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Introduction

In recent research that explains the role of learning in labor market outcomes, authors typically assume uncertainty about a worker’s ability when he is first hired. The worker’s ability is gradually revealed while working in a firm. Within this literature, there are two main categories of papers: those that assume that learning is public, and those that assume that learning is private. In public (symmetric) learning models, during a worker’s career, signals of ability are revealed publicly to all the firms in the market. Examples of public learning models include: Harris and Holmstrom’s model of insurance, Gibbons and Farber’s dynamic model of wage determination, and Gibbons and Waldman’s model which combines public learning, job assignment and on-the-job-training. In private (asymmetric) learning models, the employer has better information about his workers’ productivity than other firms in the market. Examples include: Waldman’s model in which job assignments are used as a signal of workers’ abilities, Greenwald’s model of adverse selection, Milgrom and Oster’s discrimination model, and Bernhardt’s model of promotions dynamics. Public learning models rely on the extreme assumption that employers and outside firms learn instantaneously about workers’ abilities. Therefore, they predict that workers with similar abilities will earn similar wages. This is inconsistent with evidence of wage variation of workers with similar abilities. Asymmetric learning models predict that wages will not depend directly on productivity. Therefore, they are inconsistent with evidence of large variation in wages and wage residuals within job levels. This paper combines the two approaches by assuming that employers have private information, as assumed in private learning model, yet the results show that private information about a worker’s productivity is gradually revealed to the market through wages, at a time lag. As a result, information about a worker’s ability becomes public knowledge over time. Thus, my model suggests that some fraction of wage growth over a worker’s career can be attributed to outside firms’ gradually learning about a workers productivity. The model is consistent with much of the empirical evidence on wage dynamics in internal labor markets. Moreover, learning in this “hybrid” model explains features of wage dynamics that neither an asymmetric learning model nor a symmetric learning model can explain.

1 See Medoff and Abraham(80).
While asymmetric learning models are consistent with some empirical findings, they are inconsistent with the large variation in the wage and wage growth across and within job levels (Baker, Gibbs and Holmstrom, 1994a, b, (BGH)). Among the findings by BGH that are inconsistent with predictions of asymmetric learning models are the findings that wage residuals vary substantially within a job level, that an individual’s wage growth within a job level varies substantially, and that promotion premiums explain only a modest portion of the average wage difference across job levels. Gibbons and Waldman (1999) show that a model which combines symmetric learning, on-the-job-training and job assignment can explain many of BGH findings. However, their model cannot explain wage variation within a job level, for workers who receive similar performance ratings.

This paper assumes that learning is private in nature: only the employer and the worker learn the worker’s productivity and skills. However, during a worker’s career, firms learn at a time lag about the worker’s productivity as they observe the worker’s wages. In order for firms to be able to infer a worker’s productivity by observing his wages, wages need to depend directly on the worker’s productivity. This contrasts existing models of private learning. Wages in this paper depend on the worker’s productivity directly because I assume that workers can renegotiate their wages anytime before production takes place, and because I impose the condition that in equilibrium wages are renegotiation-proof. This assumption that workers can renegotiate their wages is appealing when contracts are incomplete in the sense that they cannot bind workers to the relationships.\(^2\).

The goal of the paper is three fold. First, I show how information is conveyed to the market through wages. Wages are determined by bargaining between the worker and the employer under the assumption that the ability of the worker is a private information of the employer and the worker. Both the employer and the worker are impatient. Therefore, delaying production and payment is costly to both the employer and the worker. I identify the conditions under which the bargaining outcome depends directly on the workers’ productivity, and not only on publicly observable characteristics (which determine the outside option of the worker). I assume that the employer and the worker make sequential offers, and model bargaining as an infinite alternating

\(^2\)Stole and Zweibel (1996) consider a variety of application of intrafirm bargaining when there is no uncertainty regarding workers’ productivity.
offers game (Rubinstein Bargaining).

The logic behind the result that wages depend directly on the worker’s productivity is as follows: the employer earns larger profits (informational rents) in the proceeding periods if the worker accepts a wage offer which equals his outside option, and does not “reveal” the worker’s productivity. However, the worker will not accept immediately an offer which does not depend directly on his productivity. As the bargaining horizon becomes arbitrarily large, the gains in the proceeding period become arbitrarily small. Therefore, the bargaining outcome is similar to the bargaining outcome in Rubinstein Bargaining: agreement is reached with no delay, and the surplus is split according to the player’s time preferences (when $\delta \to 1$, the surplus is split equally).

Second, I analyze the different sources of wage growth over time. In particular, I analyze the effect of learning on wages over time. I divide “learning” about productivity into two types. One type is “private” learning. This is when the employer and the worker privately learn the worker’s skills and comparative advantage. The other type is “public” learning. This is when outside firms learn indirectly (through wages and job assignment) about the worker’s productivity. In this model, one cause of wage growth is private learning. Consequently, workers are sorted to jobs according to comparative advantage. The worker and the employer bargain over the wages, and the worker receives a portion of the surplus (the worker’s MRP). After observing the wage of the worker in the job (the bargaining outcome), an outside firm offers a wage that is equal to the worker’s MRP. The worker’s wage grows again; this is the portion of the wage growth that is due “public” learning. Comparing my results to other models, in symmetric learning models, when new information about the worker’s MRP is revealed, it is revealed to all the firms in the market. Therefore, the wage increases once, and is equal to the MRP of the worker. In asymmetric learning models, wages depend only on publicly observable characteristics. New information, which is only revealed to the employer and the worker, does not cause wages to grow directly. Only when the worker is promoted, and his publicly observable characteristics change, does his wage grow. However, the wage growth does not depend directly on his productivity.

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3 Only, when the outside option in the bargaining doesn’t bind.
Third, I provide a theoretical explanation to a number of empirical findings. First, real wages are not downwards rigid. There are two different causes of wage declines in this model. One cause is that “revealing” information to the market can be costly to workers. In some cases, the bargaining outcome, in which the worker receives a fraction of the surplus (his MRP), is lower than his outside option. However, accepting the bargaining wage, which depends directly on the worker’s productivity will enable firms to infer his productivity; this will increase the worker’s wage in the proceeding periods. Therefore, the worker will experience a one-period wage decrease. There is an additional source of wage decreases in the model. When workers are first hired, the information is symmetric. After the worker is hired, the employer gains private information and therefore, earns informational rents. The equilibrium entry level wage includes the expected profit from a worker that is hired. If a worker’s productivity realization is “low”, firms will not earn informational rents, and the worker will be paid a lower wage; this is a typical result in asymmetric learning models. In symmetric learning models, the causes of real wage reductions are different from the two causes in this paper. In symmetric learning models, firms only observe a noisy signal about the worker’s ability and update their expectations of the worker’s MRP. Therefore, when the output realization is “low”, workers experience a wage reduction.

A second finding which I explain is the diversity in wage levels and wage growth within a job. As in the symmetric learning models, one source of variation in wages and wage growth is that workers are heterogeneous with respect to their abilities. In my model there are few sources of wage differences between workers in the same job level who are equally productive in the job. One source is differences in publicly observable characteristics or in productivity in a different job than the job they are currently in. This can occur when some worker’s outside option in the bargaining binds and for some not. In this case, wages of some workers will reflect their productivity, while other workers’ wages will reflect publicly observable characteristics only. Wages can be also different for

\footnote{All of the findings are from BGH. However, other empirical papers support some of the evidence.}
\footnote{this is the only source that of wage variation in symmetric learning models in which productivity growth depends on ability}
\footnote{The outside option is affected by observable characteristics, which can include productivity in a different job if the worker was previously employed in. It happens when the wage in that job depended on his productivity.}
workers with similar productivity in all job levels and their publicly observable characteristics. The reason for that is difference in ages (more precisely, the time left before the worker retires). This can happen in cases when “revealing” productivity is costly. In these cases, younger workers have more time left to work hence, they benefit more from a wage that will “reveal” their productivity to firms in the market. In comparison, the only source of wage variation in asymmetric learning models is a difference in observable characteristics.

