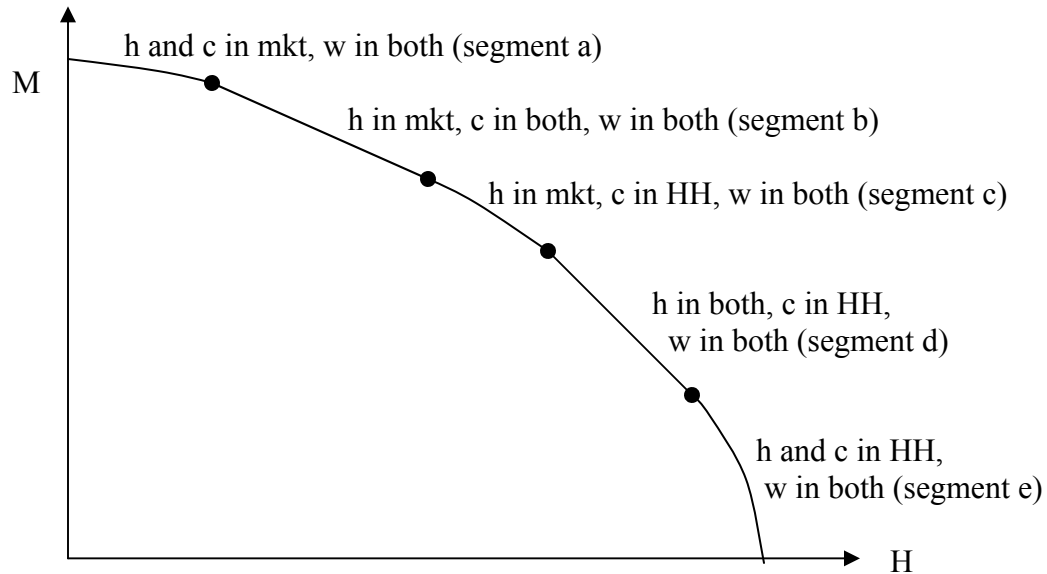
where  $x$  will be given below. For each of the following cases (with different values of  $x$ ), can the match structure  $(M1-F1, M2-F2)$  be supported by some vector of shares  $(\theta_{11}^*, \theta_{22}^*)$ ? [Following the notation from lecture, let  $\theta_{ii}^*$  denote the share of the surplus received by  $M_i$  in match  $M_i-F_i$ , so that  $1-\theta_{ii}^*$  is the share received by  $F_i$ .] If so, give one possible vector of shares that will support this match structure. [HINT: You only need to give one.] If not, explain why this match structure cannot be supported.

a)  $x = 5$

b)  $x = 6$  [HINT: Assume that the relevant inequalities are weak (i.e.,  $\leq$  instead of  $<$ ).]

c)  $x = 7$

1) [30 pts] The overall PPC for the household is:

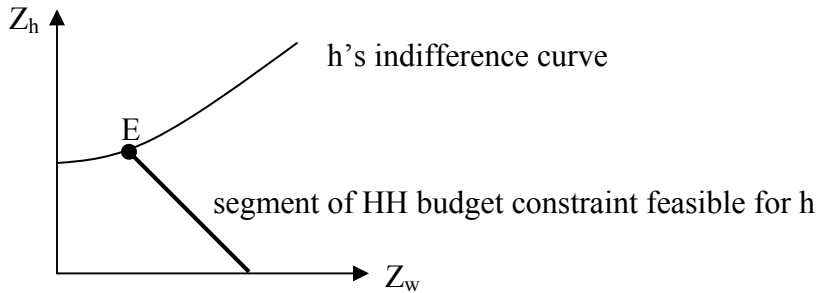


To construct the combined PPC, you might first obtain the vertical intercept by adding together the total market production of the husband and wife and child assuming that all three spend all of their time in that sector. Starting from that point, if the household wanted to consume at least some of the household good, the initial production would be undertaken by the wife who initially (when  $H = 0$ ) has the comparative advantage at producing the household good. (Graphically, at the vertical intercepts of the individual PPCs, the wife's PPC is the flattest.) As the wife begins to produce some of the household good (moving along her individual PPC), we obtain the first (curved) segment of the overall PPC (labeled "segment a"). Eventually, if the wife continues to increase her production of the household good, the slope of her PPC becomes equal to the slope of the child's PPC. At this point, the child would begin to produce some of the household good (moving along his/her individual PPC while the wife remains split between sectors), and we obtain the second (linear) segment of the overall PPC (labeled "segment b"), which has a slope equal to the slope of the child's individual PPC. If the child continues to increase production of the household good, eventually he/she spends all of her time in household production. Assuming that the household wants to consume even more of the household good, the wife will again increase her production of this good (again moving along her PPC), and we obtain the (curved) segment c of the overall PPC. Following this same logic, we can trace out the remainder of the overall PPC.

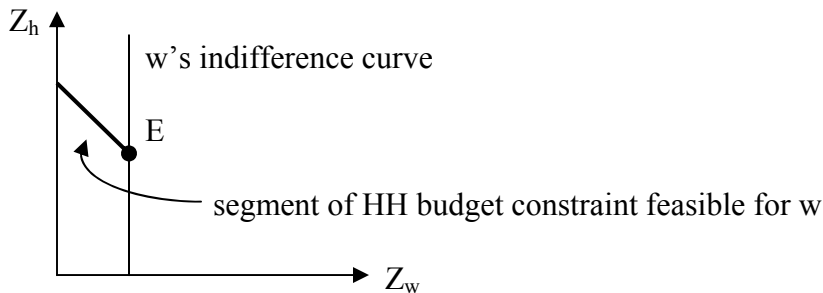
Given the division of labor along each segment of the overall PPC, we find that a) is *impossible*, b) is *possible* [on segment d], c) is *impossible*, and d) is *impossible*.

2) [15 pts] Because neither spouse is altruistic, both are at corner solutions. Transfers are set to zero, and the consumption of each spouse is equal to own income (i.e.,  $Z_w^* = I_w$  and  $Z_h^* = I_h$ ). Graphically,

*h's optimal solution*



*w's optimal solution*



On both graphs, E denotes the endowment point  $(I_w, I_h)$ . Note that the husband's indifference curves are upward sloping (given his envy) while the wife's indifference curves are vertical (given her selfishness).

