## Economics 311 Midterm Exam

John Kennan, October 21, 2014
Time allowed: 75 minutes
IMPORTANT: Explain your answers carefully. A good diagram is often more effective than a lot of words (but you must explain what the diagram means). You get no credit for unsupported assertions or guesses. Write as if you are trying to convince an intelligent person who does not already know the answers. If your answers would not convince such a person, it will be assumed that you do not fully understand the answers.

1. Five people go to dinner at a restaurant. They all have the same preferences, over food, $f$, and other stuff, $y$, represented by the utility function

$$
u(f, y)=f y
$$

They all have the same amount of money to spend, $I$. The price of food is $p$ (relative to other stuff).
(a) Suppose each person orders independently, and each person pays an equal share of the total cost of the meal. Find a Nash equilibrium of this game (where each person's strategy is the cost of the food that this person orders).
i. Suppose the others spend $X$ (in total) If this person orders food costing $x$, the payment for each person will be $\frac{x+X}{n}$ (with $n=5$ )
so

$$
\max _{x} x\left(I-\frac{x+X}{n}\right)
$$

or

$$
\max _{z} z\left(I_{0}-z\right)
$$

where $z=\frac{x}{n}$ (this person's share of the cost )
and $I_{0}=I-\frac{X}{n}$
(this person's remaining money after they've paid their share of everyone else's food). Then the optimal choice is

$$
\hat{z}=\frac{I_{0}}{2}
$$

so

$$
\begin{aligned}
\hat{x} & =n \frac{I_{0}}{2} \\
& =\frac{n I-X}{2}
\end{aligned}
$$

But in a Nash equilibrium $X=(n-1) \hat{x}$, so

$$
2 \hat{x}=n I-(n-1) \hat{x}
$$

and then

$$
\hat{x}=\frac{n}{n+1} I
$$

So if $n=5$ then each person spends $\frac{5}{6}$ of their income on food, instead of $\frac{1}{2}$. And utility is $\frac{5}{36}$ compared with $\frac{1}{4}=\frac{9}{36}$ in the case where each person pays for their own food.
(a) Compare the Nash equilibrium outcome with the outcome when each person pays separately for their own meal.

$$
\begin{aligned}
& \max _{f} f y \\
& p f+y=I
\end{aligned}
$$

Substitute using the budget constraint

$$
\max _{x} x(I-x)
$$

$x=p f$ (expenditure on food - or just say $p=1$, so $x$ and $f$ are the same thing)
A quadratic function, zero at $x=0$ and at $x=I$, positive in between symmetric in $x$ and $I-x$, maximal at $x=\frac{I}{2}$.
This is a dominant strategy for each person, so this is the Nash equilibrium
2. An expected utility maximizer with constant relative risk aversion and wealth $w$ buys $\alpha$ units of insurance at price $q$ against a loss $D$ that occurs with probability $\pi$, where $q \geq \pi$. Find $\alpha$.
(a) The first-order condition is

$$
\frac{u^{\prime}(w-D+\alpha-\alpha q)}{u^{\prime}(w-\alpha q)}=\frac{q}{1-q} \frac{1-\pi}{\pi}
$$

In the CRRA case the utility function is $u(x)=\frac{x^{1-\varrho-1}}{1-\varrho}$ so $u^{\prime}(x)=x^{-\varrho}$ so

$$
\frac{w-D+\alpha-\alpha q}{w-\alpha q}=A
$$

where

$$
A=\left(\frac{q}{1-q} \frac{1-\pi}{\pi}\right)^{-\frac{1}{\rho}}
$$

Then

$$
\frac{D-\alpha}{w-\alpha q}=1-A
$$

and

$$
D-\alpha=(1-A) w-\alpha(1-A) q
$$

and the optimal choice of $\alpha$ is given by

$$
\begin{aligned}
\alpha & =\frac{D-(1-A) w}{1-(1-A) q} \\
& <\frac{D-(1-A) D q}{1-(1-A) q} \\
& =D
\end{aligned}
$$

If $q=\pi$ then $A=1$ so $\alpha=D$. If $q>\pi$ then $A<1$ so $\alpha<D$ (it is assumed that wealth is sufficient to purchase full insurance, meaning that $w$
In the case of log utility,

$$
\begin{aligned}
1-A & =1-\left(\frac{1-q}{q} \frac{\pi}{1-\pi}\right) \\
& =\frac{q-q \pi-\pi+q \pi}{q(1-\pi)} \\
& =\frac{q-\pi}{q(1-\pi)}
\end{aligned}
$$

so

$$
\begin{aligned}
\frac{\alpha}{D} & =\frac{1-\frac{q-\pi}{q(1-\pi)} \frac{w}{D}}{1-\frac{q-\pi}{q(1-\pi)} q} \\
& =\frac{q(1-\pi)-(q-\pi) \frac{w}{D}}{q(1-\pi)-(q-\pi) q} \\
& =\frac{q(1-\pi)-(q-\pi) \frac{w}{D}}{q(1-q)} \\
& =\frac{1-\pi-\left(1-\frac{\pi}{q}\right) \frac{w}{D}}{1-q}
\end{aligned}
$$

3. Say whether the following assertions are true, false or uncertain, and explain why.
[Hints: (1) most true-false questions are false; (2) this exam was written by someone who knows (1)].
(a) A firm uses 10 units of labor and 20 units of capital to produce 10 units of output. The marginal product of labor is 0.5 . If there are constant returns to scale the marginal product of capital must be 0.25 .
i. True. $F(\alpha K, \alpha L)=\alpha F(K, L)$. Differentiating with respect to $\alpha$ gives $K F_{K}(\alpha K, \alpha L)+L F_{L}(\alpha K, \alpha L)=F(K, L)$, for any $\alpha$. Then setting $\alpha=1$ gives $K F_{K}(K, L)+L F_{L}(K, L)=F(K, L)$. Using the numbers given for $K, L, F(K, L)$ and $F_{L}(K, L)$, this equation is $20 F_{K}(K, L)+10 \times .5=10$, and this implies $F_{K}(K, L)=\frac{5}{20}$.
(b) A movie theater which sets admission prices in such a way that many seats remain empty cannot be maximizing profits.
i. False. If marginal revenue is zero at the point where the last seat is sold, then selling more seats reduces revenue, and therefore reduces profits. For example, if the demand curve is $p=20-\frac{q}{5}$, and if there are 100 seats, then filling every seat would mean giving the tickets away for free. If marginal cost is zero in this example, the optimal price is $p=10$, implying that 50 seats are left empty.