A third finding is that the average wage increase upon promotion is modest relative to the difference in average wages across jobs (BGH find that they can differ by a factor of five or more). This model’s prediction is consistent with the findings. The increase in the wage upon promotion is the bargaining wage, which is a portion of the worker’s MRP minus the previous wage. However, firms make inferences about a worker’s productivity after the worker is promoted and they observe the wage. This causes an additional wage increase (due to the “public” learning). In asymmetric models typically wages increase only upon promotion. In Gibbons and Waldman’s model of symmetric learning, wage growth after promotion is associated with on-the-job-training.

The forth finding is the “Green card” effect: workers who earn less than the average earn a larger wage increase. Controlling for performance rating, workers with relative low wages receive a larger percent of wage increase; in the lower job levels, it is also a larger wage increase in real wage\(^7\). This model predictions are consistent to some extent with this finding. In the model controlling for ability, wages are one of the following three. 1) If the worker was recently promoted, then the bargaining wage can be a fraction of the worker’s MRP; 2) if the worker works for more than a period then his wage can be equal his MRP in the job, and 3) wage can depends only on the worker’s publicly observable characteristics. Those who receive a wage that is equal their MRP have the highest wage. Those who receive a wage that does not depend directly on their productivity, but depend on their productivity in the previous level, receive the second highest pay. Those who were recently promotes, and receive a fraction of their MRP have the lowest pay. The model predicts that only those who receive the lowest wage will experience a wage growth. Gibbons and Waldman’s

\(^7\)BGH concluded that this is an evidence to administrative pressure to reduce pay dispersion, and that the pressure might be stronger in lower levels.
model can only explain this finding if human capital increases in ability in a decreasing rate. They conclude that the effect that BGH find is too strong to only be explained by concavity of human capital accumulation⁸.

Other empirical findings which are consistent with this model's prediction are that there is an overlap in the wage distributions for adjacent job levels, and that workers who are promoted, come from different parts of the wage distribution in their job prior to promotion. This is simply because this model is a model in which workers are assigned to jobs according to their comparative advantage.

The paper is organized as follows: section one describes the economy. Section two analyzes the basic model with one simple job and one complex job. The section explains how private information about workers’ abilities is revealed to the market, analyzes sources of wage growth, and provides explanation to wage decreases. Section three extends the model in the second section. It adds a third level complex job. The aim of the section is to explain empirical evidence.

1. The Model

In this section I describe a model of the competitive economy. I make the following assumptions:

1. The price of the homogeneous good is normalized to one.
2. There is free entry into the market.
3. Each firm has a job ladder which consists of three different jobs: \( j = 1, 2, 3 \); \( j = 1 \), is a job with simple tasks; \( j = 2, 3 \) are complex jobs and \( j = 3 \) is a higher level job in the hierarchy.
4. Technology: the production function is linear. The output produced by a firm is the sum of the workers’ productivity in each job level.
5. Each worker is characterized by three random variables, \( \psi, \theta_2, \) and \( \theta_3 \), which are the marginal productivity in each job: \( 1, 2, 3 \) respectively. Each worker is equally productive in all firms.
6. I assume that if the ability of a worker is unknown to the firm and the worker, it is always optimal to assign the worker to a low type job.

⁸Incorporating on-the-job-training in this model can potentially explain a strong “Green card” effect without assuming concavity.
7. Workers have a reservation utility of zero and display no disutility from effort.

8. A worker’s career lasts $S$ periods. Workers who enter the labor market when they are older have shorter careers.

9. Contracts are incomplete in the sense that they cannot bind workers\(^9\) to the relationship. Each individual worker can renegotiate his wage any time before production takes place.

10. The bargaining procedure is a common knowledge in the economy.

11. Workers and firms are risk neutral.

12. If a worker left a firm and is employed in another firm, he cannot return later to work in the former employing firm.

Assumptions 1, 4 and 5 imply that a worker’s ability parameter in each job is also his MRP in that job. Assumption 6 is made for simplicity. Assumption 7 implies that the labor supply is perfectly inelastic and that job assignment does not affect a worker’s utility; only the wage affects the utility of workers. Assumption 12 ensures that all the competing firms have the same information regarding the worker’s abilities. It is made to preclude the possibility that workers switch firms in order to convey information about their abilities to more than one firm.

I describe next the way the worker, the employer and outside firms learn about a workers ability, the nature of jobs in a firm and the bargaining.

*learning in the economy*: when the worker is first hired, the information is symmetric and incomplete: all the firms observe the productivity of the worker in the low level job $\psi$, and no one (firms and workers) observe the productivities in the high level jobs, $\theta_2, \theta_3$ (only the joint distribution). After the worker works for one period in the firm, both the worker and the employer learn privately the worker’s productivity in the complex jobs. This information is unavailable to the outside firms, and the output in the complex jobs is unobserved by firms. This assumption rules out the possibility of contingent contracts. $S$ is the number of (“production”) periods left before the worker retires.

\(^9\)The results in the paper do not change if contracts do not commit firms to the relationship either.
Jobs: I assume that in each job level \((j + 1)\) the tasks are more complex, and mistakes are more costly than in the job level \((j)\). Hence, each worker needs to spend time in a lower level job before he is assigned to a higher level job\(^{10}\). In addition, the information about a worker’s ability/productivity in a job: \(j\), is revealed while working in a lower level adjacent job: \(j - 1\). This is because adjacent jobs are more similar in their tasks.

Bargaining: If a worker is assigned to a high level job, he can renegotiate his wage any time before production takes place. If the worker and the employer enter renegotiation, production is delayed until they reach an agreement. If they do not reach an agreement, they keep making offers unless the worker accepts an outside offer. I denote the bargaining discount factor (I assume both the worker and the employer have the same discount factor) by \(\delta_b\). \(T\) denote the number of periods before an agreement is reached (to distinct from \(S\) which is the number of “production periods” in a worker’s career). The length of each period of production is constant. If an agreement is reached after \(x\) units of time, it delays the beginning of the following period of production by \(x\).

Let \(\delta\) denote the discount factor of a “production” period.

I assume that the bargaining is an infinite Rubinstein Bargaining, with an arbitrary small periods of time between the offers\(^{11}\). The assumption implies that it is not important which party makes the last offer. The assumption that the length of the periods between offers is arbitrary small means that it is not important to the result who makes the first offer.

2. The basic learning model:

For simplicity I assume that there are only two types of jobs in the economy: low level job and a high level job. Section three extends the analysis to 3 jobs. I describe next the timing of the game and the information available to each player at each informational node.

2.1 The timing and information structure in the model:

\(^{10}\)Relaxing the assumption will not change the results, but will complicate the equilibrium analysis.

\(^{11}\)I choose to model the bargaining as an infinite horizon model, because I assume that there is no formal procedure that is known to both the employer and the worker, and hence there is no final period that enters their strategic considerations.
First period: the worker works in a low type job in the firm. By the end of the first period, the employer and the worker both know the ability parameters of the worker in both the high and low type jobs. Firms in the market observe the worker's productivity in low type job only, and the wage; observing the wage outside firms make a wage offer.