Yes, the wife would take this action. Because there is no altruist in the household, the Rotten Kid Theorem does not apply. The wife would always take any action that would increase her own income (and hence consumption) regardless of the impact on the husband's (or total household) income.

3a) [6 pts] That match structure is not stable. M5 and F1 would both prefer to leave current partner for each other. (Note both conditions  $m_{51} > m_{52}$  and  $f_{15} > f_{16}$  hold.)

3b) [12 pts] Using the Gale-Shapley algorithm with men choosing, the stable match structure is  $\{M1-F6, M2-F5, M3-F2, M4-F3, M5-F1, M6-F4\}$ . When women choose, the stable match structure is  $\{M1-F6, M2-F5, M3-F1, M4-F3, M5-F2, M6-F4\}$ .

3c) [12 pts] Applying the Gale-Shapley Theorem, we know that the stable match structure generated when men choose yields the highest possible equilibrium utility for each man. Thus, M1 could never have an equilibrium payoff higher than 7. Similarly, from the G-S Theorem, we know that the stable match structure generated when women

choose yields the lowest possible equilibrium utility for each man. Thus, M1 could never have an equilibrium payoff lower than 7. Thus, in any equilibrium match structure, M1 must receive exactly 7, which can occur only if he is matched with F6. Using this reasoning, any equilibrium match structure must contain the pairs M1-F6, M2-F5, M4-F3, and M6-F4. Thus, the only variation across match structures comes from the matching of M3 and M5 with F1 and F2. Since there are only 2 possible ways to match these pairs – either {M3-F1, M5-F2} or else {M3-F2, M5-F1} – there are only 2 possible match structures.

4a) [7 pts] The match structure {M1-F1, M2-F2} cannot be supported because it does not maximize aggregate surplus. The other possible match structure {M1-F2, M2-F1} generates aggregate surplus  $3 + 4 = 7$ , which exceeds  $1 + 5 = 6$ . [Given this result, you did not need to waste time attempting to solve for a vector of shares to support the match structure. But if you tried, you found that the inequalities  $3 < \theta_{11}^* 1 + (1-\theta_{22}^*) 5$  and  $4 < \theta_{22}^* 5 + (1-\theta_{11}^*) 1$  led to the inconsistent conditions  $\theta_{22}^* < \theta_{11}^*/5 + 2/5$  and  $\theta_{22}^* > \theta_{11}^*/5 + 3/5$ .]

4b) [9 pts] The match structure {M1-F1, M2-F2} can be supported. (Note that both match structures now generate aggregate surplus 7.) To keep the M1-F2 match from occurring, the equilibrium shares must satisfy

$$s_{12} \leq \theta_{11}^* s_{11} + (1-\theta_{22}^*) s_{22} \quad \text{which becomes} \quad 3 \leq \theta_{11}^* 1 + (1-\theta_{22}^*) 6;$$

to keep the M2-F1 match from occurring, these shares must satisfy

$$s_{21} \leq \theta_{22}^* s_{22} + (1-\theta_{11}^*) s_{11} \quad \text{which becomes} \quad 4 \leq \theta_{22}^* 6 + (1-\theta_{11}^*) 1.$$

We thus obtain  $\theta_{22}^* \leq \theta_{11}^*/6 + 1/2$  and  $\theta_{22}^* \geq \theta_{11}^*/6 + 1/2$ , and thus the match structure is supported by any pair  $(\theta_{11}^*, \theta_{22}^*)$  that satisfies the equation  $\theta_{22}^* = \theta_{11}^*/6 + 1/2$ . For instance, the match structure is supported by the pair  $(\theta_{11}^* = 1/2, \theta_{22}^* = 7/12)$ . Note that the pair  $(\theta_{11}^* = 1/2, \theta_{22}^* = 1/2)$  does **not** support the match structure.

4c) [9 pts] The match structure {M1-F1, M2-F2} can be supported. (Note that it now generates aggregate surplus 8, and so is the only match structure that can be supported.) To keep the M1-F2 match from occurring, the equilibrium shares must satisfy

$$3 \leq \theta_{11}^* 1 + (1-\theta_{22}^*) 7;$$

to keep the M2-F1 match from occurring, these shares must satisfy

$$4 \leq \theta_{22}^* 7 + (1-\theta_{11}^*) 1.$$

We thus obtain  $\theta_{22}^* \leq \theta_{11}^*/7 + 4/7$  and  $\theta_{22}^* \geq \theta_{11}^*/7 + 3/7$ . Any pair  $(\theta_{11}^*, \theta_{22}^*)$  consistent with these inequalities would support the match structure. For instance, you could set  $(\theta_{11}^* = 1/2, \theta_{22}^* = 4/7)$ . In this case, allowing the inequalities to be weak, the pair  $(\theta_{11}^* = 1/2, \theta_{22}^* = 1/2)$  also support the match structure.

Answer all questions. 100 points possible.

1. [18 points] Two individuals (A and B) are involved in a negotiation. The efficiency frontier for this negotiation is determined by the equation

$$2 U_A + U_B = 100$$

where  $U_A$  is A's utility level and  $U_B$  is B's utility level. Given the threat points  $T_A = 20$  and  $T_B = 10$ , how much utility does each individual receive if the negotiated outcome is determined by the Nash bargaining solution? How does your answer change if the threat points are  $T_A = 15$  and  $T_B = 15$ ? [HINT: I am looking for *numerical* solutions.] Briefly discuss the concept of a "threat point" in a bargaining situation.

