

Do **FOUR** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 30/25/25/20 or 40/20/20/20).

1. An isolated economy produces just one crop, corn, using a fixed quantity of land and variable quantities of labor and tractors. Labor is also used to produce the tractors.
  - (a) Which of the following conditions are needed for Pareto optimality:
    - i. The marginal rate of technical substitution of tractors for labor in the production of corn equals the marginal product of labor in the production of tractors.
    - ii. The marginal utility of leisure equals the marginal product of labor in corn multiplied by the marginal utility of corn, for each individual.
    - iii. The marginal rate of substitution of leisure for corn is equal for all consumers.
 Explain your answers in detail.
  - (b) Which of the above conditions would be satisfied in a competitive market economy? How?
  
2. A monopolist faces two types of consumers, with demand curves given by  $p_1 = 3 - q_1$  and  $p_2 = 4 - 2q_2$ . There are 100 consumers of each type. The (constant) marginal cost of production is 1. Does the profit-maximizing two-part tariff involve charging a unit price below marginal cost, once the consumers have paid a fixed fee? Explain why or why not.
  
3. Is it possible to find a two-good utility function such that the two goods are complements? If so, find one; if not, explain why none can be found.
  
4.
  - (a) State the First Theorem of Welfare Economics for an exchange economy.
  - (b) Give an example in which the First Theorem does not hold, and explain which assumptions of the theorem are violated in your example.
  - (c) State the Second Theorem of Welfare Economics for an exchange economy.
  - (d) Give an example in which the Second Theorem does not hold, and explain which assumptions of the theorem are violated in your example.
  
5.
  - (a) State and prove Hotelling's Lemma.
  - (b) Show that a factor of production is inferior if and only if an increase in the price of that factor reduces marginal cost.
  
6. Consider an exchange economy with two goods, fish and cheese, and two people, One and Two. One is endowed with .5 pounds of fish and .5 pounds of cheese, and Two has the same endowment. One's utility function is  $u(f,c) = fc$ , and Two's utility function is  $u(f,c) = f^2 + c^2$ .
  - (a) Can you find a competitive equilibrium for this economy? If so, is it Pareto optimal? If not, explain which assumptions of the First Welfare Theorem are violated.
  - (b) What is the set of Pareto optimal allocations in this economy? Can all these allocations be supported as competitive equilibria? Explain.
  
7. State whether the following assertions are true, false or ambiguous, and explain why.
  - (a) Constant relative risk aversion implies that the demand for insurance is a decreasing function of wealth.
  - (b) If a state seeks to attract new industries by subsidizing production costs, it is more effective to subsidize either labor or capital costs than to allocate the same total to a mixture of labor and capital subsidies
  - (c) A firm uses 10 units of labor and 20 units of capital to produce 10 units of output. The marginal product of labor is 0.5. If there are constant returns to scale the marginal product of capital must be 0.25.

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1. A factor of production is called **inferior** if the conditional demand for that factor falls as output is increased while factor prices are held constant.
- (a) Draw an isoquant map showing a technology with an inferior factor of production.
- (b) Do you think it is possible that an increase in the price of some factor of production might cause a profit maximizing firm to **increase** output? Explain.
- (c) Analyze the following argument:

"In consumer theory it can happen that an increase in the price of some good leads to an increase in the quantity demanded (if the good is inferior and the income effect of the price increase is stronger than the substitution effect). Similarly it can happen that an increase in the price of a factor of production leads to an increase in the quantity of that factor demanded by a profit-maximizing firm. This can happen if the higher factor price leads to a reduction in output, and if the factor is inferior, so that the reduction in output leads to an increase in the use of this factor."

2. (a) Fill in the missing arguments, and explain why the equation is true:

$$\frac{\partial e(p,?)}{\partial p_i} \frac{\partial v(p,?)}{\partial Y} + \frac{\partial v(p,?)}{\partial p_i} = 0$$

where  $e(\cdot, \cdot)$  is the expenditure function of a utility-maximizing consumer, and  $v(\cdot, \cdot)$  is the indirect utility function.

- (b) Suppose a consumer has the following utility function

$$U(x_1, x_2, \dots, x_k) = \alpha x_1 + u_2(x_2) + u_3(x_3) + \dots + u_k(x_k)$$

where  $\alpha$  is a positive parameter, and the functions  $u_i(\cdot)$  are all concave.

- i. Is it true that  $x_1$  absorbs all income effects: the income elasticity of demand for each of the other goods is zero? Explain.
- ii. Is it true that the cross price elasticities of demand are all zero? Explain.

3. Consider a one-period pure exchange economy with three agents (1,2 and 3) and three goods ( $x, y$  and  $z$ ). Agent  $i$ 's consumption vector is  $(x_i, y_i, z_i)$ . Agent 1,2 and 3's endowment vectors are  $(x^e, 0, 0)$ ,  $(0, y^e, 0)$  and  $(0, 0, z^e)$  (each agent is endowed with only one type of good). The agents' utility functions are as follows:

$$U_1(x_1, y_1, z_1) = x_1(y_1 + z_1)$$

$$U_2(x_2, y_2, z_2) = x_2 y_2$$

$$U_3(x_3, y_3, z_3) = x_3 z_3.$$

- (a) Compute a Walrasian equilibrium for this economy under the condition that good  $z$  cannot be traded.
- (b) Assume now that there exists a complete set of markets in all three goods. Compute a Walrasian equilibrium for this case.
- (c) Can you Pareto-rank the equilibria which you found? If so, in which order? Explain your findings.
4. State whether the following assertion is true, false or ambiguous, and **explain why**.

"There are many firms which produce wooden chairs, and many firms which produce wooden tables. If these firms are all separate, and if they all maximize profits, taking prices as given, then the equilibrium cannot be efficient. This is because when the chair firms increase output they bid up the price of wood, which reduces the profits of the table-producing firms. But the chair firms ignore the effect of their output decisions on the profits of the table firms. An efficient equilibrium would be achieved if all of the firms produced both tables and chairs."

5. Consider the function

$$v(p_1, p_2, p_3, y) = \frac{y - p_1 - 3p_2 - 2p_3}{(p_1)^2 (p_2)^3 (p_3)^5}$$

Is this a valid indirect utility function for a consumer who buys three commodities at prices  $p_1, p_2, p_3$ , with an income of  $y$ ? If so, find the indirect utility function. If not, explain why.

6. A profit maximizing monopoly firm sells to many identical buyers. The marginal cost of production is a constant,  $c$ , with  $c < .5$ . Each buyer's demand function is  $p = 1 - q$ . The firm decides to charge a two-part tariff, so that a consumer who buys a quantity  $q$  must pay the amount  $y = a + bq$ . How would the firm choose  $a$  and  $b$ ?
- (a) Now suppose a second firm enters the industry. This firm has the same production cost as the first firm. Can you find a Nash equilibrium in which each firm chooses a two-part tariff which is optimal, given the other firm's two-part tariff?