## Economics 703 Midterm Exam

John Kennan, September 5, 2018
Answer all 5 questions
Time allowed: 2 hours

1. Show that if $f$ is differentiable on an interval with $f^{\prime}(x) \neq 1$, then $f$ can have at most one fixed point.
2. Show that the following sequence is bounded

$$
x_{n}=\left(1+\frac{1}{n}\right)^{n}, n=1,2, \ldots
$$

3. Show that if the function $f: \mathbb{R} \rightarrow \mathbb{R}_{++}$is continuous on an interval $[a, b]$, then the reciprocal of this function $\left(\frac{1}{f}\right)$ is bounded on this same interval.
4. Suppose

$$
A=\{f: \mathbb{R} \rightarrow \mathbb{R}, f \text { concave, } f(1)=1, f(3)=5, f(4)=6\}
$$

Solve the following equations

$$
\begin{aligned}
\sup \{f(2) \mid f \in A\} & =u \\
\inf \{f(2) \mid f \in A\} & =v
\end{aligned}
$$

5. A consumer has an income of $\$ 300$ per week, which is spent entirely on two goods, food $(f)$, measured in pounds, and gas $(g)$, measured in gallons. The consumer's utility function is

$$
u(f, g)=\sqrt{\frac{f}{100}}+\log (g)
$$

(a) If the price of food is $\$ 1$ per pound, and the price of gas is $\$ 4$ per gallon, what is the optimal (i.e. utility-maximizing) consumption plan?
(b) If the price of gas rises to $\$ 10$ per gallon, does the consumer buy more food?