2. [29 points] Suppose there are 3 individuals (A, B, C) and 4 possible social outcomes (w, x, y, z). Further suppose that each individual's (strict) preference ordering over outcomes is given by the following table.

		individuals		
		A	B	C
rank	1	w	x	y
	2	x	y	z
	3	y	z	w
	4	z	w	x

a) Using the *Condorcet procedure*, give the social preference order. [HINT: You can either draw a graph (with nodes and directed edges) or list the pairwise comparisons that constitute the social preferences.] Are the social preferences transitive? Briefly explain. Which outcomes are contained in the Condorcet set (i.e., the "top cycle")?

b) As discussed in lecture, an agenda follows the "amendment procedure" when one pair of outcomes is compared, the winner of that pairwise contest is then compared to a third outcome, the winner of that pairwise contest is then compared to a fourth outcome, etc. Assuming that voting is *sincere*, is it possible to construct an agenda using the amendment procedure so that outcome z is the ultimate winner? If so, give an agenda (i.e., the sequence in which outcomes are compared) that leads to z as the ultimate winner. If not, briefly explain why there is no agenda with z as the winner.

c) Consider the following agenda (which follows the amendment procedure): w is compared to x; the winner is then compared to y; the winner is then compared to z. Which outcome will be the ultimate winner if voting is *sophisticated*? Explain why. [HINT: Use a diagram.]

3. [24 points] Modifying the example used in the previous question, suppose that each individual's (strict) preference order is now given by the following table.

		individuals		
		A	B	C
rank	1	w	y	z
	2	y	x	w
	3	x	z	y
	4	z	w	x

a) Use the *Borda count* procedure to determine the social preference order. For the present example, is this social preference order transitive? Will the social preference order *always* be transitive (given any possible combination of individual preference orders as “inputs”) using the Borda count procedure? Briefly explain.

b) Now use the Borda count procedure to determine the social preference order given the individual preferences orders from *question 2* (on the previous page of the exam).

c) Comparing the social preference orders derived in parts (a) and (b) above, does the Borda count procedure satisfy condition I (Independence from Irrelevant Alternatives) from Arrow's Impossibility Theorem? Briefly explain.

4) [29 points] Consider a university with 40,000 students. Each school year, students are required to pay student fees to support various student organizations. Some students would prefer lower fees (and less support for student organizations) while other students would prefer higher fees (and greater support for student organizations). Suppose that student fees could be set at  $x \in \{0, 100, 250, 600, 1000\}$ . Given these 5 possible outcomes, the following table shows the number of students,  $N(x)$ , for whom outcome  $x$  is their most preferred alternative (i.e., “ideal point”).

student fee level [x]	number of students with this ideal point [N(x)]
\$0	15,000
\$100	9,000
\$250	5,000
\$600	4,000
\$1000	7,000

Thus, there are 15,000 students whose ideal point is \$0; there are 9,000 students whose ideal point is \$100; etc. Finally, suppose that all students have *single-peaked* preferences.

[question 4 continued...]

4a) Suppose the *Condorcet procedure* is used to determine the level of student fees. Without working out the entire social preference order (you don't yet have quite enough information), will this social preference order be transitive? Explain. Which outcome will beat all other outcomes? Explain.

b) Further suppose that each student  $i$ 's utility function is given by

$$U_i(x) = c - |x - x_i|$$

where  $x$  is the student fee level,  $x_i$  is the student's ideal point, and  $c$  is a constant. Consider a student  $i$  whose ideal point is  $x_i = 100$ . Give this student's preference order (ranking) of the five outcomes. Now give the preference order for a student  $i$  whose ideal point is  $x_i = 600$ .

c) Instead of using the Condorcet procedure, suppose that student fees are determined by the Student Government president (whose sole responsibility is to set the level of student fees). The Student Government president is elected by the student body. Suppose that two candidates are running for this office: student A whose own ideal point is  $x_A = 0$ , and student B whose ideal point is  $x_B = 1000$ . The candidates' utility functions take the same form as the utility functions of the other students (as described in part b). Thus, the utility of each candidate depends on his/her ideal point and on the fee level  $x$  that is ultimately implemented by the winner of the election. Further suppose that each candidate can credibly commit to implement (assuming he/she wins the election) some fee level  $x$  that differs from his/her own ideal point.

If candidate B commits to set student fees at \$1000, what is candidate A's best response? (i.e., what fee level would candidate A optimally commit to?) Alternatively, if candidate A commits to set fees at \$0, what is candidate B's best response? In Nash equilibrium (assuming both candidates simultaneously commit to fee levels), what student fee will be implemented by the winner of the election? Briefly discuss.

**Econ 451      Exam 2      Fall 2007      Solutions**

1) [18 pts] The Nash bargaining solution sets utility levels to maximize the objective function  $(U_A - T_A)(U_B - T_B)$ . Rewriting the equation for the efficiency frontier as

$$U_B = 100 - 2U_A$$

and substituting it into the objective function, we obtain

$$(U_A - T_A)(100 - 2U_A - T_B)$$

which (given that  $T_A$  and  $T_B$  are constants) is simply a function of  $U_A$ . Differentiating with respect to  $U_A$  and setting this derivative equal to zero, we obtain

$$(U_A - T_A)(-2) + (1)(100 - 2U_A - T_B) = 0$$

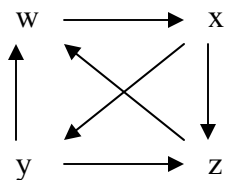
which can be rewritten as  $U_A = 25 + (1/2)T_A - (1/4)T_B$

Given the threat points  $T_A = 20$  and  $T_B = 10$ , we thus obtain  $U_A = 32.5$  and  $U_B = 35$ .

Given the threat points  $T_A = T_B = 15$ , we thus obtain  $U_A = 28.75$  and  $U_B = 42.5$ .

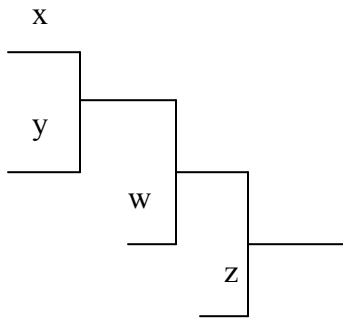
An individual's threat point is the payoff that the individual would receive if the negotiation broke down completely (i.e., it is the individual's "best alternative to negotiated agreement"). The Nash bargaining solution presumes that the negotiated outcome depends on the relative strength of the individuals' threat points (even though these "threats" are never actually carried out).

