Saturday May 9, 1998, 2-4pm. **Economics 713: Final Exam** John Kennan

Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince an intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

1. A vineyard sells grapes to a single winery. The winery's profit is $\pi(a,x) = a^2 - 6a - 9 + 2ax + 6x - x^2$, where x is the quantity of grapes used, and a is either 1 or 4. The vineyard does not know a, but knows that the two possible values are equally likely. Both sides maximize expected profits.

a. The vineyard makes offers that the winery must accept or reject. If an offer is rejected, the vineyard can make another offer. And even if an offer is accepted, the vineyard can make another offer. What kind of offers will the vineyard make?

b. What will the final outcome be?

2. An economy contains many identical consumers, with utility functions $u(x) = \log(x_0) + \log\left(\sum_{i=1}^{N} \sqrt{x_i}\right)$. Each consumer is endowed with some quantity of good 0, and the other goods are produced using identical technologies which require α units of x_0 to get started, and c units of x_0 for each unit of x_i that is produced. Good i is produced by a single firm that maximizes profits. The number of possible goods, N, is big relative to the number of consumers. There is free entry in the production of all goods.

a. How many goods will be produced in equilibrium?

b. Is the equilibrium Pareto optimal?

3. Consider an economy in which farming is the only occupation. All arable land lies along a linear river, and is divided into 2000 farms, arranged symmetrically on each side of the river. Up-river the weather is not so warm, and the beaches are further away, so each farmer would prefer to be closer to the sea, *ceteris paribus*. There are 105 identical farmers, each of whom can work just one farm, and each farm yields y pounds of food. Each farmer's utility function is $u(c,x) = c^{\alpha} x^{-\alpha}$, where c is consumption of food, α is positive, and x is distance from the coast, in farms (x=1,2,...,1000). Each farmer owns one share in each farm.

An economist uses cross-section data to estimate a linear regression equation in which distance from the coast is the dependent variable, and the explanatory variables are the log of consumption and a constant term. Interpret the results.

4. Suppose there are two consumption goods with production functions $Q_1 = \sqrt{K_1 L_1}$, $Q_2 = \dfrac{1}{\frac{1}{K_2} + \frac{1}{L_2}}$ where K_1 is the amount of capital used in the production of good 1, etc. The economy is competitive.

a. Find the relationship between the factor price ratio and the efficient capital-labor ratio in each industry.

b. Find the relationship between the price ratio of the consumption goods and the factor price ratio.

c. Suppose now that there are two countries, A and B, each having the technologies described above. A is endowed with 2 units of K and 1 units of L, and B is endowed with 1 units of K and 2 units of L. Initially the economies are isolated from each other, and the price ratio p_1/p_2 is 1/4 in A and 1/3 in B. Then trade in the consumer goods is permitted, but **not** in the factors of production. Describe the pattern of trade, and the changes in the factor prices in the two countries.

5. Suppose there are I voters with preferences over a set X that are single-peaked with respect to a linear order $\geq$. Prove that there is some alternative in X that cannot be defeated by any other alternative by pairwise majority voting. Give an example.

6. Consider an economy in which there are equal numbers of two kinds of workers, a and b, and two kinds of jobs, good and bad. Some workers are qualified for the good job, and some are not. Employers believe that the proportion of a-workers who are qualified is 2/3 and the proportion of b-workers who are qualified is 1/3. If a qualified worker is assigned to the good job the employer gains $1000, and if an unqualified worker is assigned to the good job the employer loses $1000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t^2. The probability that an unqualified worker will have a test score less than t is $1-(1-t)^2$. Employers are subject to a rule that requires the proportion of a-workers assigned to the good job to be the same as the proportion of b-workers. Otherwise employers maximize expected profits.

a. Find the profit-maximizing policy for an employer.

b. Test your policy as follows. If you are told that a worker has just barely passed the test (and you are not told whether the worker is an a-type or a b-type), what is the probability that the worker is qualified? Is it the case that such a worker is a fair bet from the employer's point of view? If not, should the policy be changed?