Second period: at the beginning of the first period the employer assigns the worker to either a low or a high type job. If the worker is assigned to a high level job, then the employer and the worker bargain over the wage, observing the outside firms wage offers. If they agreed on a wage, the worker works for a period. At the end of the second period, after production took place, firms in the market observe the worker's job assignment and wage in the second period, and make a wage offer. The employer then makes a counteroffer.\textsuperscript{12}

As long as the worker didn't accept any outside offer, the employer can match any offer. I assume that if the worker accepts an outside firm's offer and works for that firm, he cannot return to work for his former employer afterwards. Since I assume that if a worker quits he cannot return to work for the former employer the first assumption simplify the equilibrium analysis. It rules out the possibility of a market of unemployed workers.\textsuperscript{13}

An additional assumption that simplify the off-equilibrium-path analysis is that outside firm cannot observe the offers directly. They can only observe a worker's job assignment if he actually works in the job, and observe wages if they were actually paid rather than the wage offer itself.

2.2.1 Equilibrium analysis: I will solve next for a perfect bayesian equilibrium in which wages are renegotiation-proof. Therefore, in equilibrium, no party has incentives to enter renegotiation.

I characterize the equilibrium wage in each period of the \((S)\) “production” periods.

Definition 1: A “revealing wage” equilibrium, is an equilibrium in which the wage in the complex job depends directly on the MRP of the worker in that job.\textsuperscript{14}

\textsuperscript{12}I assume that outside firms make an offer and that the employer makes the last offer. This assumption is made in order to simplify the exposition. All the results hold if each firm can always match an offer.

\textsuperscript{13}Greenwald (86) analyzes an adverse selection model of labor markets.

\textsuperscript{14}Since the wage contains the information on the ability in the job it reveals publicly the worker’s ability in the complex job. This implicitly contains the assumption that outside firms know the bargaining procedure.
**Definition 2:** A “non-revealing wage” equilibrium, is an equilibrium in which the wage in the complex job depends only on publicly observable characteristics, and do not contain information on the MRP of the worker in that job. An equilibrium consists of outside firms’ wage offers and beliefs about the worker’s productivity at the beginning of each period, the bargaining outcome and the employer’s counteroffer if an outside firm makes an offer, and the worker’s choice of an offer.

Next, I describe the equilibrium strategies:

2.1 *Equilibrium strategies* $s \geq 2$:

I denote the equilibrium bargaining outcome, if agreement is reached, as $w^s_b$, and specify the equilibrium strategies of each player for any possible bargaining outcome. Then, I will solve for the equilibrium bargaining outcome given the equilibrium strategies.

*The worker’s strategy:* each worker’s equilibrium strategy is to accept the highest offer, and accept the employer’s offer if there is a tie. If the highest offer is made by more than one outside firm, then the worker chooses which offer to accept randomly.

*The employer’s strategy:* (period $s$ job assignment, wage offers during bargaining and a counteroffer if an outside firm makes an offer). Employer’s wage offer (counteroffer) if agreement, $u^s_e$, has been reached

$$W^e_s = \begin{cases} \max[w^M_s, w^b_s] & \text{if } \theta \geq w^M_s, \\ w^e_s & \text{otherwise} \end{cases}$$  \hspace{1cm} (2)

$w^M_s$ is the highest offer made by an outside firm Notice, that $w^M_s$ is made by the outside firm at the beginning of period $s$, before it observes the wage that the worker is actually paid in that period.

The employer’s job assignment if $J_{s-1} = L$ (recall that demotions are ruled out by assumption).

$$J^e_s = \begin{cases} H & \text{if } \theta \geq \psi, \\ L & \text{otherwise} \end{cases}$$  \hspace{1cm} (3)

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If the offer is made while the employer and the worker bargain and agreement hasn’t been reached then the employer’s counteroffer is : 

$$W^c_s = \begin{cases} \max[w^M_s, w^b_s] & \text{if } \theta \geq w^M_s, \\ w^c_s & \text{otherwise} \end{cases}$$  \hspace{1cm} (1)

where $w^c_s$ is the highest offer made by the employer during the bargaining if agreement hasn’t been reached. To simplify the analysis, I assume that outside firms make one offer at the beginning of each period. Notice that until the beginning of the following period the firm doesn’t have new information.
The employer assigns the worker to a job according to the worker’s comparative advantage.

An outside firm’s strategy:

In the beginning of the period, firms only observe previous job assignments, wages and $\psi$. If the worker wasn’t assigned previously to a high level job then, outside firms offer a contract of a low type job and a wage equal to the MRP in the low type job, I denote by $w_s$, past wage realizations:

$$\{w^M_s(\psi, w_1, \ldots, w_{s-1}) = \psi, J^M_2(\psi, w_1, \ldots, w_s) = L\}. \quad (4)$$

If the worker was assigned previously to a high level job, then outside firms offer the worker a high level job assignment. The wage offer depends on the worker’s observed wage history. There are two cases:

1) If the wage history in the high level job contains a wage such that:

$$2w_{s-k}^b > \psi, \text{ and } w_{s-k}^b \neq \psi \text{ then the wage offer and beliefs are:}$$

$$w^M_s = 2w_{s-k}^b \text{ and } \hat{\theta}_s = 2w_{s-k}^b. \quad (5)$$

2) If at least one of the conditions in 1) are not satisfied and the highest wage of a worker in a high level job is not larger than $\psi$ (if the wage is larger than $\psi$ than the conditions in 1) are satisfied). The wage offer and beliefs are:

$$w^M_s = \psi \text{ and } \hat{\theta}_s = E[\theta|\theta^* \geq \theta \geq \psi]. \quad (6)$$

Intuition: On the equilibrium path, the bargaining outcome is $\frac{\theta}{2}$ or $\psi$. The condition that $2w_{s-k}^b > \psi$ is the condition that the worker is only promoted if he has a comparative advantage and hence, $\theta \geq \psi$. In all other cases, firm do not believe that they can infer the worker’s actual productivity from the wage. Therefore, they offer a wage equals to the lowest possible MRP of the worker.\(^{16}\) If the highest wage is equal to the highest outside option, then outside firms can only infer

\(^{16}\)In this model when firms are uncertain about a worker’s productivity, they offer a wage that equals the lowest MRP of the worker when the expected MRP is larger. This is in contrast to previous results in asymmetric learning models (an example is Waldman(84)). The reason for the difference in the results is that I assume that the employer can match the offer, while in asymmetric learning model it is typically assumed that outside firms make the final offer.
that the MRP of the worker is larger than this wage offer (otherwise, the employer will not match the offer). If the highest offer is \( \psi \), than outside firms believe that the minimal worker’s MRP is \( \psi \) since the worker was promoted.

**Proof** See appendix A.

2.3 Bargaining:

I analyze next the equilibrium bargaining outcome, taking the equilibrium strategies and beliefs (for any possible bargaining outcome) which I specified above, as given.

Consider the final “production” period \( S \): if the worker’s productivity in the complex job, is unobserved by the market\(^{17} \): the employer and the worker bargain over a surplus \( \theta \). The worker’s “threat” point (the outside option, in case that the negotiation breaks down) is \( w^M_S \) (which is equal \( \psi \) on the equilibrium path), the worker’s publicly observable characteristics. The employer’s “threat” point is zero. This is a standard infinite horizon Rubinstein Bargaining \(^{18} \), and the share of the party that makes the first offer is:

\[
\theta \frac{1}{1 + \delta_b} ; \text{ the surplus of the second party is: } \theta \frac{\delta}{1 + \delta_b}.
\]

When the length of the time between offers is arbitrarily small, and the time preferences of the parties are similar (equal discount factors), the surplus is split equally, and each party receives: \( \frac{\theta}{2} \). The outside option only affect the bargaining outcome if it binds. However, in this model the bargaining outcome affect the payoffs of the worker and the employer in the future periods. I will next show that the bargaining outcome in the model will be the same as in the standard Rubinstein Bargaining.