2a) [14 pts] Given 4 outcomes, the Condorcet procedure requires 6 pairwise comparisons, leading to the collective preferences  $\{wPx, yPw, zPw, xPy, xPz, yPz\}$ . Alternatively, using a graph (where a directed edge from  $x$  to  $y$  denotes  $xPy$ ),



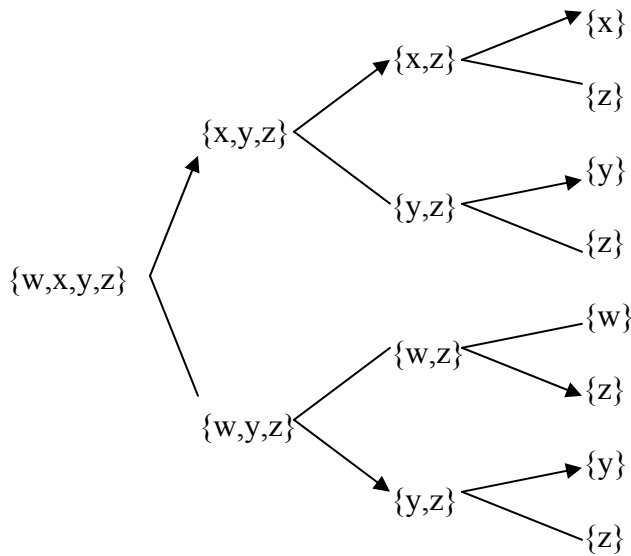
These social preferences are not transitive. Transitive preferences cannot include voting cycles (such as  $wPx$  and  $xPz$  and  $zPw$ ). All of the outcomes  $\{w, x, y, z\}$  are in the Condorcet set. [To see this, you could find a (Hamiltonian) path which begins at each node and passes through the other nodes. E.g., for node  $z$ , the path  $z \rightarrow w \rightarrow x \rightarrow y$ . Alternatively, you can see that there is no proper subset of  $\{w, x, y, z\}$  such that no outcome outside the subset can beat any outcome in the subset.]

b) [6 points] Yes, you can construct such an agenda. For instance,



Given this sequence,  $x$  will beat  $y$ ;  $w$  will beat  $x$ ;  $z$  will beat  $w$ . Note that I've simply reversed the order of the Hamiltonian path that starts at node  $z$  given in my solution to part (a) above.

c) [9 points] To answer this question, it is helpful to represent the agenda using the following tree diagram (giving the set of "live options" following each possible vote).



Applying backward induction, we can first determine the outcome of every final (pairwise) contest. For instance, if the final contest is between  $x$  and  $z$ , the winner would be  $x$  (indicated by the arrow from  $\{x,z\}$  to  $\{x\}$ ). Moving backwards, we can then determine the winners in the previous round. For instance, if the second-round contest is between  $w$  and  $y$ , sophisticated voters would understand that they are "really" voting between  $z$  (which would beat  $w$  in the final round) and  $y$  (which would beat  $z$  in the final round), and hence  $y$  would win (indicated by the arrow from  $\{w,y,z\}$  to  $\{y,z\}$ ). Moving further backwards to consider the first-round contest, sophisticated voters would understand that they are "really" voting between  $x$  and  $y$ . Thus, given the agenda structure,  $x$  would beat  $w$ , then  $x$  would beat  $y$ , then  $x$  would beat  $z$ . Thus,  $x$  is the ultimate winner.

3a) [9 pts] Given 4 outcomes, the Borda count assigns 3 points to each voter's most preferred outcome, 2 points to each voter's second-ranked outcome, 1 point to each voter's third-ranked outcome, and 0 points to each voter's least preferred outcome. Votes are then summed over all voters to determine the social preference order. For the example given, point totals for each outcome would be:

$$\begin{array}{ll} w \text{ receives } 3+0+2 = 5 \text{ points} & x \text{ receives } 1+2+0 = 3 \text{ points} \\ y \text{ receives } 2+3+1 = 6 \text{ points} & z \text{ receives } 0+1+3 = 4 \text{ points} \end{array}$$

Thus, the social preference order is  $yPwPzPx$ . The social preference order generated by the Borda count is always transitive because the ranking is determined by comparison of point totals (and the mathematical relation "is greater than" on any set of integers is necessarily transitive).

b) [6 pts] Applying the Borda count to the example in question 2, we obtain:

$$\begin{array}{ll} w \text{ receives } 3+0+1 = 4 \text{ points} & x \text{ receives } 2+3+0 = 5 \text{ points} \\ y \text{ receives } 1+2+3 = 6 \text{ points} & z \text{ receives } 0+1+2 = 3 \text{ points} \end{array}$$

Thus, the collective preference order is now  $yPxPwPz$ .

c) [9 pts] No, the Borda count does not satisfy condition I. Comparing the two collections of preference orders, note that  $y$  is the "irrelevant alternative." Each individual ( $A, B, C$ ) has the same ranking of the other alternatives ( $w, x, z$ ) in both problems. Thus, if condition I was satisfied, the social preference order must also give the same ranking of ( $w, x, z$ ) in both cases. But comparing parts (a) and (b), we see that outcome  $x$  has moved from last place (behind both  $w$  and  $z$ ) to second place (ahead of both  $w$  and  $z$ ).

4a) [12 pts] Yes, the social preference order will be transitive. Given the single-peaked preference condition, the Condorcet procedure always generates a transitive social preference order. The Condorcet winner (i.e., the outcome that beats all other outcomes) is \$100. This is because \$100 is the ideal point for the median student. (To determine the median, imagine lining up all students in order from those who want to pay the least to those who want to pay the most. The student at the 50<sup>th</sup> percentile – the 20,000<sup>th</sup> student in line – is the median.) Black's Median Voter Theorem states that the median voter's ideal point cannot be beaten by any other outcome (i.e., it has an empty winset).

4b) [6 pts] For a student  $i$  with ideal point  $x_i = \$100$ , the utility of each outcome would be

$$\begin{array}{ll} U_i(0) = c - 100 & U_i(100) = c \\ U_i(250) = c - 150 & U_i(600) = c - 500 \\ U_i(1000) = c - 900 & \end{array}$$

Thus, this student's preference order is  $\$100 P \$0 P \$250 P \$600 P \$1000$ .

For a student  $i$  with an ideal point  $x_i = \$600$ , the utility of each outcome would be

$$\begin{aligned} U(0) &= c - 600 & U(100) &= c - 500 \\ U(250) &= c - 350 & U(600) &= c \\ U(1000) &= c - 400 \end{aligned}$$

Thus, this student's preference order is  $\$600 \text{ P } \$250 \text{ P } \$1000 \text{ P } \$100 \text{ P } \$0$ .

