There is a partial answer to parts a-c at
http://www.ssc.wisc.edu/~jkennan/teaching/answerkeys/Selected%20(Partial)%20Answers.pdf
(question #51)
(This has been available to the students for a long time.)

The equilibrium with complete markets is pareto optimal, by the first welfare theorem.

The equilibrium when z is not traded is not pareto optimal. For instance, transfering 3's endowment to 1 would make 1 better off; a tiny transfer of the x good from 1 to 2 and 3 would leave everyone better off.

The core is the set of pareto optimal allocations that are not blocked by {1,2} or by {1,3}. No individual is in a position to block anything in the nonnegative orthant, since the utility under autarky is the utility of consuming nothing. The {2,3} coalition can't block anything either, for the same reason.

Pareto optimality requires $x_2 = \frac{1}{2} y_2$ and $x_3 = \frac{1}{2} z_3$ (with $z_2 = y_3 = 0$).

The utility possibility frontier for {1,2} is $\sqrt{U_1} + \sqrt{U_2} = 1$, so anything inside this can't be in the core. This reduces to the condition $z_3 \le 2-\sqrt{2}$. Similarly, $y_2 \le 2-\sqrt{2}$, or else the allocation is inside the utility possibility frontier for {1,3}.

So the core is the (2-dimensional) set
$\{(x,y,z): y_2 \le 2-\sqrt{2},\ z_3 \le 2-\sqrt{2},\ x_2 = \frac{1}{2} y_2,\ x_3 = \frac{1}{2} z_3,\ z_2 = 0,\ y_3 = 0,\ x_1 = 1-x_2-x_3,\ y_1 = 1-y_2,\ z_1 = 1-z_3\}$.
The allocation in part (d) is in this set.