I denote the present value of the expected payoffs in future periods of the employer and the worker, for any given wage bargaining outcome \( w_b^s \), in period \( s < S \), given the equilibrium strategies in the game, as: \( x^e_s(w_b^s) \), \( x^w_s(w_b^s) \), respectively. For notational simplicity, I will use: \( x^e_s, x^w_s \) instead.

In order to find the solution to the bargaining between the worker and the employer, I first solve for a finite bargaining with \( T \) rounds. I assume that the bargaining breaks down only if the worker accepts an outside firm offer. Otherwise, the employer can always match any outside offer, and the

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\(^{17}\) Otherwise, the worker is paid his MRP.

\(^{18}\) when the outside options do not bind.
bargaining between the employer and the worker continues until the worker accepts another offer, however, by assumption outside firms only make offers at the beginning of each period. Recall that until the worker produces and gets paid, outside firms cannot observe the bargaining outcome or the offers.

Consider the period before the final production period, $S - 1$: (Assume that outside firms cannot observe the worker’s productivity in the high level job. Otherwise there is no bargaining and the worker is paid $\theta$).

Consider a finite alternating offers bargaining, in which the employer makes the first offer and the employer makes the last offer.

In the final round of the bargaining, $T$, the employer makes an offer: $w_b$ that the worker accepts. Then the employer’s expected payoff is: $x_e^s(w_b)$.

**Solution for the (equilibrium) alternating bargaining offers:**

In the last round the employer offers $w_{S-1}^M$, and the worker accepts. The value of this wage offer to the employer is $\pi_e^b = \theta - w_{S-1}^M + x_e^{S-1}(w_b)$. Solving backwards, in the first period the worker makes an offer of a wage:

$$w_{b}^{S-1} = \theta \frac{1 - \delta_b^T}{1 + \delta_b} - \delta_b^{T-1}(x_e^{S-1}(w_b))$$

(7)

The employer accepts the offer, and his share is $\pi_b^{S-1} = \theta - w_b^{S-1}$. When the bargaining horizon is arbitrarily large, the employer’s and the worker’s share is:

$$\lim_{T \to \infty} \pi_b^{S-1} = \theta \frac{\delta_b}{1 + \delta_b}$$

and

$$\lim_{T \to \infty} w_b^{S-1} = \theta \frac{1}{1 + \delta_b}$$

. Define: $w_{\infty}(S - 1) \equiv \lim_{T \to \infty} w_b^{S-1}$, and $\pi_{\infty}^{S-1} \equiv \lim_{T \to \infty} \pi$.

Taking the limit of the time between offers to 0 will yield:

\footnote{According to the equilibrium strategies, if $w_b = \psi$ then $x_e^{S-1}(w_b) = \theta - \psi + \theta - w_b^S$. If $w_b \neq \psi$ than the worker’s outside option in the next period is: $\text{Max}\{2w_b^{S-1}, \psi, w_{S-1}^M\}$. On-the equilibrium-path the worker will earn in the next period: $\theta$ and the employer’s profit will be zero.}
\[ \lim_{t_0 \to 1} \Pi_\infty (S - 1) = \frac{\theta}{2} \text{ and } \lim_{t_0 \to 1} W_\infty (S - 1) = \theta \frac{\theta}{2}. \]

The “compensation” that the worker needs to pay the employer if the worker is paid his MRP and reveal to outside firms his productivity, goes to zero when the bargaining horizon is arbitrarily large.

**Claim 1**: in any production period \( S - k \), the bargaining outcome is similar as in period \( S - 1 \) (the conditions to when the outside option binds are different).

**Proof**: the value of an outcome \( u_b^{S-k} \), to the employer in future periods is: \( x_e^{S-k} \). Solving backwards, yield the same solution form as in 7, with a different (finite) value of the second term. However, when the bargaining horizon is arbitrarily large the value of the payoffs is arbitrarily small. Q.E.D.

If the worker makes the last offer the bargaining outcome when the horizon is infinite, is arbitrarily large\(^{20}\).

The fact that different bargaining outcomes yield different payoffs in the continuation game doesn’t affect the bargaining outcome. This is because the process is potentially infinite. Therefore, the present value of any payoff, after the production period is arbitrarily small. Therefore, the bargaining outcome is similar to the Rubinstein bargaining.

The bargaining outcome itself will be different than splitting the surplus to half. However, since the bargaining procedure is a common knowledge, firms can infer the productivity if it different from the outside option. This will change the wage in the first period in a high level job, but afterwards ability is revealed, and the wage will be equal to the worker’s MRP.

*When does the bargaining outside option binds?*-conditions for “non-revealing” equilibrium.

I solve next for the conditions under which the outside option binds (so the wage depends on the outside offer rather than the worker’s MRP in the high type job), in each production period.

\(^{20}\)The difference is that the employers future value is zero, because in the final period the worker offers a wage of \( \theta \), since the employer’s outside option is zero.
Result 1: The types who receive a wage that depends directly on their productivity \( (\frac{\theta}{2}) \) satisfy:

\[
\frac{\theta}{w_{s-1}^M} \geq \frac{\frac{1-\delta^{s-1}}{1-\delta}}{\frac{1-\delta^{s-1}}{1-\delta} - 0.5} \tag{8}
\]

Otherwise they receive a “non-revealing” wage, which is the highest outside option: \( w_{s-1}^M \).

Define

\[
\theta^* \equiv \frac{\frac{1-\delta^{s}}{1-\delta}}{\frac{1-\delta^{s}}{1-\delta} - 0.5} w_{s-1}^M \tag{9}
\]

This is the cutoff level of ability in the complex job when the outside option is \( w_{s-1}^M \). Above this level, the wage is a “revealing” wage. Below this level, it is a “non-revealing wage”.

Claim 2: in equilibrium, if a worker receives a “revealing” wage it will occur in the first period in which he works in the high level job.

Proof: immediate. The derivative of \( \theta^* \) with respect to \( s \) is positive. This is because at any given period the cost of “revealing” ability is the same while the benefit is larger, the more time the worker will work before he retires.

Proof of Result 1: Consider bargaining in the final “production” period: The worker can either receive a “separating wage”, \( \frac{\theta}{2} \), or his outside option: \( w_{s-1}^M \). Therefore, the condition in period \( S \) is:

\[
\frac{\theta}{w_{s-1}^M} \geq 2.
\]

Similarly, in any period period \( S - k \), the worker can receive a “separating wage” of \( \frac{\theta}{2} \), and in the proceeding period he will receive \( \theta \). The alternative is to receive his outside option in the current and the following periods (as proved in the claim, we only need to consider the possibility of “revealing” the wage immediately and the possibility of not “revealing” the ability at all). Therefore, depending whether the present value of earnings is larger determines whether the wage is the outside option or not. The condition that the present value of earning is larger if the worker receives a “non-revealing” wage is:

\[
\frac{\theta}{w_{s-1}^M} \geq \frac{1+\delta+\ldots+\delta^k}{0.5+\delta+\ldots+\delta^k} = \frac{\frac{1-\delta^{k+1}}{1-\delta}}{\frac{1-\delta^{k+1}}{1-\delta} - 0.5}.
\]

Q.E.D.

Next, I discuss the how private information about the worker’s productivity is revealed to firms in the market, and discuss the different sources of wage growth in the model.
Proposition 1—Characterization of the equilibrium outcome:

A) All the workers that have a comparative advantage in the high level job, \( \theta \geq \psi \) are assigned to the high level job in the second period.

B) All the workers stay with the first period employer.

"Revealing" equilibrium outcome: the case in which \( \theta \geq \theta^* \):

C) The outside option at the beginning of the second period is \( \psi \). For any \( s > 2 \), the outside option is: \( \theta \).