4c) [11 pts] If B commits to \$1000, A's best response is to commit to \$0. (A can win the election while sticking to his/her own ideal point.) If A commits to \$0, B's best response is to commit to \$100. (B needs to move far enough to the "left" to win the election. Otherwise, A will win and set the fee of \$0.) In Nash equilibrium, the winner of the election will set a fee of \$100. Intuitively, the candidates are "pulled" to the median voter by the need to win the election.

[It was certainly not necessary to construct the following payoff matrix, but this may help clarify the game played by the candidates. I have constructed the matrix so that A is the row player, B is the column player, and thus payoffs are given as (A's payoff, B's payoff). Each player's strategy is the fee level to which he/she commits. Payoffs associated with best responses are underlined. Note that there are actually two Nash equilibria: (A commits to \$0, B commits to \$100) and (A commits to \$100, B commits to \$100). However, in either Nash equilibrium, the fee is set to \$100.

B	0	100	250	600	1000	
A	0	A or B wins Fee is \$0 ( <u>c</u> , c-1000)	B wins 25 to 15 sets fee \$100 ( <u>c-100</u> , <u>c-900</u> )	A wins 24 to 16 sets fee \$0 ( <u>c</u> , c-1000)	A wins 29 to 11 Sets fee \$0 ( <u>c</u> , c-1000)	A wins 29 to 11 Sets fee \$0 ( <u>c</u> , c-1000)
100	A wins 25 to 15 sets fee \$100 (c-100, <u>c-900</u> )	A or B wins Fee is \$100 ( <u>c-100</u> , <u>c-900</u> )	A wins 24 to 16 Sets fee \$100 (c-100, <u>c-900</u> )	A wins 29 to 11 Sets fee \$100 (c-100, <u>c-900</u> )	A wins 29 to 11 Sets fee \$100 (c-100, <u>c-900</u> )	
250	B wins 24 to 16 sets fee \$0 ( <u>c</u> , c-1000)	B wins 24 to 16 Sets fee \$100 ( <u>c-100</u> , c-900)	A or B wins Fee is \$250 (c-250, <u>c-750</u> )	A wins 29 to 11 Sets fee \$250 (c-250, <u>c-750</u> )	A wins 33 to 7 Sets fee \$250 (c-250, <u>c-750</u> )	
600	B wins 29 to 11 Sets fee \$0 ( <u>c</u> , c-1000)	B wins 29 to 11 Sets fee \$100 ( <u>c-100</u> , c-900)	B wins 29 to 11 Sets fee \$250 (c-250, c-750)	A or B wins Fee is \$600 (c-600, <u>c-400</u> )	A wins 33 to 7 Sets fee \$600 (c-600, <u>c-400</u> )	
1000	B wins 29 to 11 Sets fee \$0 ( <u>c</u> , c-1000)	B wins 29 to 11 Sets fee \$100 ( <u>c-100</u> , c-900)	B wins 33 to 7 Sets fee \$250 (c-250, c-750)	B wins 33 to 7 Sets fee \$600 (c-600, c-400)	A or B wins Fee is \$1000 (c-1000, <u>c</u> )	

Answer all questions. 100 points possible.

1. [30 points] Bisin and Verdier (*Quarterly Journal of Economics* 2000) develop a model of the socialization process in which individuals acquire either trait A or trait B. Following their paper (and the class lecture), let  $q_t^i$  denote the proportion of the population with trait  $i$  ( $= A$  or  $B$ ) in period  $t$ , and note that  $q_t^A + q_t^B = 1$  for all  $t$ . Traits may be acquired from parents (through vertical socialization) or from other members of the society (through oblique socialization). Each parent has one child, and attempts to instill her own trait in the child. Vertical socialization succeeds with probability  $\tau^A$  for parents with trait A, and succeeds with probability  $\tau^B$  for parents with trait B. If vertical socialization fails, then the child adopts the trait of a “cultural parent” selected randomly from the population (i.e., the child adopts trait A with probability  $q_t^A$  and adopts trait B with probability  $q_t^B$ ). As discussed in lecture, the dynamics of intergenerational cultural transmission is thus given by the equation

$$q_{t+1}^A = q_t^A [\tau^A + (1-\tau^A)q_t^A] + (1-q_t^A) [(1-\tau^B)q_t^A]$$

which simplifies to  $q_{t+1}^A = q_t^A + (\tau^A - \tau^B) q_t^A (1-q_t^A)$ .

a) Suppose that the direct socialization probabilities ( $\tau^A$  and  $\tau^B$ ) are constants. In particular, suppose that  $\tau^A = 0.6$  and  $\tau^B = 0.4$ . Further suppose that the population is composed of 10% As and 90% Bs in period 1 (i.e., suppose  $q_1^A = 0.1$ ). What proportion of the population holds each trait in period 2? in period 3? What proportion of the population holds each trait in the long run (as  $t$  becomes very large)? Use an equation or graph to explain.

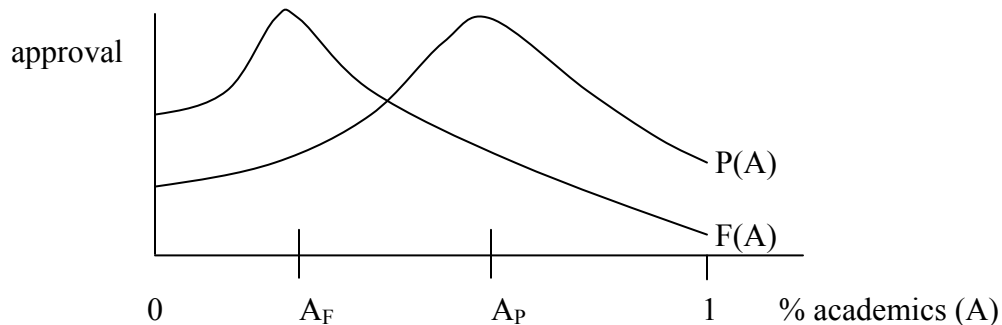
b) Now suppose that each parent with trait A chooses  $\tau^A$  to maximize her expected utility. Each of these parents receives utility level  $V^{AA}$  if her child acquires trait A, and receives  $V^{AB}$  if her child acquires trait B. The parent’s cost of vertical socialization is given by  $(\tau^A)^2$ . Write the parent’s expected utility function, and then solve the parent’s utility maximization problem to determine her optimal socialization choice ( $\tau^{A*}$ ).

