Economics 703 Midterm Exam

John Kennan, September 5, 2018

Answer all 5 questions Time allowed: 2 hours

- 1. Show that if f is differentiable on an interval with $f'(x) \neq 1$, then f can have at most one fixed point.
- 2. Show that the following sequence is bounded

$$x_n = \left(1 + \frac{1}{n}\right)^n, n = 1, 2, \dots$$

- 3. Show that if the function $f : \mathbb{R} \to \mathbb{R}_{++}$ is continuous on an interval [a, b], then the reciprocal of this function $\left(\frac{1}{f}\right)$ is bounded on this same interval.
- 4. Suppose

$$A = \{f : \mathbb{R} \to \mathbb{R}, f \text{ concave}, f(1) = 1, f(3) = 5, f(4) = 6\}$$

Solve the following equations

$$\sup \{ f(2) \mid f \in A \} = u$$

$$\inf \{ f(2) \mid f \in A \} = v$$

5. A consumer has an income of \$300 per week, which is spent entirely on two goods, food (f), measured in pounds, and gas (g), measured in gallons. The consumer's utility function is

$$u\left(f,g\right) = \sqrt{\frac{f}{100}} + \log\left(g\right)$$

- (a) If the price of food is \$1 per pound, and the price of gas is \$4 per gallon, what is the optimal (i.e. utility-maximizing) consumption plan?
- (b) If the price of gas rises to \$10 per gallon, does the consumer buy more food?