D) The second period wage is \( w_2 = \frac{\theta}{2} \), and the wages for \( s > 2 \) are: \( w_s = \theta \).

"Non-revealing" equilibrium outcome: the case in which \( \theta < \theta^* \):

E) All the workers stay with the first period employer.

F) The outside option at the beginning of the each period is \( \psi \).

G) The wage is \( w_s = \psi \) for all \( s \geq 2 \).

Proof: follows immediately from the equilibrium strategies. A proof for the equilibrium strategies is in appendix A.

In the model, information is revealed directly only to the employer and to the worker. Outside firms learn about a worker’s productivity only through wages\(^{21}\). Some workers who are promoted experience wage growth in both periods. There are two sources of wage growth. The first is due to the “private” learning. After the worker and the employer observe the worker’s productivity, the employer learns the worker’s comparative advantaged and assigns the worker to the job in which his MRP is larger. Then, as an outcome of the bargaining, the worker receives a portion of the surplus he produces. In the following period, potential employers observe the wage and the worker’s job assignment. Since outside firms observe \( \psi \), and know that the bargaining outcome is that the employer and the worker split the surplus equally, they infer that the worker’s MRP is twice the wage he received. Then, in the following period the worker is paid his MRP. This is the second source of wage growth.

\(^{21}\) Since workers never leave the employing firm, no other firm observes directly the worker’s productivity.
wage growth.

2.4 First Period Equilibrium Wages:

At the beginning of the first period the information is symmetric. Since I assume free entry, an equilibrium wage should clear the market, and a firm that hires a worker expects zero profit over time. Imposing the zero profit condition:

$$E\Pi(\psi) = \psi - w_1(\psi) + \delta[Pr(\text{"revealing wage"}) \frac{\hat{\theta}_2^r}{2} + Pr(\text{"non-revealing wage"}) (\hat{\theta}_2^n - \psi) \frac{1 - \delta^S}{1 - \delta}] = 0$$

$Pr(\text{"revealing wage"})$ | $Pr(\text{"non-revealing wage"})$, is the probability of receiving a “revealing wage”/“non-revealing wage”, conditional on being promoted. $\hat{\theta}_2^r$ / $\hat{\theta}_2^n$ is the expected productivity in the complex job conditional on receiving a revealing/nonrevealing wage, respectively. From this equation one can solve for the first period wage.

Next, I analyze the entry level wages. I assume that $\theta$ and $\psi$ are independent. The first period wage has two components. One is the MRP in low level jobs, the other is the expected payoff from future periods.

Solving for $w_1(\psi)$ from the equation above:

$$W_1(\psi) = \psi + \delta(1 - F(\psi))[(1 - F(\theta_2^s(\psi))) \frac{\hat{\theta}_2^r}{2} (\psi) + F(\theta_2^s(\psi))(\hat{\theta}_2^n (\psi) - \psi) \frac{1 - \delta^S}{1 - \delta}]$$

$F$ is the cdf of $\theta$. $\theta$ is independent of $\psi$ by assumption (made for simplicity). $\hat{\theta}_2^s (\psi)$, is the expected productivity in the complex job conditional on receiving a”non-revealing” wage. $\hat{\theta}_2^r (\psi)$, is the expected productivity in the complex job conditional on receiving a”revealing” wage. Define $k_0 = \frac{1 - \delta^S}{1 - \delta}$ and $\hat{\theta}_s = k_1 \psi$, where $k_1$ is the constant in equation 9. Writing explicitly the expression for the entry level wage:

$$\frac{w_1(\psi) - \psi}{\delta} = k_0 \int_{\psi}^{k_1 \psi} f(\theta) d\theta (\frac{f_{k_1 \psi} \theta f(\theta) d\theta}{f_{\psi} \theta f(\theta) d\theta} - \psi) + \frac{1}{2} \int_{k_1 \psi} f(\theta) d\theta$$

The right hand side in 10 is the expected future profit of the employer in the proceeding period, if he hires a worker with productivity in low level jobs, $\psi$. The expected profit is different for workers with different $\psi$ although $\theta$ is independent of $\psi$. 

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Proposition 2.

1) If $\theta$ and $\psi$ are independent random variables (with continuous densities) then first period wage is increasing in $\psi$, in a decreasing rate. This is because the employer’s expected profit from future periods is monotonically decreasing in $\psi$.

2) When $\psi = \bar{\theta}$, where $\bar{\theta}$ is the highest $\theta$ in the distribution, assuming that it is finite, the first period wage is $\psi$, and the expected future payoff of the employer is zero.\textsuperscript{22}

Proof: In order to prove the claim, I will show that the derivative of the the expected profit, the RHS in equation 10 of negative. Surprisingly enough, the derivative is simple. The derivative is not negative for any constants $k_0, k_1$. However, given the relationship between these parameters in the model, the derivative is always negative. Taking a derivative of the RHS of equation 10 gives the following expression:

$$-k_0 \int_{\psi}^{k_1} f(\theta) d\theta + f(k_1 \psi) \psi \left[-k_0 k_1 + k_0 k_1^2 - \frac{1}{2} k_1^2 \right] < 0$$

The expression: $[-k_0 k_1 + k_0 k_1^2 - \frac{1}{2} k_1^2] = 0$.

Part 2) is trivial when $\psi = \bar{\theta}$. In this case, the probability that the worker will be promoted is zero. Therefore, the employer will not have any informational rent. When $\bar{\theta} = \infty$ then only when the distribution has a finite expectation the expected profit is zero when $\psi = \infty$. Q.E.D

\textsuperscript{22}If it is infinite, then if the distribution has finite first moment then when $\psi \to \infty$, the wage approaches $\psi$. 

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Figure 1: The first period wage as a function of publicly observable characteristics.

Example (figure 1):

Figure one displays the entry level wage as a function of $\psi$. The productivity in the complex job ($\theta$) is independent of the observed productivity in the simple job, $\psi$, and $\theta$, distribute uniformly on [0.5, 1.5]. $S = 10$ and $\delta = 0.7$. As shown above, the wage is concave in general.

The analysis of the entry level wage shows that workers who differ by their productivity in low level, but have the same potential of the productivity in high level jobs, earn different wages, not only due to differences in the productivity in the low level jobs. The reason to the differences is that the employer expects a different profit in the future. The productivity in low level jobs affects the probability of promotion (which declines in $\psi$), and the probability of “revealing” and “non-revealing” equilibrium outcome. Since, the composition of workers in high level jobs that will have a “revealing” and “non-revealing” wages is different for different $\psi$, the expected profit of the employer in each scenario is different.

2.5 Next, I discuss the different sources of wage reductions.

Proposition 3:

A) All the workers that are not promoted experience a wage reduction after the first period: $w_2 - w_1 = \text{or differ in any publicly observable characteristics which is relevant to productivity in low level jobs}$
\( \psi - w_1 \leq 0 \). The decline is smaller for workers with larger \( \psi \). For any \( s > 2 \) the change in the wage is zero.

B) Worker that are assigned to the high level job in the second period, for which \( \theta^* > \theta \), experience either a wage decline or a wage increase in the second period, depending on the sign of \( w_1(\psi) - \frac{\theta}{2} \). All these workers experience a wage increase in the third period of: \( \frac{\theta}{2} \).

C) Workers that are assigned to the high level job in second period, for which \( \theta^* \leq \theta \), experience wage decline in the second period: \( w_2 - w_1 = \psi - w_1 \leq 0 \), and a wage change of zero in the proceeding periods.

Proof: these results follow immediately from proposition 1.