c) Suppose that the vertical socialization probability for parents with trait A is given by the optimal choice  $\tau^{A*}$  from part (b), while the vertical socialization probability for parents with trait B is simply fixed at  $\tau^B = 1/2$ . Substituting these probabilities into the intergenerational dynamics equation (given above), determine the long-run outcome (as a function of  $V^{AA}$  and  $V^{AB}$ ) assuming that the initial  $q_0^A$  is strictly between 0 and 1. How does the long-run proportion of As depend on the difference ( $V^{AA} - V^{AB}$ )? Briefly discuss the intuition behind this result.

2. [10 points] The concept of “religious capital” was introduced by Iannaccone (*Journal for the Scientific Study of Religion* 1988). What is the religious-capital explanation for low religious mobility (i.e., low rates of switching between denominations)? Briefly discuss one other empirical fact about religious practice that might be explained by the religious-capital approach.

3. [30 points] Iannaccone’s “Church and Sect” model (*American Journal of Sociology* 1986) might be applied to the choices facing high-school students. Suppose that each student divides his time between academics and sports, setting some level of conduct  $A$  between 0 and 1, where  $A$  denotes the proportion of time spent on academics.

Further suppose that students receive social approval from two reference groups – parents and friends. Let  $P(A)$  and  $F(A)$  represent the levels of social approval from these groups as a function of the student’s choice  $A$ . The student’s utility depends positively on both of these levels, and might be written as  $U(P(A), F(A))$ . Further assume that  $P(A)$  is maximized at  $A_P$ ,  $F(A)$  is maximized at  $A_F$ , and  $A_P > A_F$  as in the figure below.



a) Without further information about the student’s preferences, can you say anything about the student’s optimal choice  $A^*$ ? How does this optimal choice depend on the difference between  $A_P$  and  $A_F$ ? How does this optimal choice depend on the parent’s tolerance of sports? Briefly illustrate your answers using a production possibilities curve diagram. [HINT: Be sure to label the axes of the PPC curve.]

b) Now suppose the student can join one of two group of friends – either “jocks” or “nerds.” (The student cares only about social approval from the group he’s joined.) Jocks like sports and thus have a lower value of  $A_F$ . For the nerds,  $A_F$  is close to (but still less than)  $A_P$ . What would you predict about the absolute levels of approval offered by the jocks and nerds? In which group would you expect more diversity among group members? Again, briefly explain using a PPC curve.

4. [30 points] Suppose that you have to take an exam on Wednesday. You have 4 hours of free time on Monday and another 4 hours of free time on Tuesday. You may spend your free time in one of two ways: studying or watching television. Suppose that your utility depends upon both your exam score and the amount of television watched each day. Further suppose that you always discount tomorrow's utility using the discount factor  $\beta$ . More formally, your utility (viewed from Monday) may be written

$$U = v_M t_M + \beta v_T t_T + \beta^2 R(s_M, s_T)$$

where  $t_M$  = time spent watching television on Monday  
 $t_T$  = time spent watching television on Tuesday  
 $s_M$  = time spent studying on Monday  
 $s_T$  = time spent studying on Tuesday  
 $v_M$  = value (per hour) of watching television on Monday  
 $v_T$  = value (per hour) of watching television on Tuesday  
 $R(s_M, s_T)$  = exam score as a function of  $s_M$  and  $s_T$   
 $\beta$  = discount factor

and your time constraints are  $t_M + s_M = 4$  and  $t_T + s_T = 4$  .

Finally, assume that the functional form for the exam score is given by

$$R(s_M, s_T) = 2[\ln(s_M) + \ln(s_T)] .$$

[HINT: Given this functional form, you have enough information to derive precise *numerical* solutions for the questions below. Recall that  $d(\ln(u))/dx = (1/u)(du/dx)$ .]

- Assuming that you don't discount the future ( $\beta = 1$ ) and that television is equally interesting both nights ( $v_M = v_T = 1$ ), what is your optimal time allocation decision each day? Does study time increase or decrease from Monday to Tuesday?
- Now suppose that your discount factor is  $\beta = 1/2$  (while still assuming  $v_M = v_T = 1$ ). What is your optimal time allocation each day? Does total study time ( $s_M + s_T$ ) rise or fall compared to part (a)? Briefly explain.
- Finally, suppose that television is less interesting on Monday night (so that  $v_M = 1/2$ ) but great on Tuesday night (so that  $v_T = 2$ ). Assuming that  $\beta = 1$ , what is your optimal time allocation each day? How does this compare to your solution in part (a)? Briefly explain.

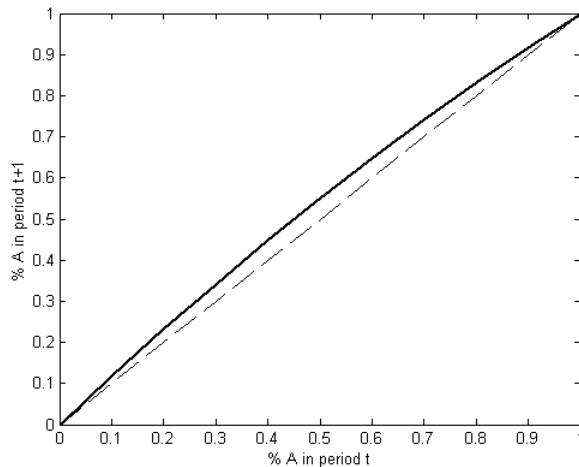
1a) [xx pts] Substituting  $\tau^A = 0.6$  and  $\tau^B = 0.4$  into the intergenerational dynamics equation, we obtain

$$q^A_{t+1} = q^A_t + (0.2) q^A_t(1-q^A_t)$$

Thus,  $q^A_1 = 0.1$  implies  $q^A_2 = (0.1) + (0.2)(0.1)(0.9) = 0.118$

$$q^A_3 = (0.118) + (0.2)(0.118)(0.882) = 0.1388$$

Plotting  $q^A_{t+1}$  as a function of  $q^A_t$ , we obtain the graph below, which reveals that the proportion of As will continue to rise until everyone in the population has trait A.



