Saturday May 9, 1998, 2-4pm. **Economics 713: Final Exam** John Kennan

Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince an intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

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1. An economy contains many identical consumers, with utility functions $u(x) = \log(x_0) + 2\log\left(\sum_{i=1}^{N} \sqrt{x_i}\right)$. Each consumer is endowed with the same quantity of good 0, and the other goods are produced using identical technologies which require α units of x_0 to get started, and c units of x_0 for each unit of x_i that is produced. The number of possible goods, N, is big relative to the number of consumers.

a. A planner seeks to maximize social welfare, by regulating the product prices. Each product is produced by a monopolist, and the planner cannot drive profits below zero. Lump sum transfers are not feasible. What prices should the planner set?
b. What is the increase in social welfare using the optimal prices, relative to the alternative of unconstrained monopolistic competition?

2. An economics professor suddenly finds that he has bought one more house than he needs. He wishes to auction the extra house so as to maximize expected profit. There are three potential buyers, with independent valuations of the house which are distributed between 0 and $300,000, according to the following law: the probability F(v) of having a valuation no higher than v is given by $F(v) = \sqrt{\dfrac{v}{300{,}000}}$.
The professor has no other use for the house.

For reasons best known to himself, the professor has decided to conduct a sealed-bid auction, with the understanding that the house will be awarded to the highest bidder, in exchange for a payment equal to the *lowest* bid.

a. How much will the buyers bid for the house?
b. What advice would you give this professor, in case he finds himself in this position again?

3. Suppose there are two consumption goods with production functions $Q_1 = \sqrt{K_1 L_1}$, $Q_2 = \dfrac{1}{\frac{1}{K_2} + \frac{1}{L_2}}$ where K_1 is the amount of capital used in the production of good 1, etc. The economy is competitive.

a. Find the relationship between the factor price ratio and the efficient capital-labor ratio in each industry.
b. Suppose now there are two small countries, A and B, each having the technologies described above. A is endowed with 4 units of K and 24 units of L, and B is endowed with 12 units of K and 2 units of L. There is free trade in the consumer goods, but the factors of production cannot move between countries. The prices of the consumer goods, established in the world market, are $p_1 = 3$ and $p_2 = 16$. Taking these prices as given, find the competitive equilibrium in each country.

c. In your equilibrium, are factor prices equal across these two countries? If so, explain why; if not, explain why not.

4. Consider an isolated economy which produces just one crop, corn, using a fixed quantity of land and variable quantities of labor and tractors. Labor is also used to produce the tractors.

a. Which of the following conditions are needed for Pareto optimality:
   i. The marginal rate of technical substitution of tractors for labor in the production of corn equals the marginal product of labor in the production of tractors.
   ii. The marginal utility of leisure equals the marginal product of labor in corn multiplied by the marginal utility of corn, for each individual.
   iii. The marginal rate of substitution of leisure for corn is equal for all consumers.

Explain your answers in detail.

b. Which of the above conditions would be satisfied in a competitive market economy? How?

5. Consider a one-period pure exchange economy with three agents (denoted 1,2 and 3) and three goods (denoted x,y and z). Agent i's consumption vector is (x_i,y_i,z_i). Agent 1,2 and 3's endowment vectors are $(x^e,0,0)$, $(0,y^e,0)$ and $(0,0,z^e)$ respectively (each agent is endowed with only one type of good). Their preferences can be represented by utility functions, as follows:

$$U_1(x_1,y_1,z_1) = x_1(y_1+z_1)$$
$$U_2(x_2,y_2,z_2) = x_2y_2$$
$$U_3(x_3,y_3,z_3) = x_3z_3.$$

a. Find a Walrasian equilibrium for this economy under the condition that good z cannot be traded.
b. Assume now that there exists a complete set of markets in all three goods. Find a Walrasian equilibrium for this case.
c. Can you Pareto-rank the equilibria which you found? If so, in which order? Explain your results.

6. Consider an economy in which there are equal numbers of two kinds of workers, a and b, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains $2,000, and if an unqualified worker is assigned to the good job the employer loses $14,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t^2. The probability that an unqualified worker will have a test score less than t is $1-(1-t)^2$.

There is a fixed wage premium of $42,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and $20,000. This distribution is the same for a-workers and b-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

a. Can you find an equilibrium in which there are more a-workers than b-workers in the good jobs?
b. Now suppose that employers are required to assign the same proportion or a-workers and b-workers to the good jobs. If there is no change in the workers' investment behavior, what standards will the employers use?