The model shows that workers' who are not promoted experience a wage reduction in the second period. BGH find that workers who are not promoted, experience, on average, real wage decline. Further, about 15 percent of the workers in a typical cohort experience real wage decline ten years out. In this model, wage can decrease for two reasons. One, is because after the employer realizes that a worker has a comparative advantage in low level jobs, he expects no future profits. If the worker has a comparative advantage in low level jobs, the the private information that the employer has, has no value. I comparison, in symmetric learning models, real wage declines result from uncertainty about a worker's productivity. Workers in these models are paid the expected MRP. Therefore, if the productivity realization is smaller than the expected productivity, firms update their expectations regarding the worker's productivity, and that can cause a wage decline. The result in our model is typical to asymmetric learning models, because in future periods employers earn informational rents. Hence, when a worker is hired, the entry level wage partially reflects the expected profit of the employer.

The second source of wage decline is that it is costly for some worker’s to publicly reveal their ability. This is because the bargaining outcome can be smaller than the worker’s outside option. However, accepting a wage that is equal the outside option, will not reveal the worker’s ability since outside firms’ offers do not depend directly on the productivity of the worker.
3. A Model with two complex Jobs (and one simple job):

I extend the basic model by adding another higher level, complex job: \( j = 3 \), to the job ladder. I assume that the ability in the third job is revealed to both the employer and the worker while working in the second job. The worker can renegotiate his wage any time before production takes place, in the same manner described in the one complex job model. The aim of this section is first, to compare age-earnings profiles of workers with different abilities. Secondly, examine wage variation within job levels, especially wages of workers with similar productivities. Lastly, it analyzes wage variation across and within jobs, and explain empirical findings in BGH.

**Result 2:** Characterization of a “revealing wages” in the third job, \( j = 3 \):

1) A worker that was assigned to job 3 earns:

\[
\begin{align*}
    w^3_3 &= \begin{cases} 
        \frac{\theta_3}{\mu}, & \text{if } \frac{\theta_3}{\mu} \geq A(\delta, s - 2) \\
        \mu, & \text{otherwise}
    \end{cases} \\
    \mu &= \begin{cases} 
        \theta_2, & \text{if } \theta_2 \in I \\
        \psi, & \text{otherwise}
    \end{cases} \\
    A(\delta, s - 2) &= \frac{1 - \delta^{s-2}}{1 - \delta} \frac{1 - \delta^{s-2}}{1 - \delta - 0.5}
\end{align*}
\]

"Proof": the proof is identical to the proof in the one complex job model. The difference is that it is only possible to be in the third job, \( j = 3 \), after two periods in the firm. As in the previous model, if a worker reveals his productivity in the third job, it happens in the third period (first period in the job). This is because the cost of “revealing” ability is constant, while the benefit increases in the number of periods left. Note, that the outside option depends on whether the second period wage revealed the productivity in the second job or not.

In order to simplify the exposition, I assume that both the employer and the worker learn the abilities in the complex jobs, \( \theta_3 \) and \( \theta_2 \), before they negotiate the second period wage.

I denote by \( \theta_3^*, \theta_3^{**} \), the cutoff level of productivity in the third job, \( j = 3 \), if the outside option is \( \psi, \theta_2 \), respectively. Workers who are promoted to the third level job can have different wage. Workers
for which the realization of productivity in the higher level job, \((j = 3,\) is such that, \(\theta^*_3 \geq \theta_3,\) earn their outside option. Workers for which \(\theta^*_3 \leq \theta_3 \leq \theta^{**}_3,\) will earn a wage equals to the outside option if the outside option is \(\theta_2,\) but will earn \(\theta_2 + \frac{\theta_3}{2},\) if the outside option is \(\psi.\) Workers for which \(\theta^{**}_3 \leq \theta_3,\) will earn \(\frac{\theta_2}{2}.\)

\[
\theta^*_3 = \psi \times A(\delta, s - 2) \tag{12}
\]

\[
\theta^{**}_3 = \theta_2 \times A(\delta, s - 2) \tag{13}
\]

Next, I characterize which workers (among workers who have a comparative advantage in the third level job), will receive a wage that depends on their productivity in the second level job, while working in the second level job\(^{24} \).

**Result 3:** Among the workers with a comparative advantage in the third level job, the following workers will receive a wage that depends on their ability in the second level job while working in the second level job. A necessary condition is that the outside option \(\psi\) doesn’t bind, \(\theta_2 \geq \psi A(\delta, S - 1)\).

(a) If \(\frac{\theta_2}{2} - \psi \geq 0,\) then regardless of the realization of \(\theta_3,\) the wage in the second level job depends directly on \(\theta_2.\)

Otherwise, if \(\frac{\theta_2}{2} - \psi < 0\) (in this case, it is costly to “reveal” \(\theta_2)\) then:
(b) if \(\theta_3 < \theta^*_3.\) The worker will reveal \(\theta_2\) but will not reveal \(\theta_3.\)
(c) if \(\theta^{**}_3 > \theta_3 > \theta^*_3\) and \(\theta_2 \left(\frac{1-\delta^{s-2}}{1-\delta} - 0.5\right) - \delta \theta_3 \left(\frac{1-\delta^{s-1}}{1-\delta} - 0.5\right) \geq \psi\)

In this case, \(\theta^{**}_3 > \theta_3 > \theta^*_3,\) if the worker earns \(\psi\) in the second level job, he will receive a wage that depends directly on \(\theta_3\) in the third level job, but if he receives a wage that depends directly on \(\theta_2\) in the third level job, he will receive \(\theta_2 \times A(\delta, s - 2)\). Under the conditions in c\) the worker reveals \(\theta_2\) and does not reveal \(\theta_3.\)

In all the other cases, the second period wage is a “non-revealing” wage, \(\psi.\)

**Proof:** a) In that case, there is no cost to revealing the productivity in the second level job. Therefore, the outside option does not bind, and the worker receives a “revealing” wage.

\(^{24}\)recall that I assume that workers learn their abilities in both the third and second level jobs at the beginning of the second period, before they bargain on the wage. However, all workers need to work for a period in the second level job before they are assigned to the third level job.
b) The worker will not reveal his productivity in the third job. Therefore, it is similar to the one complex job case (it needs to satisfy, $\frac{\theta_s}{\psi} \geq A(\delta, s - 1)$).

c) In this case, if the worker does not reveal his productivity in the second job, he will reveal his ability in the third job, in the next period. If he reveals his ability in the second job (which is costly), he will not reveal his ability in the third job (this is optimal). Hence, the condition for revealing his second job productivity is: $\frac{\theta_s}{\psi} + \delta \theta_2 + \ldots + \delta^{s-2} \theta_2 > \psi + \delta \frac{\theta_s}{\psi} + \delta^2 \theta_3 + \ldots + \delta^{s-2} \theta_3$. Rearranging the above equation gives the condition in c). Q.E.D

**Proposition 4:**

Define: $\Delta_s \equiv w_s - w_{s-1}$. The following are the age-earnings profiles in the economy for the different types of workers that are assigned to the third level job25:

Case 1) If $\frac{\theta_s}{\psi} \geq \psi$:

a) if $\theta_3 < \theta_3^{**}$ then:

$$w_1 = \psi, \ w_2 = \frac{\theta_1}{\theta_2}, \ w_3 = \ldots = w_s = \theta_2.$$  

$$\Delta_3 = \frac{\theta_1}{\theta_2}, \ \Delta_4 = 0.$$  

$$\Delta_3 = \frac{\theta_1}{\theta_2}, \ \Delta_4 = 0.$$  

b) if $\theta_3 > \theta_3^{**}$ then:

$$w_1 = \psi, \ w_2 = \frac{\theta_1}{\theta_2}, \ w_3 = \frac{\theta_3}{\theta_2}, \ w_4 = \ldots = w_s = \theta_3.$$  

$$\Delta_3 = \frac{\theta_1}{\theta_2} - \frac{\theta_3}{\theta_2}, \ \Delta_4 = \frac{\theta_1}{\theta_2}.$$  

**CASE 2)** If $\frac{\theta_s}{\psi} < \psi$:

a) if $\theta_3 > \theta_3^{**}$ then,

$$w_1 = \psi, \ w_2 = \psi, \ w_3 = \frac{\theta_1}{\theta_2}, \ w_4 = \ldots = w_s = \theta_3.$$  

The wage growth in periods 3 and 4: $\Delta_3 = \frac{\theta_1}{\theta_2} - \psi$, $\Delta_4 = \frac{\theta_1}{\theta_2}$, respectively.

b) if $\theta_3 < \theta_3^{**}$ then,

---

25 For those who do not have a comparative advantage in the third level job the age-earnings profile is similar the one in the two-jobs model.
w_1 = \psi, \ w_2 = \frac{\theta_2}{2}, \ w_3 = \ldots = w_S = \theta_2.