b) [xx pts] The parent's utility function is given by

$$EU_A(\tau^A) = [\tau^A + (1-\tau^A)q^A] V^{AA} + (1-\tau^A)(1-q^A) V^{AB} - (\tau^A)^2$$

Differentiating with respect to  $\tau^A$ , we obtain  $(1-q^A)(V^{AA} - V^{AB}) - 2\tau^A = 0$

and thus  $\tau^{A*} = (1/2)(1-q^A)(V^{AA} - V^{AB})$ .

c) [xx pts] Substitution into the intergenerational dynamics equation gives

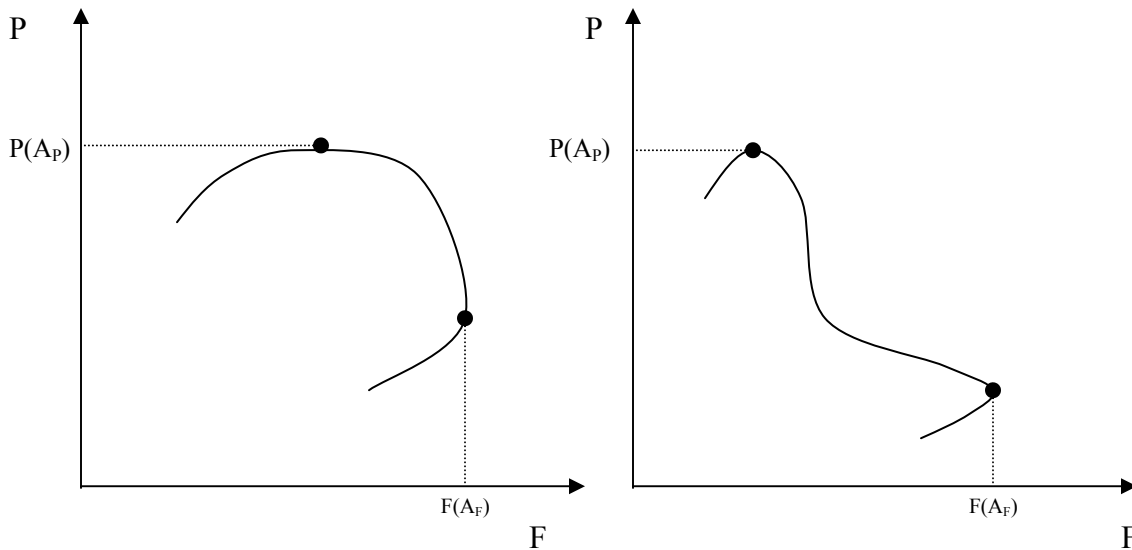
$$q^A_{t+1} = q^A_t + (1/2)[(1-q^A)(V^{AA} - V^{AB}) - 1] q^A_t(1-q^A_t)$$

The long run outcome is given by  $[(1-q^A)(V^{AA} - V^{AB}) - 1] = 0$

Thus, in the long run, the proportion of As is  $q^A = 1 - (1/(V^{AA} - V^{AB}))$  which is increasing in A's "cultural distaste" for Bs. Intuitively, the stronger this "cultural distaste," the more effort As will make at vertical socialization of their children.

2) [10 pts] Children develop religious capital in their parents' denomination. Because this capital is (at least somewhat) denomination-specific, switching to a new denomination would cause loss of religious capital (and hence utility from religious consumption). To the extent that switching does occur, individuals will tend to switch to denominations "close" to their parents' denomination (since this would imply less-than-complete loss of the initial religious capital). Iannaccone (1988) further argues that the religious-capital perspective could also explain (1) low conversion age [to obtain benefits of religious investment over longer time horizon], (2) low rates of interdenominational marriage [to obtain economies of scale in household production of religion], and (3) positive relationship between intradenominational marriage and participation [since same-denomination households are more efficient at producing religion].

3a) [15 pts] The student would always choose  $A^*$  between  $A_F$  and  $A_P$ . (Otherwise, the student would be able to increase social approval from *both* friends and parents.) If  $A_F$  and  $A_P$  are close and the parents are tolerant of sports, the student's PPC is concave (with respect to the origin). In this case, the student would choose a value of  $A^*$  somewhere between  $A_F$  and  $A_P$ . (The precise optimal solution would depend on the slope of the student's indifference curves.) If  $A_P$  and  $A_P$  are far apart or if the parents are intolerant of sports (where intolerance is reflected by a rapid decrease in  $P(A)$  as  $A$  falls below  $A_P$ ), the student's PPC becomes convex (with respect to the origin). In those cases, the student would choose  $A^*$  close to one of the extremes (near  $A_F$  or near  $A_P$ ). Graphically,



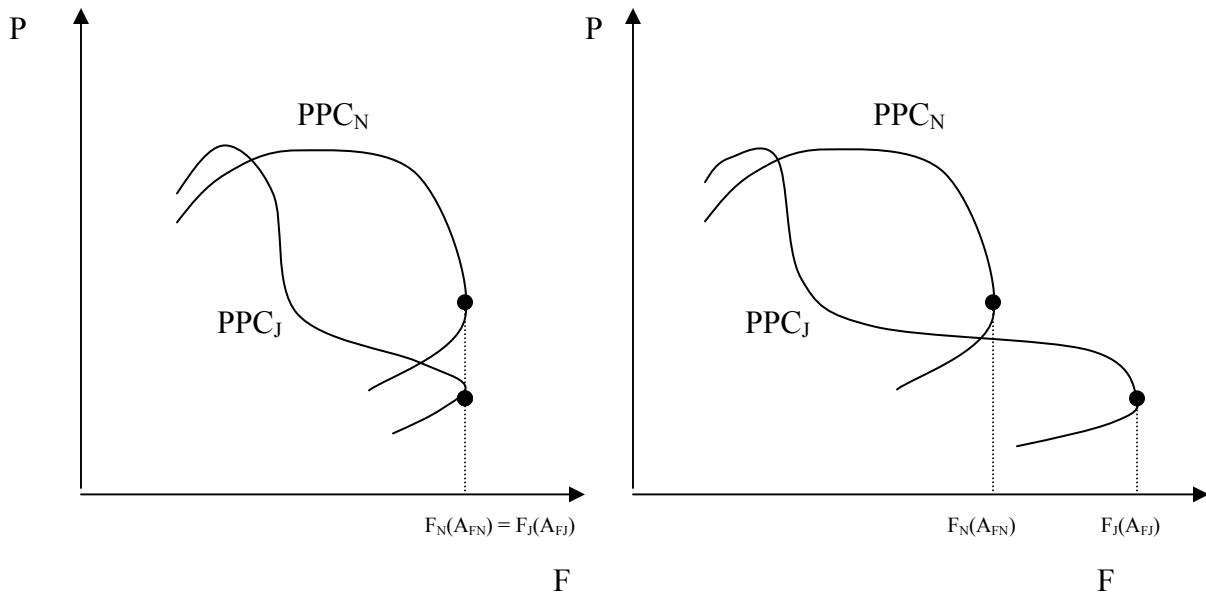
PPC is "bowed outwards" when  $A_F$  is close to  $A_P$ , and parents are tolerant of sports