\Delta_3 = \frac{\theta_2}{2}, \ \Delta_1 = 0.

c) if \ \theta_3^{**} > \theta_3 > \theta_3^{*} \text{ then,}

w_1 = \psi, \ w_2 = \psi, \ w_3 = \frac{\theta_3}{2}, \ w_4 = \ldots = w_S = \theta_3.

The wage growth in periods 3 and 4: \ \Delta_3 = \frac{\theta_2}{2} - \psi, \ \Delta_4 = \frac{\theta_2}{2}, \ \text{respectively.}

\text{c1) If } \psi - \frac{\theta_2}{2} \geq \theta_2(\frac{1-\delta^{s-1}}{1-\delta} - 0.5) - \delta \theta_3(\frac{1-\delta^{s-2}}{1-\delta} - 0.5) \text{ then}

\Delta_3 = \frac{\theta_2}{2} - \psi

\Delta_4 = \frac{\theta_2}{2}.

\text{c2) If } \psi - \frac{\theta_2}{2} < \theta_2(\frac{1-\delta^{s-1}}{1-\delta} - 0.5) - \delta \theta_3(\frac{1-\delta^{s-2}}{1-\delta} - 0.5) \text{ then}

w_1 = \psi, \ w_2 = \frac{\theta_2}{2}, \ w_3 = \ldots = w_S = \theta_2. \ \Delta_3 = \frac{\theta_2}{2}, \ \Delta_4 = 0.

“\text{Proof}”: \ This \ is \ just \ a \ combination \ of \ results \ 5 \ and \ 6.

BGH find that promotion premiums explains only a modest part of the differences in average wage, across job levels. In this model, workers that are promoted to the third job \( j = 3 \), and workers who are promoted to the second job, but are not promoted to the third job, always experience a larger wage increase in the period after the promotion. However, workers who are promoted to the third job, but do not reveal their ability in the third job, experience a larger increase upon promotion. The range of “types” that experience larger increase when promoted is bounded. Although, I cannot determine whether the average increase in wage that is associated with promotion is modest relative to the average wage difference across jobs, without making further assumptions on the distributions, this model seems consistent with the evidence.

BGH finds a strong “green card effect”, workers with the same performance rating receive a larger percent of wage increase than workers who earn relatively larger wages. In this model, workers with similar abilities earn either: \( \frac{\theta_2}{2}, \ \theta_3, \ \psi \) or \( \theta_2 \). Those who earn the highest wage are the workers with longer tenure in the job that “revealed” their ability, \( \theta_3 \). Those who earn either \( \psi \) or \( \theta_2 \), are in the middle. They earn more than \( \frac{\theta_2}{2} \) but less than \( \theta_3 \). Finally, new promotees who reveal their abilities
earn the lowest wage: $\frac{\theta_2}{2}$. However, those who earn the lowest wage, also experience the largest wage increase\textsuperscript{26}. The model also supports BGH’s finding that workers that earn a relative high wage before they are promoted (relative to the average wage in the job prior to promotion), earn a relatively low wage (comparing to the average wage in the new job) shortly after they are promoted. This model is consistent with this finding as well.

3.1 \textit{Comparative statics:}

In this section I analyze changes in wages as a function of ability in the third level job, provide examples, and analyze how wage growth vary when the productivity in the third job is similar, but other characteristics are different.

First, I fix $\psi$, and $\theta_2$, and analyze how wages in the second, third, and forth period vary with the productivity in the third job, $\theta_3$. (across and within job levels job). Let $i$ and $j$ be two workers that have different realizations of productivity in the third job: $\theta_3^i > \theta_3^j$.

CASE 1: if $\frac{\theta_3^i}{2} - \psi \geq 0$ then:

a) If $\theta_3^i > \theta_3^j > \theta_2 A(\delta, s - 2)$, both the wage increase upon promotion ($\frac{\theta_3^i}{2} - \frac{\theta_3^j}{2}$), and the wage increase a period after promotion, are larger for the worker with the higher ability: $\Delta_3^i > \Delta_3^j$, and $\Delta_4^i > \Delta_4^j$.

b) If $\theta_3^i > \theta_3^* > \theta_3^j$, then the wage increase in period 3 might be smaller or larger for the high productivity worker. This is because only the high productivity worker reveals his ability in job 3, and pays the cost of publicly revealing his ability.

\[
\Delta_3^i - \Delta_3^j = \frac{\theta_3^i}{2} - \theta_2.
\]

\[
\Delta_4^i - \Delta_4^j = \frac{\theta_3^i}{2}.
\]

CASE 2: if $\frac{\theta_3^i}{2} - \psi \leq 0$, only workers that will not reveal their ability in the third job, reveal their ability in the second job. If $\theta_3^i > \theta_3^* > \theta_3^j$ then:

\[
\Delta_3^i - \Delta_3^j = \frac{\theta_3^i}{2} - \theta_2 - \psi < 0.
\]

\[
\Delta_4^i - \Delta_4^j = \frac{\theta_3^i}{2}.
\]

In this case the high productivity worker experiences a lower wage increase in period three (the third

\textsuperscript{26}BGH do not provide the information on tenure in the job when the analyze the “green card” effect.
period wage can be smaller or larger for the high type worker, but a larger wage increase in the forth period.

Example:
Let $S = 10$, $\delta = 0.7$, $\theta_2 = 0.9$.

CASE 1: $\frac{\theta_3}{2} \geq \psi$. (See figures 2 and 3). All the workers that are promoted to the second job receive a "revealing" wage. Only workers for which $\theta_3 \geq \theta_{3**}$ receive a wage which depends directly on $\theta_3$. Figure 2, shows that all the workers with a productivity in the third level job below $\theta_{3*}$, receive a wage increase in the third period, and no wage increase in the forth period. Figure 3 describes the same for CASE 2. In this case some workers that are promoted to the third job, and have lower productivity in the third job (do not receive a "revealing" wage in the third job), will earn a larger wage, and experience larger wage increase than more productive workers, although they are similar with respect to their other characteristics. However, the wage level and growth are larger in the forth period for the more productive workers. Therefore, in the third period, we observe workers with lower performance ratings and higher wages. This is inconsistent with symmetric learning models’ predictions. If all the firms have the same information about a worker’s ability, and the worker is equally productive in all firms, then the wage should be similar to workers with the same productivity, in these models.
Figure 2: Wage changes in periods 3 and 4 ($\Delta_3$, $\Delta_4$) as a function of ability in the third job ($\theta_3$). Case 1: $\frac{\theta_3}{2} \geq \psi$.

Figure 3: Wages in periods 3 and 4 as a function of ability in the third job ($\theta_3$). Case 1: $\frac{\theta_3}{2} \geq \psi$. 
Figure 4: Wage changes in periods 3 and 4 as a function of ability in the third job. Case 2: $\frac{\theta_3}{2} < \psi$.