PPC is "bowed inwards" when  $A_F$  is far from  $A_P$ , or when parents are intolerant of sports

depending on student's indifference map, choose  $A^*$  between  $A_F$  and  $A_P$

depending on students' indifference map, choose  $A^*$  near  $A_F$  or near  $A_P$

3b) [15 pts] Applying Iannaccone's theory to this problem, we might view the parents as mainstream society, nerds as a church, and jocks as a sect. If both groups offered similar levels of social approval (i.e.,  $F_N(A_{FN}) \approx F_J(A_{FJ})$ ) then no one would ever become a jock because jocks would receive approval only from friends, while nerds would receive approval from both friends and parents. Thus, if the jocks group is to attract any members, the jocks must offer higher levels of social approval. In Iannaccone's terms, because jocks require rejection of parents, they must provide their own substitute society.



In this case, the jocks and nerds offer the same maximum level of social approval. Consequently, no one would ever join the jocks. (No matter what the shape of the student's indifference curves, the student would receive higher utility by joining the nerds.)

In this case, the jocks offer a higher level of social approval. Now, students with steeper indifference curves (who place more weight on friends' approval) might optimally choose to join the jocks while students with flatter indifference curves (who place more weight on parents' approval) would choose to join the nerds.

Following Iannaccone's argument, we would expect more diversity among the nerds (who might choose  $A^*$  anywhere between  $A_P$  and  $A_{FN}$  depending on their preferences) while the jocks would be less diverse (all choosing  $A^*$  near  $A_{FJ}$ ). Note that a student would never join the jocks and then choose  $A^*$  near  $A_P$  since the student would have been better off joining the nerds.

4) [30 pts] This problem is an application of the Azzi-Ehrenberg model: watching television corresponds to working; studying for the test corresponds to religious participation; the test score corresponds to the afterlife reward.

Formally, you should choose  $t_M$ ,  $s_M$ ,  $t_T$ , and  $s_T$  to maximize  $v_M t_M + \beta v_T t_T + \beta^2 R(s_M, s_T)$  given the time constraints  $t_M + s_M = 4$  and  $t_T + s_T = 4$ . Substituting the constraints into the utility function, and given the functional form for  $R$ , you should choose  $t_M$  and  $t_T$  to

$$\text{maximize } v_M t_M + \beta v_T t_T + \beta^2 2[\ln(4-t_M) + \ln(4-t_T)].$$

To solve this problem, we differentiate the utility function with respect to  $t_M$  and  $t_T$ , setting each derivative to zero to find the optimal choices  $t_M^*$  and  $t_T^*$ :

$$\partial U / \partial t_M = v_M + \beta^2 2 (1/(4-t_M))(-1) = 0 \quad \text{which implies } t_M^* = 4 - 2\beta^2/v_M$$

$$\partial U / \partial t_T = \beta v_T + \beta^2 2 (1/(4-t_T))(-1) = 0 \quad \text{which implies } t_T^* = 4 - 2\beta/v_T$$

Using the time constraints, we can then obtain  $s_M^* = 4 - t_M^*$  and  $s_T^* = 4 - t_T^*$ .

a) Given  $\beta = 1$  and  $v_M = v_T = 1$ ,

$$t_M^* = 4 - 2(1)^2/1 = 2 \quad \text{which implies } s_M^* = 2$$

$$t_T^* = 4 - 2(1)/1 = 2 \quad \text{which implies } s_T^* = 2$$

Intuitively, given no discounting and the same value of watching television (“wage”) in both periods, the optimal decision is to smooth study time (“religious participation”) across the two days. Thus, study time the same on Monday and Tuesday.

b) Given  $\beta = 1/2$  and  $v_M = v_T = 1$ ,

$$t_M^* = 4 - 2(.5)^2/1 = 3.5 \quad \text{which implies } s_M^* = 0.5$$

$$t_T^* = 4 - 2(.5)/1 = 3 \quad \text{which implies } s_T^* = 1$$

Intuitively, given a positive discount factor and the same value of watching television (“wage”) in both periods, the optimal decision is to increase study time (“religious participation”) as the test (“afterlife reward”) grows closer. Thus, study time is now higher on Tuesday than Monday. Compared to part (a), total study time falls from 4 to 1.5 reflecting the fact that utility from the test (which occurs later, on Wednesday) is now discounted relative to television watching (which occurs earlier, on Monday and Tuesday).

c) Given  $\beta = 1$ ,  $v_M = 1/2$ , and  $v_T = 2$ ,

$$t_M^* = 4 - 2(1)^2/(1/2) = 0 \quad \text{which implies } s_M^* = 4$$

$$t_T^* = 4 - 2(1)/2 = 3 \quad \text{which implies } s_T^* = 1$$

Intuitively, given no discounting and an increase in the value (“wage”) over time, the optimal decision is to spend less time watching (“working”) when the value (“wage”) is low and more time watching (“working”) when the value (“wage”) is high. Compared to part (a), study time is now higher on Monday than Tuesday.