Next, I analyze the differences in wage growth when the productivity in the third job is similar, but the workers have different observable characteristics (I compare case 1, in which $\frac{\theta_3}{2} - \psi \geq 0$ with case 2, in which $\frac{\theta_3}{2} - \psi < 0$): Figure 6 shows the difference in wage growth at the third period. Workers with lower $\psi$ (observable characteristics), will receive a “revealing” wage for lower values of $\theta_3$, compared to workers with higher $\psi$, and identical $\theta_3$. All the workers with low $\psi$, experience smaller wage change with promotion.

Figure 7 shows that some workers with low $\psi$ revealed their ability in the third job, while workers with the same productivity in the third job and higher $\psi$ did not; the workers that revealed their ability in the third job experience a larger wage increase. Note, that the life-time earnings of the workers with higher $\psi$ is (weakly) larger than the life-time earnings of workers with higher $\psi$ (and identical abilities in the complex jobs).
Figure 5: Wages in periods 3 and 4 as a function of ability in the third job. Case 2: \( \frac{\theta_3}{2} < \psi \).

Figure 6: Wage growth in the third period as a function of ability in the third job: comparison of workers with low and high outside option.
The analysis shows that the “hybrid” model generates a large variation in wages and wage growth within and across jobs. One source of variation is the heterogeneity of workers’ abilities. This is also the source of variation in wages in the symmetric learning model. In asymmetric learning model wage do not depend on ability directly. In the “hybrid” model, workers which are equally productive in a job can have a different wages. This is due to differences in productivities in other jobs and age which determine the workers’ outside options. This is typical to asymmetric learning models, in which wages depend only on workers’ outside option, which is only a function of publicly observable characteristics. Symmetric learning models do not capture variation in wages of workers with the same productivity in a job. This is consistent with evidence that performance ratings are not always strongly correlated with earnings.

Conclusions:

This paper shows how abilities of workers can be publicly revealed, when the abilities are a private information of the employer. What enables the revelation of private information through wages is that workers and employer bargain over wages. What drives the bargaining outcome is that both the employer and the worker are impatient.

In the model, wages grow partially because workers learn their comparative advantage over time,
and are sorted to jobs, in which they have comparative advantage. The second source of wage growth in the model is that over time, workers’ abilities are revealed to firms in the market, which use wages to make the inference.

The models has features of both private and public learning models. Therefore, it is consistent with evidence that cannot be explain using the existing public learning models or private learning models.
Appendix A: Proof that the equilibrium strategies constitute P.B.E:

In order to prove that the equilibrium strategies above and beliefs constitute P.B.E. I show that no player can deviate profitably from his equilibrium strategies given his beliefs at any informational node, and that the beliefs satisfy bayes’ law on-the-equilibrium-path.

Worker: clearly, choosing the highest offer, \(\max[u_b^h, w_s^M, u_e^h]\), (where \(u_b^h\) is only available if the worker is assigned to a high level job) maximizes the worker’s payoffs.

Employer: counteroffer strategy: the strategy in equation 2 maximizes the employer’s profit. If outside firms offer a wage that exceeds the worker’s MRP, it is optimal to offer a lower wage since the employer has no incentive to retain the worker in a wage that exceeds his MRP. If the offer is lower than the worker’s MRP, then the employer’s payoff is \(\theta - \max[u_b^h, w_s^M] > 0\). This is the lowest wage that the employer needs to pay in order to retain the worker. Clearly the counteroffer strategy maximizes the employer’s profit.

Job assignment strategy: The strategy in equation 3 maximizes the employer’s profit. Assigning a worker with a comparative advantage in low type job to a high type job is clearly not a profitable deviation.

Assigning a worker with a comparative advantage in high level job to a low level job is not a profitable deviation either. The profit if a worker with \(\theta > \psi\) is promoted is \(\frac{\theta}{2}\) in a “revealing” equilibrium, and \((\theta - \psi)(\sum_{s=1}^{S-1}\delta)\) in a “non-revealing” equilibrium. Assigning the worker to a low type job instead, yields zero profit at first. Since we only check for a single deviation, if \(s < S - 1\) then this worker will be promoted in the proceeding period. The payoff from deviation is \(\delta\frac{\theta}{2}\) in “revealing” equilibrium. Hence the profit from deviation minus the profit if plays the equilibrium strategy is: \(\frac{\theta}{2} \times \delta - 1 < 0\) in a “revealing equilibrium”. The payoff if deviating in a “non-revealing” equilibrium is: \(-\delta(\theta - \psi) < 0\) (because after the single deviation the employer continues according to his equilibrium strategy). Clearly, not assigning workers to jobs in which they have a comparative advantage in, is not a profitable deviation.

Outside firm’s strategy: I first show that if the firm observe only low level history of job assign-
ments and wages, then the strategy in 4 is optimal.

The equilibrium offer in equation 4, yields zero expected payoff. Since the market is competitive any wage lower than $\psi$ will yield zero profit.

Claim: any offer larger than $\psi$ is not a profitable deviation. To see that, consider the following cases:

1. If $W_s^M > \theta$, the employer, who knows the worker’s ability, will not match the offer and the firm will raid the worker. In this case, the expected payoff is negative.

2. If $W_s^M \leq \theta$ the employer will match the offer, and the worker will stay with the employer. The continuation payoff is the same as if the firm offers $\psi$. Therefore, any wage offer that is larger than the wage offer in 4 is not a profitable deviation. Notice, that this is optimal even though the beliefs are that the the worker’s productivity is larger than $\psi$.

The beliefs: $\hat{\theta}_2 = E[\theta|\psi]$, $\hat{\theta}_s = E[\theta|\theta < \psi]$, $s > 2$ satisfy bayes’ law. In the second period this is the expected productivity of a random worker at the end of period one. In any period after the second period, only workers that have comparative advantage in low level jobs were not promoted at the beginning of period two, to a high level job. Hence, on-the-equilibrium path, the expected MRP in high level jobs, $\theta$, of workers in low level jobs is indeed $\hat{\theta}_s = E[\theta|\theta < \psi]$.

Next, I prove that the outside firm, cannot deviate profitably from the equilibrium strategy, when it observe that the worker was assigned to a high level job.

Case 1) the wage history in the high level job contains a wage such that:

$2u^k_{t-1} > \psi$ and $u^k_{t-1} \neq \psi$ and $u^h_{t-1} > w^M_s$. Since firm believe that $\hat{\theta} = 2u^h_{t-1}$ there is no profitable deviation from the strategy in 5.

The beliefs satisfy bayes’ law. On-the-equilibrium-path. Those promoted have $\theta \geq \psi$ (since employers assigns workers according to their comparative advantage), and $\theta \geq \theta^n$, since workers with ability $\theta \geq \theta^n$ they receive bargaining wage of $\frac{\theta}{2}$. Hence, the beliefs are consistent with the players’ strategies and satisfy bayes’ law.

Case 2) the condition of case 1 isn’t satisfied, and the highest wage of a worker in a high level job satisfies is $\psi$. In this case offering $\psi$ is optimal. As in the case in which firms do not observe any
wage in the high level job, the firm cannot infer from the wage the true productivity of the worker. In this case, a firm can raid the worker only with a wage that exceeds the worker’s MRP. Hence, the expected payoff from any offer above \( \psi \), which is the lowest possible productivity of a worker who was promoted is negative.

Clearly, the beliefs satisfy bayes’ law. On-the-equilibrium-path only workers with \( \psi \leq \theta \leq \theta^* \) receive a wage \( \psi \), hence, the expected value of \( \theta \) is the one specified in 6.
References


